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Synchronous vs asynchronous behavior of Hopfield's CAM neural net

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The performance of Hopfield's neural net operating in synchronous and asynchronous modes is contrasted. Two interconnect matrices are considered: (1) the original Hopfield interconnect matrix; (2) the original Hopfield interconnect matrix with self-neural feedback. Specific attention is focused on techniques to maximize convergence rates and avoid steady-state oscillation. We identify two oscillation modes. Vertical oscillation occurs when the net's energy changes during each iteration. A neural net operated asynchronously cannot oscillate vertically. Synchronous operation, on the other hand, can change a net's energy either positively or negatively and vertical oscillation can occur. Horizontal oscillation occurs when the net alternates between two or more states of the same energy. Certain horizontal oscillations can be avoided by adopting appropriate thresholding rules. We demonstrate, for example, that when (1) the states of neurons with an input sum of zero are assigned the complement of their previous state, (2) the net is operated asynchronously, and (3) nonzero neural autoconnects are allowed, the net will not oscillate either vertically or horizontally.

I. Introduction

The neural network model of a content addressable memory (CAM) proposed by Hopfield[1] has stirred great interest in the optical and the signal processing communities. The model has been implemented both electronically and optically.[2–4]

In this paper, we contrast the performance of a Hopfield-type neural net for synchronous and asynchronous operation. We show, for example, that for synchronous operation, Hopfield's net can oscillate in the steady state and that the oscillation can be avoided by slightly altering the neural operation. Previously, these results have been observed empirically.[1,5,6] Asynchronous operation of the net, on the other hand, always results in a stable steady state when the neural threshold function is properly defind and nonzero autoconnects are used. Use of nonzero neural autoconnects also results in a net that converges faster than when zero autoconnects are used. This is true for both asynchronous and synchronous operations. Thus, in general, asynchronous implementation of nets with nonzero autoconnects have the best convergence properties. Better convergence, however, does not necessarily imply better (or worse) steady-state accuracy.

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II. Preliminaries

Hopfield's model for a content addressable memory consists of a set of identical processing nodes (or neurons) with intensive pairwise interconnects. The neuron model proposed by Hopfield uses two state neurons. The state of each neuron depends on the states of the other neurons in the network. Other models also allow autoconnects (e.g., self-neural feedback). We consider the discrete time rather than the analog neuron model.

Let the total number of the neurons in the network be L. The states of the L neurons can therefore be described by an L-tuplet $\mathbf{v}$, each element of which is binary: either 0 or 1. The strength of the interconnects is represented by an $L \times L$ interconnect matrix $\mathbf{T}$. The ijth element of $\mathbf{T}$, t_{ij}, represents the strength of the interconnect between the ith and the jth neurons. Let $\{\mathbf{f}_n | 1 \le n \le N\}$ denote a set of binary library vectors (or memories). We begin formation of $\mathbf{T}$ by defining the library matrix of binary bipolar (−1,+1) elements:

$$\mathbf{F} = [2\mathbf{f}_1 - \mathbf{1}_L | 2\mathbf{f}_2 - \mathbf{1}_L | \ldots | 2\mathbf{f}_N - \mathbf{1}_L], \tag{1}$$

where $\mathbf{1}_L$ is the L-tuplet with every element equal to one. Hopfield's interconnect matrix follows as[7]

$$\mathbf{T} = \mathbf{T}_0 = \mathbf{F}\mathbf{F}^T - N\mathbf{I}_L, \tag{2}$$

where the superscript T denotes matrix transposition, $\mathbf{I}_L$ is the $L \times L$ identity matrix, and the 0 subscript denotes that the interconnect matrix $\mathbf{T}$ has a zero diagonal (i.e., no autoconnects). If on the other hand we allow autoconnects, the interconnect matrix is then simply

$$\mathbf{T} = \mathbf{T}_\varnothing = \mathbf{F}\mathbf{F}^T. \quad (3)$$

The $\mathbf{T}_0$ and $\mathbf{T}_\varnothing$ matrices will be referred to as zero autoconnects (ZA) matrix and the nonzero autoconnects (NZA) matrix, respectively.

With $\mathbf{v}_0$ as the vector of initial neural states, the network generates a sequence, $\mathbf{V}$, of binary vectors either synchronously or asynchronously:

$$\mathbf{V} = \{\mathbf{v}_0,\mathbf{v}_1,\ldots,\mathbf{v}_k,\mathbf{v}_{k+1},\ldots\}. \quad (4)$$

A. Asynchronous Mode

In the asynchronous mode of operation, only one of the L neurons is free to change state at a time:

$$v_{k+1,i} = \begin{cases} v_{k,i}; & i \neq m, \\ \mu[G_{k,i}]; & i = m, \end{cases} \quad (5)$$

where

$$G_{k,i} = \sum_{j=1}^{L} t_{ij} v_{k,j},$$

$$\mu(x) = \begin{cases} 1; & x > 0, \\ 0; & x < 0. \end{cases}$$

By this definition, asynchronicity does not necessarily imply randomness. The neurons, for example, could fire periodically one at a time in sequence (i.e., $m = 1,2,\ldots,L$) and fit this definition of asynchronicity.

B. Synchronous Mode

In the synchronous mode of operation, all neurons are free to change state in each iteration:

$$v_{k+1,i} = \mu[G_{k,i}]; \quad i = 1 \text{ to } L, \quad (6)$$

or in vector form:

$$\mathbf{v}_{k+1} = \boldsymbol{\mu}[\mathbf{G}_k], \quad (7)$$

where $\boldsymbol{\mu}$ performs a unit step operation on each element of $\mathbf{G}_k$ and $\mathbf{G}_k = \mathbf{T}\mathbf{v}_k$. We denote the ZA and NZA versions by $\mathbf{G}_k$ and $\mathbf{G}_k^\varnothing$, respectively.

In both modes of operation, the sequence $\mathbf{V}$ usually stabilizes at some particular binary vector $\mathbf{v}_s$ after a finite number of iterations. In particular, $\mathbf{v}_s = \boldsymbol{\mu}[\mathbf{T}\mathbf{v}_s]$. Ideally, $\mathbf{v}_s$ is that library vector closest to v_0 in the Hamming sense.

III. Convergence of Hopfield's Algorithm

In this section, we consider the convergence of Hopfield's algorithm, performed both synchronously and asynchronously. In either case, we define an energy function of an L-dimensional binary vector $\mathbf{v}$:

$$E = -\frac{1}{2}\mathbf{v}^T\mathbf{T}\mathbf{v}.$$

Since $\mathbf{v}$ is binary, the energy has a lower bound:

$$E \geq -\frac{1}{2}\sum_{i=1}^{L}\sum_{j=1}^{L}|t_{ij}| = E_{\min} \geq -\frac{1}{2}NL^2.$$

The energy of the neural net at the kth iteration is denoted by

$$E_k = -\frac{1}{2}\mathbf{v}_k^T\mathbf{T}\mathbf{v}_k; \quad \mathbf{v}_k \in \mathbf{V}. \quad (8)$$

The corresponding energy change at this iteration is

$$\Delta E_k = E_{k+1} - E_k. \quad (9)$$

To calculate ΔE_k, we define $\boldsymbol{\Delta}_k$ as the state transition vector:

$$\boldsymbol{\Delta}_k = \mathbf{v}_{k+1} - \mathbf{v}_k. \quad (10)$$

Combining Eqs. (7)–(10) yields the energy transition expression when the states of the neurons evolve from $\mathbf{v}_k$ to $\mathbf{v}_{k+1}$:

$$\Delta E_k = -\boldsymbol{\Delta}_k^T\mathbf{G}_k - \frac{1}{2}\boldsymbol{\Delta}_k^T\mathbf{T}\boldsymbol{\Delta}_k. \quad (11)$$

We now investigate the nature of $\boldsymbol{\Delta}_k$ for various cases.

A. Asynchronous Mode of Operation

If the iteration is carried out asynchronously, then

$$\Delta_{k,i} = \begin{cases} \operatorname{sgn}[G_{k,i}]; & i = m \text{ and } \mu[G_{k,m}] \neq v_{k,m}, \\ 0; & \text{otherwise}, \end{cases} \quad (12)$$

where

$$\operatorname{sgn}[x] = \begin{cases} 1; & x < 0, \\ -1; & x > 0. \end{cases}$$

Case I: ZA Model

With no autoconnects, the energy transition expression is obtained by substituting Eq. (12) into Eq. (11), with $\mathbf{T} = \mathbf{T}_0$:

$$\Delta E_k^0 = \begin{cases} 0; & \mu[G_{k,m}^0] = v_{k,m}, \\ -|G_{k,m}^0| < 0; & \mu[G_{k,m}^0] \neq v_{k,m}. \end{cases} \quad (13)$$

Case II: NZA Model

With the incorporation of autoconnects, we obtain

$$\Delta E_k^\varnothing = \begin{cases} 0; & \mu[G_{k,m}^\varnothing] = v_{k,m}, \\ -|G_{k,m}^\varnothing| - (N/2) < 0; & \mu[G_{k,m}^\varnothing] \neq v_{k,m}. \end{cases} \quad (14)$$

In both the ZA and NZA models, ΔE_k is always zero or negative:

$$E_{k+1} \leq E_k. \quad (15)$$

Since $E_k \geq E_{\min}$ for every $\mathbf{v}_k \in \mathbf{V}$, the sequence $\mathbf{V}$ must converge to some stable state $\mathbf{v}_s$, where

$$E_s \leq E_k; \quad s > k. \quad (16)$$

In general, if the energy transition is maximized at each iteration, the required number of iterations for convergence should be reduced considerably. Convergence acceleration, however, does not imply increased steady-state accuracy.

If $\mathbf{v}_s$ is stable, then

$$\mathbf{v}_s = \boldsymbol{\mu}[\mathbf{T}\mathbf{v}_s]; \quad (17)$$

$\mathbf{v}_s$ is said to be a locally stable state because generally more than one stable state exists with any given interconnect matrix $\mathbf{T}$. For example, if $\mathbf{v}_s$ is a stable state, the complement of $\mathbf{v}_s$ is also stable.[1] Neural networks of the type considered can also oscillate in steady state. If the energy remains the same throughout, the oscillation will be referred to as horizontal. Here, the net alternates between two or more states of equal energy. Vertical oscillation, on the other hand, results from alternating between states, at least two of which have different energies.

B. Synchronous Mode of Operation

Now we iterate the Hopfield's algorithm synchronously. In accordance with Eq. (6), more than one neuron may change state simultaneously. Let σ_k be the set of indices of neurons that change state in the kth iteration:

$$\sigma_k = \{i | v_{k,i} \neq \mu[G_{k,i}]\}.$$

The transition vector $\mathbf{\Delta}_k$ then follows as:

$$\Delta_{k,i} = \begin{cases} \mathrm{sgn}[G_{k,i}]; & i \in \sigma_k, \\ 0 & i \notin \sigma_k. \end{cases} \tag{18}$$

Case I: ZA Model

With the absence of autoconnects, we obtain the energy transition in this operating mode from Eq. (11):

$$\Delta E_k^0 = -\mathbf{\Delta}_k^T \mathbf{G}_k^0 - \frac{1}{2} \mathbf{\Delta}_k^T \mathbf{T}_0 \mathbf{\Delta}_k$$

$$= -\sum_{i \in \sigma_k} |G_{k,i}^0| - \frac{1}{2} \mathbf{\Delta}_k^T \mathbf{T}_0 \mathbf{\Delta}_k. \tag{19}$$

Replacing T_0 by Eq. (2) and manipulating give

$$\Delta E_k^0 = -\sum_{i \in \sigma_k} |G_{k,i}^0| - \frac{1}{2} \|F^T \Delta_k\|^2 + \frac{1}{2} N M_k, \tag{20}$$

where M_k is the cardinality of the set σ_k (i.e., the number of neurons that change state in the kth iteration). Since NM_k is positive, the energy transition in Eq. (20) can be positive. The possibility of positive energy transition explains the vertical oscillatory behavior occasionally exhibited when the algorithm is implemented in the synchronous mode.[5,6]

Case II: NZA Model

Following the same procedure for the ZA case, we use Eqs. (19) and (20) using $\mathbf{T}_\varnothing$ rather than $\mathbf{T}_0$. The energy transition expression follows as

$$\Delta E_k^\varnothing = -\mathbf{\Delta}_k^T \mathbf{G}_k^\varnothing - \frac{1}{2} \mathbf{\Delta}_k^T \mathbf{T}_\varnothing \mathbf{\Delta}_k$$

$$= -\sum_{i \in \sigma_k} |G_{k,i}^\varnothing| - \frac{1}{2} \|\mathbf{F}^T \mathbf{\Delta}_k\|^2 \leq 0. \tag{21}$$

Here, $\Delta E_k^\varnothing$ can never be positive because both terms in Eq. (21) are always negative. This negativity of the energy transition seems to provide good support for the NZA model over the ZA model in terms of vertical oscillation suppression. However, in the following section we will see that the NZA model can oscillate horizontally in the synchronous mode.

IV. Zero Input Problem

A practical problem that occurs frequently in the operation of the Hopfield model is to decide the value of a neural state, $v_{k+1,m}$, when $G_{k,m}$ is zero. We call this the zero input (ZI) problem. In the following, we resolve the problem in the context of maximizing the energy transition.

A. Asynchronous Mode of Operation

Case I: ZA Model

For the ZA model, $G_{k,m}^0 = 0$ in Eq. (13) results in an energy transition of

$$\Delta E_k^0 = -\Delta_{k,m} G_{k,m}^0 = 0. \tag{22}$$

There is no contribution to the energy transition regardless of whether the mth neuron changes its state.

Case II: NZA Model

In the NZA model, the energy transition follows from Eq. (11) as

$$\Delta E_k^\varnothing = -\Delta_{k,m} G_{k,m}^\varnothing - \frac{1}{2} N \Delta_{k,m}^2 = -\frac{1}{2} N \Delta_{k,m}^2. \tag{23}$$

If each neuron with a zero input sum responds by complementing the state, $v_{k+1,m} = \bar{v}_{k,m}$, then $\Delta_{k,m}^2 = 1$. As a result,

$$\Delta E_k^\varnothing = -\frac{1}{2} N < 0, \tag{24}$$

and the energy transition is maximized.

Thus, when the ZI problem is encountered, the NZA model always offers a negative energy transition when we complement the neuron's state. In contrast, the energy transition in the ZA model stays the same whether the neuron's state changes. In this sense, the NZA model has a faster convergence rate than the ZA model for asynchronous operation.

B. Synchronous Mode of Operation

In the synchronous mode of operation, the ZI problem may be encountered by more than one neuron simultaneously during the iteration. Denote the set of indices of neurons with zero input by

$$\beta_k = \{i | G_{k,i} \equiv 0\}.$$

Here, we only consider the specific situation when

$$v_{k+1,j} = v_{k,j}; \quad i \notin \beta_k.$$

That is, the states of all the neurons which do not encounter a zero input sum remain unchanged. The energy transition then depends only on the states of the neurons which encounter the zero input sum. Since $\mathbf{G}_k = \mathbf{O}$, Eq. (11) becomes

$$\Delta E_k = -\frac{1}{2} \mathbf{\Delta}_k^T \mathbf{T} \mathbf{\Delta}_k, \tag{25}$$

where

$$\Delta_{k,j} = 0; \quad j \notin \beta_k.$$

If one chooses to let the rest of the neurons with zero input stay at the same state, then

$$\mathbf{v}_{k+1} = \mathbf{v}_k$$

Thus, ΔE_k will be zero and there is neither a vertical improvement nor degradation to the convergence. Thus, changing the states of those neurons results in a nonzero (hopefully negative) energy transition.

Case I: ZA Model

According to Eq. (25), the energy transition for the ZA model is

$$\Delta E_k^0 = -\frac{1}{2}\Delta_k^T \mathbf{T}_0 \Delta_k.$$

If Δ_k is not a zero vector, then, with reference to Eq. (20),

$$\Delta E_k^0 = -\frac{1}{2}\|\mathbf{F}^T\Delta_k\|^2 + \frac{1}{2}N\|\Delta_k\|^2. \quad (26)$$

Note that $\|\Delta_k\|^2$ is simply the number of nonzero elements in the transition vector Δ_k.

We now examine two schemes for state transition under the ZT problem and their effects on energy transition for ZA and NZA synchronous operation.

Scheme I: Complementation

In this scheme, we complement the states of all the neurons experiencing a zero input sum:

$$v_{k+1,i} = \bar{v}_{k,i}; \quad i \in \beta_k. \quad (27)$$

The energy transition using this scheme follows as

$$\Delta E_k^0 = -\frac{1}{2}\|\mathbf{F}^T\Delta_k\|^2 + \frac{1}{2}NP_k, \quad (28)$$

where P_k is the cardinality of the set β_k. $\Delta E_k^\varnothing$ can either be positive, negative, or zero. Positive $\Delta E_k^\varnothing$ is adverse to the convergence. In the case where $\Delta E_k^\varnothing = 0$, the new transition vector will be

$$\Delta_{k+1} = -\Delta_k. \quad (29)$$

Thus $\Delta E_{k+1} = 0$, as is ΔE_{k+2}, ΔE_{k+3}, etc. These zero energy transitions thus correspond to repeat complementation of the same set of ZI neurons. The net thus locks in horizontal oscillating between two binary vectors $\mathbf{v}_a$ and $\mathbf{v}_b$, where

$$v_{b,i} = \begin{cases} \bar{v}_{a,i}; & i \in \beta_k; \\ v_{a,i}; & \text{otherwise.} \end{cases} \quad (30)$$

Neither vector is a locally stable state. The oscillation has been observed to occur many times about a locally stable point.[1,5]

Scheme II: State Hardening

One possible way to reduce the magnitude of the positive term in Eq. (28) is to complement only some of the ZI neurons. The ZI scheme suggested by Macukow and Arsenault[6] fits this description. Their ZI scheme is to harden the states of the ZI neurons at +1 regardless of what their last states are. Specifically, for $j \in \beta_k$,

$$v_{k+1,j} = \begin{cases} 1; & G_{k,j} \geq 0, \\ 0; & G_{k,j} < 0. \end{cases} \quad (31)$$

The energy transition follows as

$$\Delta E_k^0 = -\frac{1}{2}\|\mathbf{F}^T\Delta_k\|^2 + \frac{1}{2}NQ_k, \quad (32)$$

where Q_k is the number of nonzero elements of the state transition vector Δ_k. As with full complementation, $\Delta E_k^\varnothing$ can be negative. We do, however, avoid the problem that can cause horizontal oscillation in the case of ZA full complementation. Macukow and Arsenault[6] demonstrate this nonoscillatory nature empirically.

Case II: NZA Model

According to Eq. (25), the energy transition of the NZA model is

$$\Delta E_k^\varnothing = -\frac{1}{2}\Delta_k^T \mathbf{T}_\varnothing \Delta_k,$$

which is recognized as

$$\Delta E_k^\varnothing = -\frac{1}{2}\|\mathbf{F}^T\Delta_k\|^2 \leq 0. \quad (33)$$

We immediately recognize that the energy transition in the NZA model is never positive. In the following, we will apply the two schemes for zero thresholding previously discussed and investigate their behavior.

Scheme I: Complementation

From Eq. (33), the energy transition corresponding to full complementation is never positive. However, $\Delta E_\varnothing^k$ is zero when Δ_k is orthogonal to every column of the library matrix $\mathbf{F}$ (i.e., $\mathbf{F}^T\Delta_k = \mathbf{O}$). For this reason, the net can oscillate for the same reason that the ZA model can oscillate.

Scheme II: State Hardening

Again, the energy transition given by Eq. (33) is never positive. If we use this scheme of thresholding the zero input sum, the net will not oscillate horizontally.

In short, when zero thresholding is encountered by a number of neurons, one should change the states of some of those neurons rather than stay at the previous states. In the ZA model, sometimes these changes may retard the convergence or may even lock the net in vertical or horizontal oscillation. In the NZA model, the change of states never adversely affects the convergence and vertical oscillation cannot occur. To remove the possibility of horizontal oscillation, one may either (a) change the states of a partial number of neurons, or (b) operate the net asynchronously.

V. Examples

We present three examples of application of the two thresholding schemes discussed above to both the ZA and NZA models. We use the same examples used by Macukow and Arsenault[6].

Example 1:

STORED VECTORS

$\mathbf{F}_1 =$ 1 0 1 1 0 1 1 1 0 1 1 1 0 1 0 0 1 0 0 1

$\mathbf{F}_2 =$ 0 1 0 1 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1

$\mathbf{F}_3 =$ 1 1 0 0 0 1 0 1 1 1 0 1 1 0 1 1 1 1 0 0

$\mathbf{F}_4 =$ 0 1 1 1 0 1 1 0 1 0 1 0 0 0 1 0 1 1 1 0

INPUT VECTOR

$v_0 = 0\ 1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

$\mathbf{v}_0$ contains the first seven elements of $\mathbf{f}_4$.
Results:

Zero Autoconnect (ZA) Model

Thresholding schemes	$-2\Delta E_k$	Total number of iterations	Steady-state vector
(1) Complementation	+64,0,+8	4	$\mathbf{f}_4$
(2) State hardening	+64,+8	3	$\mathbf{f}_4$

Nonzero Autoconnect (NZA) Model

Thresholding schemes:	$-2\Delta E_k$	Total number of iterations	Steady-state vector
(1) Complementation	+84,+12	3	$\mathbf{f}_4$
(2) State hardening	+98	2	$\mathbf{f}_4$

Example 2:
Same as example 1, except the input vector contains only the first four elements of $\mathbf{f}_4$:

INPUT VECTOR

$v_0 = 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0$

Result:

Zero Autoconnect (ZA) Model

Thresholding schemes:	$-2\Delta E_k$	Total number of iterations	Steady-state vector
(1) Complementation	+32,+48,0,+48	5	False state
(2) State hardening	+56,+24,+8	4	$\mathbf{f}_4$

Nonzero Autoconnect (NZA) Model

Thresholding schemes:	$-2\Delta E_k$	Total number of iterations	Steady-state vector
(1) Complementation	+96,+24	3	$\mathbf{f}_4$
(2) State hardening	+96,+24	3	$\mathbf{f}_4$

Example 3:

STORED VECTORS

$\mathbf{f}_1 = 1\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 0\ 1$

$\mathbf{f}_2 = 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 1$

INPUT VECTOR

$v_0 = 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0$

$\mathbf{v}_0$ is a partial version of $\mathbf{r}_1$.
Result:

Zero Autoconnect (ZA) Model

Thresholding schemes:	$-2\Delta E_k$	Total number of iterations	Steady-state vector
(1) Complementation	+18,0,0,0,..	—	Oscillation
(2)State hardening	+42	2	$\mathbf{f}_1$

Nonzero Autoconnect (NZA) Model

Thresholding schemes:	$-2\Delta E_k$	Total number of iterations	Steady-state vector
(1) Complementation	+41	2	$\mathbf{f}_1$
(2) State hardening	+41	2	$\mathbf{f}_1$

In terms of the number of iterations and the steady-state behavior, all three examples show that the NZA model always performs better than the ZA model, and the state hardening scheme performs better than the complementation scheme.

We summarize our results in Table 1.

VI. Conclusion

We have contrasted the performance of Hopfield's neural net model operating in asynchronous and synchronous modes. From an energy transition and stability perspective, the model performs better in the asynchronous mode. The net always converges to some locally stable state. In the synchronous mode, the net may sometimes lock in oscillation. Therefore, locally stable states cannot be guaranteed in the synchronous mode. The net also performs better in the asynchronous mode when a zero neural input situation is encountered. We have also considered the incorporation of autoconnects in the model. In either mode of operation, the net performs better when autoconnects are incorporated. All these results show that the Hopfield model performs the best when the net is operated asynchronously and the autoconnects are used.

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Table I. Summary of Results

	Zero Autoconnect Model (ZA)		Nonzero Autoconnect Model (NZA)	
	Operating Mode		Operating Mode	
	Sync.	Async.	Sync.	Async.
Energy Transition: (neurons are still changing states)				
I. Input sum to any neuron is not zero	≤ 0 or > 0	< 0	≤ 0	< 0
II. Input sum to one or more neurons is zero:				
a. Complementation Scheme	≤ 0 or > 0	= 0	≤ 0	< 0
b. State-hardening Scheme	≤ 0 or > 0	= 0	≤ 0	≤ 0
Oscillation ?				
I. Vertical:				
a. Complementation Scheme	sometimes	no	no	no
b. State-hardening Scheme	?*	no	no	no
II. Horizontal:				
a. Complementation Scheme	sometimes	?*	sometimes	no
b. State-hardening Scheme	?*	no	no	no

* The ? in the table represents uncertainty.

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