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# RECONSTRUCTION AND ENHANCEMENT OF CURRENT DISTRIBUTION ON CURVED SURFACES FROM BIOMAGNETIC FIELDS USING POCS

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ABSTRACT. Reconstruction and enhancement of the current distribution on curved surfaces from biomagnetic fields using POCS is a novel idea with possible medical applications, such as reconstructions of current distribution on the heart or brain surface. An initial estimate of the current distribution is obtained by solving the biomagnetic inverse problem using pseudo-inverse techniques. The initial image vaguely resembles the original shape of the current distribution which can be improved by alternating projections assuming that images can be represented by line-like elements.

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**1 Introduction.** In an earlier paper we have shown the reconstruction and image enhancement of current distribution in a planar surface from the magnetic fields [1]. The present work is an extension to enhance the resolution of current distribution on curved surfaces using POCS (Projection Onto Convex Sets). Use of POCS is very common in 2-D for image restoration and enhancement and has been applied to various types of tomography [2]. Its application for image restoration on a curved surface is a novel idea and could have many applications, including biomagnetic reconstruction of current distribution on the heart wall or the brain surface. An initial estimate of the image on the curved surface from the magnetic fields can be obtained by use of a pseudoinverse (minimum norm) technique [3]. Inversion of biomagnetic data on a curved surface is very noisy as compared to the inversion on a planar surface because of varying vertical separation of points (pixels) on the curved surface to the biomagnetic field sample points. Thus, the initially reconstructed image has a poor resolution and a very poor resemblance to the original shape of the current distribution. The initial image is restored by the use of the method of alternating projections. A commonly used special case of alternating projections is POCS [2]. Details of the application of POCS to biomagnetic problems are given in our earlier work [1]. A brief review is given here.

**2 Application to biomagnetic image reconstruction.** The Biot-Savart law in matrix form can be written as [1]:

$$B = WD \tag{1}$$

where $B$ is the magnetic field measured at sampling points in a sampling surface, $D$ is the current distribution and $W$ is a block matrix relating the location of the current distribution to the magnetic field at sampling points in a sampling surface. In general, the number of measured magnetic field points are less than the unknown currents. Thus, it is an under-determined problem and direct inversion of the observation $B$ to the source image $D$ is not possible. However, by use of pseudoinverse techniques one could obtain a best estimate. It will be given as:

$$D \approx \hat{D} = W^T(WW^T)^{-1}B. \tag{2}$$

The image vector, $\hat{D}$, is the image closest to the true image of $D$. Note that this operation is a projection onto a convex set [2] and can be equivalently written as:

$$\hat{D} = PD = W^T(WW^T)^{-1}WD \tag{3}$$

where $P$ is the matrix that projects onto the column space, $\mathbb{C}_z$, of the matrix $W$. It is shown in Figure 1. It remains to find the vector, $\hat{D}_\perp$, such that:

$$D = \hat{D} + \hat{D}_\perp. \tag{4}$$

The vector, $\hat{D}_\perp$, lies in the orthogonal complement of the column space, $\mathbb{C}_z$. This is the space of all vectors that are orthogonal to each row of the matrix $W$. This space is denoted by $\perp \mathbb{C}_z$ and shown in Figure 1. To estimate the vector, $\hat{D}_\perp$, additional constraints must be imposed on the restored object. One of the constraints that can be imposed is the requirement that the reconstructed image be 'line-like'. The set of image vectors that satisfy the line-like constraint is denoted by $\mathbb{C}_L$ in Figure 1. Note that the desired image, $D$, lies in this set.

An iterative operation needs to be performed to obtain the true image $D$ while imposing the line-like constraint. The procedure is to first project $\hat{D}$ onto the space of line-like image vectors. This is next projected onto the orthogonal complement of the column space of $W$ resulting in $\hat{D}^1_\perp$ using the operator, $\mathcal{P}^\perp = I - P$. Here $I$ is an identity matrix. This vector, $\hat{D}^1_\perp$, is added to $\hat{D}$ to give the revised image estimate, $D^1$. The iteration is repeated to yield $D^2$, $D^3$, etc. Assuming the translated column space (linear variety set) and line-like constraint set intersect only at the desired reconstruction point then, as $n \to \infty$, we expect $\hat{D}^n_\perp \to \hat{D}_\perp$, and $D^n \to D$. This iterative procedure is shown in Figure 2.

There exist numerous methods for projecting onto line-like objects. One of the simpler ways is to compare the intensity of the given image pixel to its surrounding neighbors. If the intensity of the test pixel is greater than every neighborhood pixel then we leave the pixel as is. If the intensity of the test pixels is less than any of its neighborhood pixel, we set the intensity of the given pixel to zero. This procedure has been used earlier by us [1] and shown in Figure 3. Pixel $a$ is set to zero if the intensity of:

$$
\begin{aligned}
&(e \text{ and } b) > a\\
\text{or } &(b \text{ and } c) > a\\
\text{or } &(c \text{ and } d) > a\\
\text{or } &(d \text{ and } e) > a.
\end{aligned}
$$

Otherwise the pixel is left as is. The notation $(e \text{ and } b) > a$ means that the intensity of pixels $e$ is greater than $a$ and the intensity of pixel $b$ is greater than $a$.

**3 Reconstruction and enhancement on the curved surface.** The geometry of two parallel conductors on a parabolic curved surface was used for testing. A half paraboloid in $x$, $y$, $z$ coordinates was generated using the analytical formula of the paraboloid. A paraboloid surface

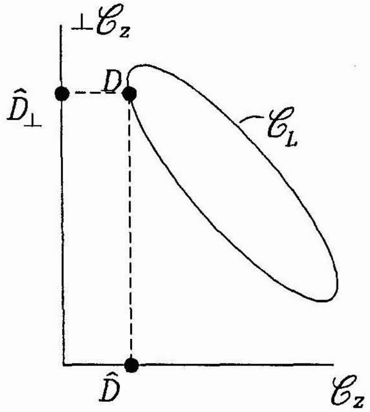


FIGURE 1: The projection of the true object, $D$, is the vector image $\hat{D}$. It is onto the column space, $\mathbb{C}_z$ of the matrix $W$. The true object, $D$, is also known to lie in a set of line-like objects, $\mathbb{C}_L$.

of $51 \times 51$ grid is used. Note the grid spacing is nonuniform. The magnetic field above the conductors in a planar sampling surface of $11 \times 11$ grid was computed by use of Biot-Savart law. Figure 4 (left) shows the paraboloid surface and the conductor layout. Figure 4 (right) shows the magnetic field profile of the component normal to the sampling surface. Positive contours are with solid lines and the negative contours are with dashed lines. A unit current of one ampere was assumed flowing in the conductors. Maximum and minimum values of the magnetic field are $+2.1$ micro tesla ($\mu T$) and $-2.1\mu T$.

The reconstructions were performed on the paraboloid surface with the same $51 \times 51$ grid. Figure 5 shows the reconstruction with the pseudo-inverse method as given by equation 2. The parallel conductor shape is barely visible in this figure. Iterative improvements are shown in Figure 6. Increasing the number of iterations from 10 to 50 shows an improvement in the shape of images. Further iterations from 50 up to 100 do not show any more improvements. The original width of the conductors is recovered and resolution is improved in going from the first 10 to 50 iterations. Some ghost images do persist, slightly displaced with respect to the original definition of the conductors. These could be suppressed by use of a threshold operation but will also reduce the original size of the current distribution.

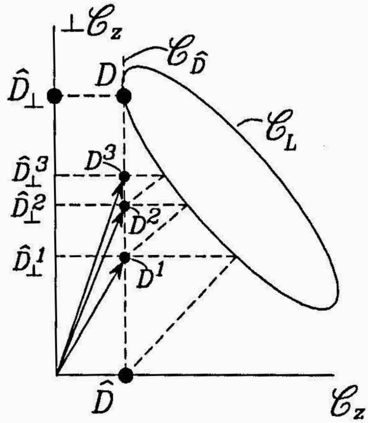


FIGURE 2: Illustration of the iterative improvements by alternating projections to reconstruct the true object $D$.

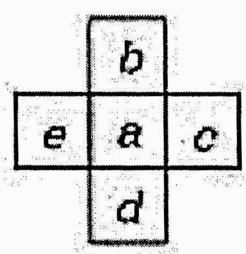


FIGURE 3: Five neighboring pixels in an image. The line-like projection is obtained by comparing the intensity of the pixel $a$ to the intensity of the surrounding pixels $e$, $b$, $c$, and $d$.

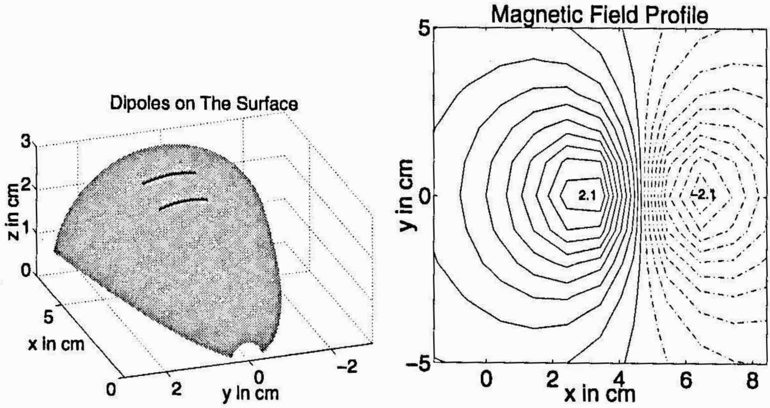


FIGURE 4: (Left) Layout of the conductors and the parabolic surface in Cartesian coordinates, (right) normal component of the magnetic field; positive contours are with solid lines and the negative contours are with dashed lines.

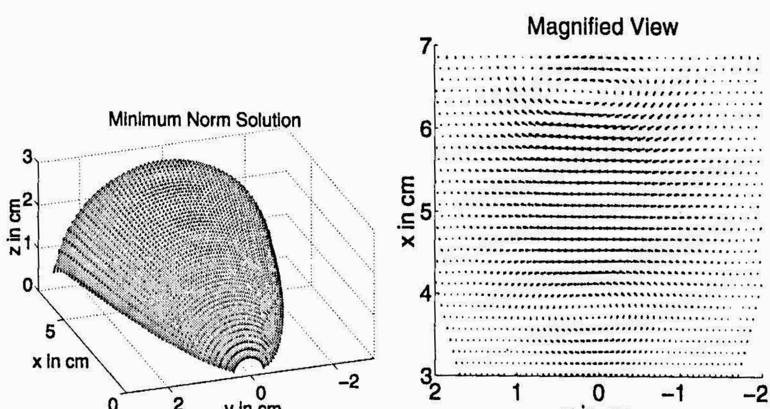


FIGURE 5: (Left) minimum norm reconstruction on the parabolic surface, (right) magnified view. Notice the shape of the conductors is barely recognizable.

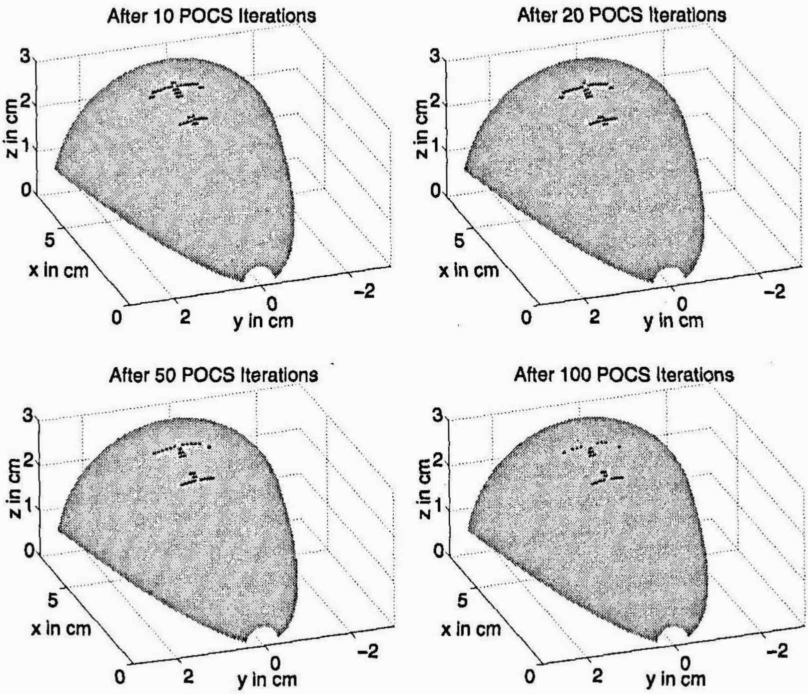


FIGURE 6: Image restoration based on alternating projections after 10, 20, 50 and 100 iterations. Notice gradual improvement in the shape of the conductors from the minimum norm solution.

**4 Discussion.** These are our preliminary results on use of POCS on curved surfaces to restore biomagnetic images. Here we have shown that the reconstruction based on a minimum norm solution provides a barely recognizable shape of the conductors on a curved surface which can be restored by use of alternating projections. Potential application of this technology will be in the reconstruction of the current distribution on the myocardial surface of heart or the brain surface. Other applications, such as improving the resolution of the surfaces in volume visualization, are also possible.

## REFERENCES

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