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Scanning technique for coherent processors

M. O. Hagler, R. J. Marks II, E. L. Kral, J. F. Walkup, and T. F. Krile

In certain linear coherent processing techniques a temporal signal is spatially encoded on an amplitude transmittance that serves as the processor input. In this paper a technique is presented whereby the temporal signal is alternately used to amplitude- and/or phase-modulate a raster scan of the processor's input plane. Using the temporal integrating and summing properties of a hologram placed in the processor's output plane, one can then regain the identical processor output that would have arisen from the spatial encoding technique. Preliminary experimental results are presented along with the theory of the input scanning technique.

I. Introduction

In certain coherent processing schemes a processor input is received as a temporal electronic signal. Conventionally, this signal is spatially encoded as a 2-D amplitude transmittance that then serves as the processor input. It is, however, usually the corresponding processor output that is of interest.

In this paper we present a scheme whereby one can achieve an identical linear processor output by utilizing the temporal signal to amplitude- and/or phase-modulate the field amplitude of an input raster scan. The time-varying field amplitude at the system's output is then temporally integrated and summed using holographic techniques. Upon playback the hologram is shown to produce a diffracted term that is identical to that which would be obtained by placing a corresponding input field amplitude transmittance mask at the processor's input. This scheme therefore eliminates the necessity of spatially encoding the input. Use of erasable photographic media suggests possible implementations near real time.

Input scanning has been used extensively in incoherent processing to add the temporal degree of freedom to the already available spatial variables. Various incoherent processors and corresponding references are given in the excellent review paper by Monahan *et al.*[1] Scanning techniques have also been extensively applied in holography[2–7] as have the effects of time-varying field amplitudes.[8–10]

R. J. Marks II is with University of Washington, Department of Electrical Engineering, Seattle, Washington 98195; E. L. Kral is with Boeing Company, Albuquerque, New Mexico 87108; and the other authors are with Texas Tech University, Department of Electrical Engineering, Lubbock, Texas 79409.

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II. Theory

We limit our scanning technique to those systems that are linear. Such systems can formally be expressed via the superposition integral

$$g(x,y) = S[u(x,y)] = \iint_{-\infty}^{\infty} u(\xi,\eta)h(x-\xi, y-\eta; \xi,\eta)d\xi d\eta, \quad (1)$$

where g is the system output corresponding to an input u into a system $S[\cdot]$. The point spread function is defined as

$$h(x-\xi,y-\eta;\xi,\eta) = S[\delta(x-\xi,y-\eta)], \quad (2)$$

where $\delta(\cdot,\cdot)$ denotes the Dirac delta. We are here using the Lohmann-Paris point spread function (impulse response) notation.[11,12]

Consider then the scanning geometry shown in Fig. 1. For the fixed value of $\eta = \eta_m$ we scan the input plane over ξ at speed v. Modeling the scanning point as an amplitude- and/or phase-modulated delta function, the field amplitude to the right of the input plane at time t is

$$u(vt,\eta_m)\delta(\xi - vt, \eta - \eta_m). \quad (3)$$

From Eq. (2) the corresponding complex field amplitude incident on the output plane is

$$u(vt,\eta_m)h(x - vt, y - \eta_m; vt,\eta_m). \quad (4)$$

Placed in the output plane is a photosensitive medium on which is also incident a planar reference beam $\exp(jk\alpha y)$, where α is a direction cosine.[9] The corresponding intensity at time t is thus given by

$$I_m(x,y;t) = |u(vt,\eta_m)h(x - vt, y - \eta_m; vt,\eta_m) + \exp(jk\alpha y)|^2. \quad (5)$$

Assuming the resulting hologram's amplitude transmittance is proportional to the exposing intensity function, we have for one scan an amplitude transmittance of[8–10]

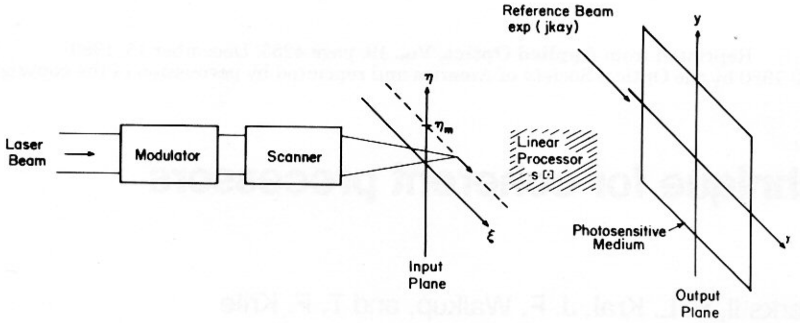

Fig. 1. General input scanning geometry.

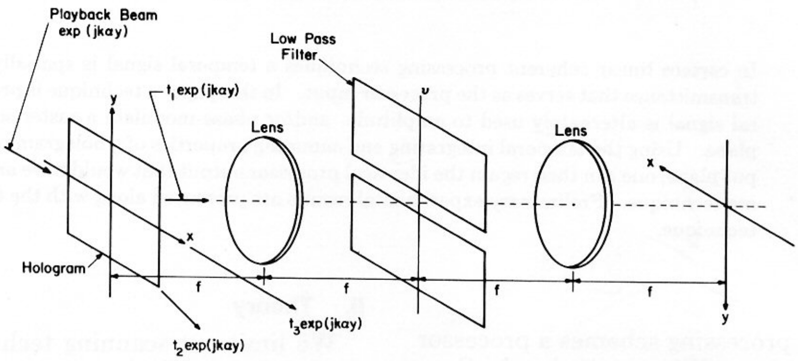

Fig. 2. Playback using a low pass filter.

$$\hat{t}_m(x,y) = \int_{t=0}^{T} I_m(x,y;t)dt, \tag{6}$$

where T is the exposure time for a single scan. For M scans corresponding to various values of η_m, the hologram's amplitude transmittance is

$$\hat{t}(x,y) = \sum_{m=1}^{M} \hat{t}_m(x,y) = t_1 + t_2 + t_3, \tag{7}$$

where

$$\left.\begin{aligned} t_1 &= \sum_m \left[\int_0^T u(vt,\eta_m)h(x - vt, y - \eta_m; vt, \eta_m)dt\right] \exp(-jk\alpha y) \\ t_2 &= t_1^*, \\ t_3 &= TM + \sum_m \int_0^T |u(vt,\eta_m)h(x - vt, y - \eta_m; vt, \eta_m)|^2 dt. \end{aligned}\right\} \tag{8}$$

Here, * denotes complex conjugate. It is the t_1 term in which we are interested. Making the variable substitution $\xi = vt$ and assuming each scan covers the entire input pupil at $\eta = \eta_m$ gives

$$t_1 = \frac{1}{v}\sum_m \left[\int_{-\infty}^{\infty} u(\xi,\eta_m)h(x - \xi, y - \eta_m; \xi, \eta_m) \times d\xi\right] \exp(-jk\alpha y). \tag{9}$$

Playback is performed as shown in Fig. 2. The playback beam gives rise to three diffracted terms. The term $t_3 \exp(jk\alpha y)$ is the zero order through beam, and $t_2 \exp(jk\alpha y)$ is the twin image conjugate component. The term of primary interest is

$$t_1 \exp(jk\alpha y) = \frac{1}{v}\sum_m \int_{-\infty}^{\infty} u(\xi,\eta_m)h(x - \xi, y - \eta_m; \xi, \eta_m)d\xi. \tag{10}$$

This expression is recognized as a semidiscrete version of the superposition integral in Eq. (1). That is, the integral over η is approximated by a summation. In some instances Eq. (10) will be an adequate approximation for the true system output.

Under certain bandlimited assumptions on the input and point spread function, we can obtain a better approximation by performing a low pass filtering operation in the y direction. This stems from space-variant system sampling theory.[13,14] If

$$\int_{-\infty}^{\infty} u(\xi,\eta) \exp(-j2\pi\eta\nu)d\eta \simeq 0 \text{ for } |\nu| > w_u, \tag{11}$$

$$\int_{-\infty}^{\infty} h(x,y;\xi,\eta) \exp(-j2\pi\eta\nu)d\eta \simeq 0 \text{ for } |\nu| > w_\nu, \tag{12}$$

then the desired low pass filter is unity over the frequency band

$$-(w_\nu + w_u) \le \nu \le (w_\nu + w_u). \tag{13}$$

As shown is Fig. 2, this filtering can be performed by conventional spatial filtering techniques.[9]

III. Experiment

To illustrate the temporal integration capabilities of the hologram, we consider the system in Fig. 3. A point source makes a single scan across the 1-D double-pulse input aperture $a(x)$. The linear processor in this example is the familiar Fourier transformer that consists of the single lens L_1. The scan is performed along the line $\eta = 0$.

Following the previous model development, the field incident on the photosensitive medium is

$$a(vt) \exp(-jkvtx/f) + \exp(jk\alpha x), \tag{14}$$

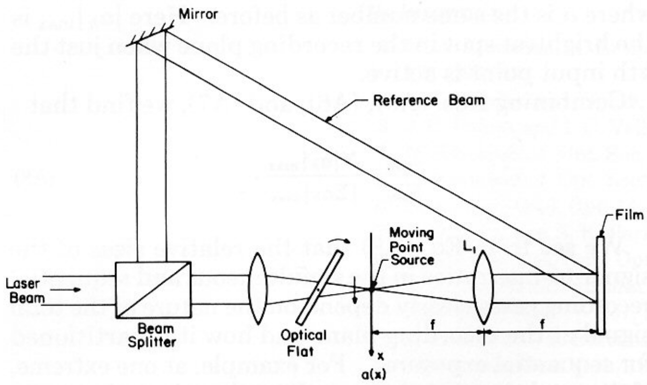

Fig. 3. Experimental scanning configuration.

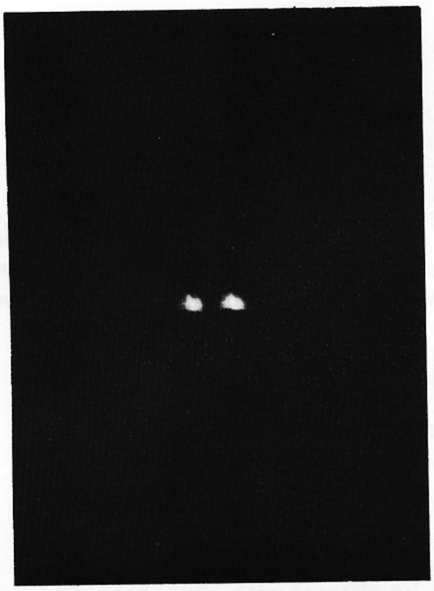

Fig. 4. Experimental output for a double slit input.

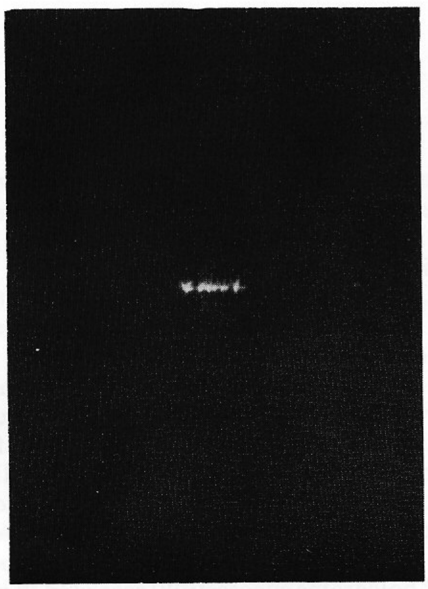

Fig. 5. Experimental output for a single slit input.

where f is the focal length of lens L_1. Under the previously stated recording assumptions, the resulting holographic field amplitude is

$$\hat{t}(x,y) = t_1 + t_2 + t_3, \tag{15}$$

where

$$\left.\begin{aligned} t_1 &= \int_{-T/2}^{T/2} a(vt)\exp(-jkvtx/f)dt\exp(-jk\alpha x),\\ t_2 &= t_1^*,\\ t_3 &= T + \int_{-T/2}^{T/2} |a(vt)|^2 dt. \end{aligned}\right\} \tag{16}$$

When the hologram is played back, the diffracted term immediately to the right of the hologram corresponding to t_1 is

$$\frac{1}{v}\int_{-\infty}^{\infty} a(\xi)\exp(-jk\xi x/f)d\xi, \tag{17}$$

where we have made the variable substitution $\xi = vt$ and have assumed the scan completely covered both input pulses. Equation (17) is recognized as the 1-D Fourier transform of $a(x)$. Thus we should be able to regain $a(x)$ by an additional Fourier transform. This is accomplished by a single spherical lens. The result of playback is shown in Fig. 4 and, as can be seen, compares quite favorably with the theory. Similar results for a single pulse (slit) input are given in Fig. 5. The somewhat discontinuous appearance of the slit may be due to the fact that the rotation of the optical flat shown in Fig. 3, which produced the scanning point source, was accomplished by hand.

IV. Conclusions

We have demonstrated a technique whereby temporal signals can be linearly processed without first being spatially encoded as an amplitude transmittance. The scheme makes use of the temporal integration and summation properties of the hologram.

This technique is potentially applicable to all linear coherent processors. By using a scanning modulated line source, it is also directly applicable to the recently presented glass of linear 1-D coherent processors.[15,16] In addition, it has been shown that when coupled with the availability of changeable Fourier plane masks, the temporal holography approach may be used to implement a 2-D space-variant processor.[17]

The scanning technique described above is, of course, subject to the diffraction efficiency limitation of sequentially recorded holograms.[18–21] It should be noted, however, that these limitations need not be severe[20,21] if appropriate exposure conditions are employed. The issue of bias-induced limitations inherent in sequentially recorded holograms as compared to those obtained in a single simultaneous recording is discussed in more detail in the Appendix.

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Appendix

When constructing sequentially recorded holograms, there is the potential for a bias build up problem and consequent reduction in diffraction efficiency. To examine the extent of this problem and lead into possible solutions proposed by other workers, we will compare the signal-to-bias ratios for the simultaneous recording case and the sequential recording case.

Assume an object composed of N points, and denote by $a_k(x,y)$ the amplitude of light on the hologram, produced by point k on the object. For simplicity the 2-D spatial function $a_k(x,y)$ will be written as a_k from now on.

In the case of simultaneous exposure of a hologram derived from all N object points, the total film exposure is:

$$\begin{aligned} E_{\text{sim}} &= T_{\text{sim}}|R_{\text{sim}} + \Sigma a_k|^2 \\ &= T_{\text{sim}}\{|R_{\text{sim}}|^2 + R_{\text{sim}}^* \Sigma a_k + R_{\text{sim}}\Sigma a_k^* + \Sigma\Sigma a_i a_j^*\}, \end{aligned} \tag{A1}$$

where T_{sim} is the exposure time, R_{sim} is the amplitude of the reference beam, and the index in all sums runs from 1 to N. The signal-to-bias ratio is then given by

$$\delta_{\text{sim}} = \left|\frac{R_{\text{sim}}^*\Sigma a_k}{|R_{\text{sim}}|^2}\right| = \left|\frac{\Sigma a_k}{R_{\text{sim}}}\right| , \tag{A2}$$

where we assume $\Sigma\Sigma a_i a_j^* \ll |R_{\text{sim}}|^2$.

For the sequential recording case the total film exposure is

$$\begin{aligned} E_{\text{seq}} &= T_{\text{seq}}\Sigma|R_{\text{seq}} + a_k|^2 \\ &= T_{\text{seq}}\{N|R_{\text{seq}}|^2 + R_{\text{seq}}^*\Sigma a_k + R_{\text{seq}}\Sigma a_k^* + \Sigma\Sigma a_i a_j^*\}, \end{aligned} \tag{A3}$$

where T_{seq} is the total film exposure time over all exposures. Note that R_{seq} is not necessarily the same as R_{sim}, but we do assume R_{seq} is the same for each individual exposure. In this case the signal-to-bias ratio is

$$\delta_{\text{seq}} = \left|\frac{R_{\text{seq}}^*\Sigma a_k}{N|R_{\text{seq}}|^2}\right| = \frac{1}{N}\left|\frac{\Sigma a_k}{R_{\text{seq}}}\right| . \tag{A4}$$

Combining Eqs. (A2) and (A4) we find that

$$\frac{\delta_{\text{sim}}}{\delta_{\text{seq}}} = N\frac{|R_{\text{seq}}|}{|R_{\text{sim}}|} . \tag{A5}$$

Now, it is customary to choose the R_{sim} such that

$$|R_{\text{sim}}| = \alpha|\Sigma a_k|_{\max}, \tag{A6}$$

where α is a number on the order of unity. Since Σa_k represents the signal in the recording plane from the entire object, $|\Sigma a_k|_{\max}$ is the brightest spot in the recording plane when all object points are active.

For the sequential recording case it is only necessary to require that

$$|R_{\text{seq}}| = \alpha|a_k|_{\max}, \tag{A7}$$

where α is the same number as before. Here $|a_k|_{\max}$ is the brightest spot in the recording plane when just the kth input point is active.

Combining Eqs. (A5), (A6), and (A7), we find that

$$\frac{\delta_{\text{sim}}}{\delta_{\text{seq}}} = \frac{N|a_k|_{\max}}{|\Sigma a_k|_{\max}} . \tag{A8}$$

We see from Eq. (A8) that the relative sizes of the signal-to-bias ratios in the simultaneous and sequential recording cases clearly depend on the nature of the total signal at the recording plane and how it is partitioned for sequential exposures. For example, at one extreme, if the total signal at the recording plane is uniform and is partitioned into disjoint spatial regions for sequential recording, then

$$|\Sigma a_k|_{\max} = |a_k|_{\max}, \tag{A9}$$

so that

$$\frac{\delta_{\text{sim}}}{\delta_{\text{seq}}} = N. \tag{A10}$$

At the other extreme, if the signal at the recording plane is uniform and is partitioned for sequential recording such that the $\{a_k\}$ are uniform and equal, then

$$|\Sigma a_k|_{\max} = N|a_k|_{\max}, \tag{A11}$$

so that

$$\frac{\delta_{\text{sim}}}{\delta_{\text{seq}}} = 1. \tag{A12}$$

As a more realistic intermediate case, we can consider the $\{a_k\}$ to be phasors with equal amplitudes and uniformly distributed random phases, in which case we would expect that on average

$$|\Sigma a_k|_{\max} = \sqrt{N}|a_k|_{\max}, \tag{A13}$$

so that

$$\frac{\delta_{\text{sim}}}{\delta_{\text{seq}}} = \sqrt{N} . \tag{A14}$$

In practical cases, therefore, we can expect the signal-to-bias ratio for the sequential case to degrade as the $\sqrt{N}$. It is important to note, however, that the degradation in δ_{seq} can be reduced significantly by optimizing R_{seq} for each exposure. Such conditions are discussed in Refs. 18–21 and lead to situations where δ_{sim} can be made equal to δ_{seq}.

To add credence to these observations, we have in our own laboratory successfully recorded 100 sequentially multiplexed holograms of the Fourier transforms of point sources for the purpose of holographically representing space-variant optical systems.[22] The system plays back well so we are confident that we are achieving adequate diffraction efficiencies. At this point the number of holograms successfully recorded in sequence has been limited mainly by our patience.

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