Astigmatic Coherent Processor Analysis

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Abstract

Certain applications in coherent optical processing of two-dimensional signals require the operation of imaging a signal in one dimension while simultaneously Fourier transforming it in the other. Such parallel operations can be performed by a number of different system designs. This paper presents a method of analyzing such systems by treating each dimension independently and using three basic component sub-systems. The method simplifies mathematical analysis of system operation and facilitates intuitive design for particular applications.

A. Introduction

An astigmatic coherent processor as discussed in this paper refers to a coherent optical system which simultaneously performs Fourier transform and imaging operations in orthogonal directions. Such processors were first used by Cutrona et al.,[1,2] and have more recently found use in generalized one-dimensional space-[3,4] and frequency-[5] variant processing, as well as in Laplace transform display,[4,6] matrix multiplication,[7] ambiguity function display,[8,9] and data retrieval systems.[10]

There are numerous ways in which such astigmatic operations can be performed. Analysis of the various possible astigmatic processor designs can be accomplished by methods such as those outlined by Goodman,[11] Iwasaki,[12] or Vander Lugt.[13] These methods, however, often involve complex mathematical manipulations of the Huygens-Fresnel diffraction integral, a process which, besides being tedious and lengthy, is subject to algebraic errors and tends to impede intuitive design. It is thus of importance to formulate a simple, straightforward method of astigmatic coherent processor analysis.

This paper presents such a method. Utilizing the separability of the Huygens-Fresnel diffraction integral in cartesian coordinates, analysis of astigmatic processors can be performed one dimension at a time. One-dimensional system decompositions are obtained which invariably fall into one of three classes: A Fourier transformer, a one-lens imaging system, or a two-lens imaging system. Once the input to output equation for each of these one-dimensional component systems has been formulated using the Huygens-Fresnel integral, the input to output relation for any two-dimensional system is obtained simply by multiplying the appropriate one-dimensional equations. The resulting relationship, however, is also valid for non-separable type inputs.

For clarity of presentation, attention will be restricted to a first order analysis. Thus, effects of lens aberrations and diffraction anomalies are not considered. Familiar assumptions such as the thin lens and paraxial approximations are inherent in the development. Complex multiplicative constants will be included due to their importance in such areas as optical feedback[14] where phased addition of wavefronts is performed.

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B. Foundations

The Huygens-Fresnel diffraction integral permits expression of the scalar field amplitude, v(x,y), on the (x,y) plane due to the field amplitude $u(\xi,\eta)$ on the (ξ,η) plane. The (x,y) plane is located a propagation distance z past the (ξ,η) plane. Then

$$v(x,y) = \frac{\exp(jkz)}{j\lambda z} \iint_{-\infty}^{\infty} u(\xi,\eta)\exp\left\{\frac{jk}{2z}\left[(x-\xi)^2 + (y-\eta)^2\right]\right\} d\xi\, d\eta \ . \tag{1}$$

Here, $k = 2\pi/\lambda$ where λ is the wavelength of the spatially coherent illumination. Consider, for now, the case where $u(\xi,\eta)$ is separable in cartesian coordinates. That is,

$$u(\xi,\eta) = u_1(\xi)u_2(\eta) \ . \tag{2}$$

Then from Eq. (1), v(x,y) is also separable, so that

$$v(x,y) = v_1(x)v_2(y) \tag{3}$$

where

$$v_1(x) = \frac{\exp(\frac{jkz}{2})}{\sqrt{j\lambda z}} \int_{-\infty}^{\infty} u_1(\xi)\exp\left[\frac{jk}{2z}(x-\xi)^2\right] d\xi$$

$$v_2(y) = \frac{\exp(\frac{jkz}{2})}{\sqrt{j\lambda z}} \int_{-\infty}^{\infty} u_2(\eta)\exp\left[\frac{jk}{2z}(y-\eta)^2\right] d\eta \ . \tag{4}$$

When these one-dimensional relations are multiplied, they give results equivalent to those of the diffraction integral of Eq. (1). With these relations we may analyze the performance of the three basic component systems required for simplified astigmatic processor analysis. The one-dimensional convex lenses in these basic systems have a transmittance of[11]

$$t(x) = \exp(jkn\Delta)\exp(\frac{-jk}{2f}x^2) \tag{5}$$

where f is the lens' focal length, n is the refractive index of the lens material, and Δ is related to the lens' maximum thickness. In applying this separation analysis method, Δ is, for a cylindrical lens, taken as the actual physical (not optical) maximum thickness of the lens; for a spherical lens, Δ is taken as one-half the maximum physical thickness.

In performing necessary integrations, the identity

$$\int_{-\infty}^{\infty} \exp(jax^2)\exp(-j\xi x)\, dx = \sqrt{\frac{j\pi}{a}}\exp(\frac{-j\xi^2}{4a}) \tag{6}$$

will be useful.[12]

1. Fourier transformer

The one-dimensional Fourier transformer is pictured in Figure 1(a). An input transmittance, $u_1(\xi)$, is placed a distance d_o in front of the lens. The field amplitude on the x plane, located a focal distance to the right of the lens, is, from Eqs. (4), (5) and (6),

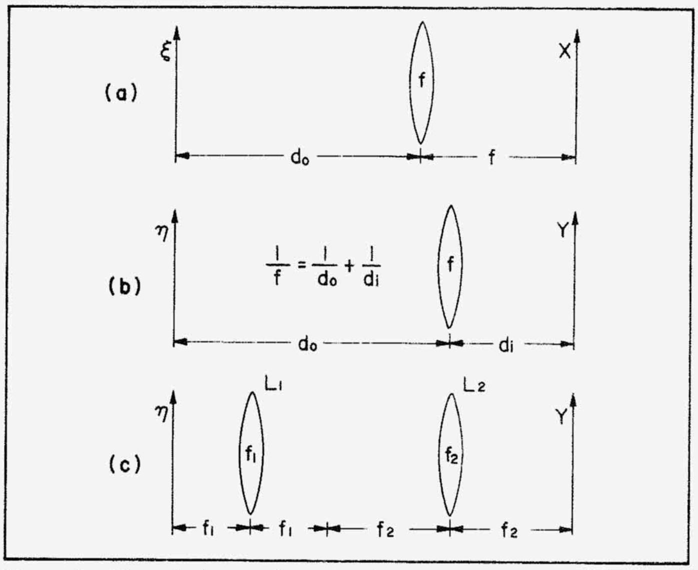

Figure 1. Three basic one-dimensional component systems. (a) Fourier transformer. (b) one lens imaging system. (c) two lens imaging system. Focal lengths are shown on the lenses.

$$v_1(x) = \frac{\exp[\frac{jk}{2}(f + d_o)]\exp(jkn\Delta)}{\sqrt{j\lambda f}} \exp[\frac{jk}{2f}(1 - \frac{d_o}{f})x^2]$$

$$\int_{-\infty}^{\infty} u_1(\xi)\exp(\frac{-jk}{f}\xi x)\, d\xi \ . \qquad (7)$$

Note that when the object is placed in the front focal plane, the quadratic phase term disappears.

2. One-Lens Imaging System

The single lens imaging system, shown in Figure 1(b), satisfies the familiar lens law

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \ . \qquad (8)$$

Again using Eqs. (4), (5), and (6), the field amplitude at d_i due to a transmittance $f_2(\eta)$ at d_o is

$$v_2(y) = \frac{-j}{\sqrt{M_1}} \exp[(\frac{jkd_i}{2})(1 + \frac{1}{M_1})]\exp(jkn\Delta)$$

$$\exp[\frac{jk}{2d_i}(1 + \frac{1}{M_1})y^2]\, u(\frac{-y}{M_1}) \qquad (9)$$

where the magnification M_1 is

$$M_1 = \frac{d_i}{d_o} \ . \qquad (10)$$

3. Two-Lens Imaging System

The simple two lens imaging system, as shown in Figure 1(c), consists of two cascaded Fourier transformers. The field amplitude along the y axis is

$$v_2(y) =$$

$$\frac{-j}{\sqrt{M_2}} \exp[jkf_1(M_2 + 1)]\exp[jkn(\Delta_1 + \Delta_2)]\, u_2(\frac{-y}{M_2}) \ . \qquad (11)$$

Here the magnification parameter is given by

$$M_2 = \frac{f_2}{f_1} \ . \qquad (12)$$

C. Example Applications

We will now illustrate application of the three one-dimensional cases to analysis of some previously used astigmatic processors. In all cases, the two perpendicular components of the system may be combined without ambiguity to arrive at the expression for the two-dimensional system's input-output relation.

1. The astigmatic processor in Figure 2(a) was used by Cu-

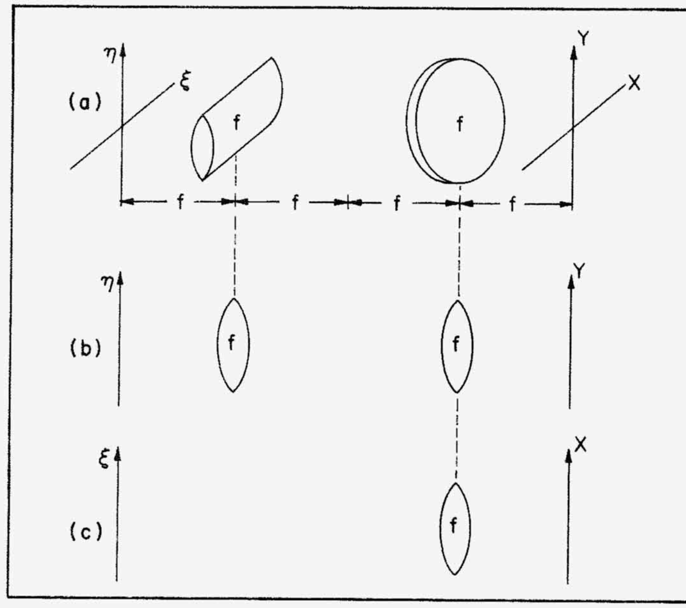

Figure 2. (a) An astigmatic processor and its (b) vertical imaging (c) horizontal Fourier transformer one-dimensional components.

trona et al.[1,2] and others.[15-16] We begin our analysis by first looking at the system in the vertical direction. The resulting one-dimensional component system, as shown in Figure 2(b), is the two-lens imaging system. The horizontal component shown in Figure 2(c), is recognized as the Fourier transformer. We thus apply the two-lens imaging system formula (Eqs. (11) (12)) with $f = f_1 = f_2$, and the Fourier transform relationship of Eq. (7) with $d_o = 3f$. The resulting horizontal and vertical functions are multiplied to obtain the two-dimensional output,

$$v(x,y) = -\sqrt{\frac{j}{\lambda f}} \exp(j4kf)\exp[jkn(\Delta_1 + \Delta_2)]\exp(\frac{-jk}{f} x^2)$$

$$\int_{-\infty}^{\infty} u(\xi,-y)\exp(\frac{-jk}{f}\xi x)\, d\xi \ . \qquad (13)$$

This system is seen to perform parallel Fourier transform and imaging operations with a quadratic phase factor in the horizontal direction. The quadratic phase factor can be removed by placing a negative cylindrical lens immediately prior to the output plane.

2. The astigmatic processor in Figure 3(a), used by Rhodes,[5]

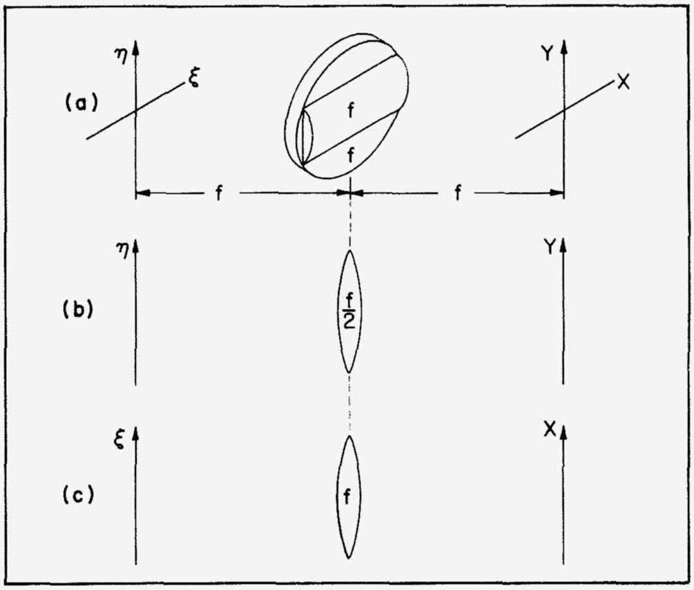

Figure 3. (a) A second astigmatic processor and its (b) vertical and (c) horizontal components.

consists of a spherical lens immediately followed by a cylindrical lens.[17] Each lens has focal length f. In the vertical direction (Figure 3(b)), the system component satisfies the lens law of Eq. (8) and is thus a single lens imaging system where the component lens has an effective focal length of f/2.[18] The horizontal component of the system (Figure 3(c)) is the Fourier transformer. Thus, substituting $d_o = d_i = f$ in Eqs. (9) (10) and $d_o = f$ in Eq. (7), the output is

$$v(x,y) = -\sqrt{\frac{j}{\lambda f}}\,\exp(j2kf)\exp[jkn(\Delta_1 + \Delta_2)]\exp\left(j\frac{k}{f}y^2\right) \int_{-\infty}^{\infty} u(\xi,-y)\exp\left(-j\frac{k}{f}\xi x\right) d\xi \quad . \qquad (14)$$

The parallel transform and imaging operations are again performed, *this time with the quadratic phase factor in the vertical* direction. Again, the phase factor could be removed by placing a cylindrical lens, this time with positive focal length, immediately in front of the output plane.

3. Both of the preceeding processors have spatially varying quadratic phase terms in their output. An astigmatic processor which does not have such a factor is pictured in Figure 4(a).[4, 7, 9] The cylindrical lens L2 has a focal length double that of lenses L1 and L3. The vertical component of the processor shown in Figure 4(b) is a two-lens imaging system. The horizontal component (Figure 4(c)) is a Fourier transformer with d_o equal to the lens' focal length which here is 2f. The processor output arises through combination of Eqs. (7) and (11). The result is

$$v(x,y) = -\sqrt{\frac{j}{2\lambda f}}\,\exp(j4kf)\exp[jkn(\Delta_1 + \Delta_2 + \Delta_3)] \int_{-\infty}^{\infty} u(\xi,-y)\exp\left(-j\frac{k}{2f}\xi x\right) d\xi \quad . \qquad (15)$$

The parallel transform/imaging operation is thus performed without a quadratic phase factor appearing in the output.

D. Conclusions

Astigmatic processors, used widely in coherent optical computing, can be analyzed one dimension at a time. The reduced systems can be recognized as belonging to one of three basic classes. The one-dimensional system equations can then be multiplied to describe the operation of the original system. This description will be valid for both separable and non-separable system inputs. This simplified method of analysis avoids cumbersome manipulations of the Huygens-Fresnel diffraction integral. The "basic component" approach also facilitates intuitive

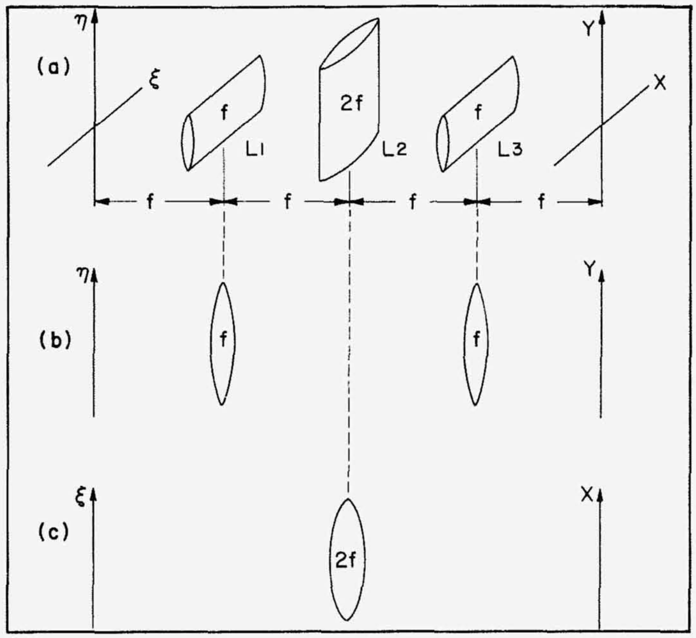

Figure 4. (a) An astigmatic processor with no quadratic phase factor in the output, and the processor's (b) vertical and (c) horizontal components.

design of an astigmatic processor for a particular application.

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17. Note that one can implement an equivalent lens transmittance to that in Figure 3(a) by appropriately placing two cylindrical lenses of focal lengths f and f/2 back to back.
18. From Eq. (5), it can be shown that the equivalent focal length, f_{eq}, of two lenses with focal lengths f_1 and f_2 placed back to back is $f_{eq} = f_1 f_2/(f_1 + f_2)$.