Editorial

Fuzzy Models—What Are They, and Why?

It is my great pleasure to welcome you to the inaugural issue of the IEEE TRANSACTIONS ON FUZZY SYSTEMS. Many of you are probably quite familiar with the basic ideas underlying fuzzy sets and systems that utilize fuzzy models. However, there will also be readers who are looking at the contents of this issue, wondering what it’s all about. For this latter group, the papers in this first issue may seem bewildering, for they are quite technical, and none are tutorial in nature. Consequently, this preface is divided into two parts. Section I contains a brief introduction to the basic ideas of our science, in hopes that this glimpse will help you understand the articles, but perhaps more importantly, pique your interest in fuzzy models. Section II gives a short précis and historical perspective of the field, and how it has led to the creation of this journal.

I. FUZZY MODELS

Fuzzy sets are a generalization of conventional set theory that were introduced by Zadeh in 1965 as a mathematical way to represent vagueness in everyday life [1]. The basic idea of fuzzy sets is easy to grasp. Suppose, as you approach a red light, you must advise a driving student when to apply the brakes. Would you say, “Begin braking 74 feet from the crosswalk”? Or would your advice be more like, “Apply the brakes pretty soon”? The latter, of course; the former instruction is too precise to be implemented.

Fuzzy interpretations of data structures are a very natural and intuitively plausible way to formulate and solve various problems. Conventional (crisp) sets contain objects that satisfy precise properties required for membership. The set of numbers H from 6 to 8 is crisp; we write H = {r ∈ R | 6 ≤ r ≤ 8}. Equivalently, H is described by its membership (or characteristic, or indicator) function (MF), m H : R → [0,1], defined as

\[ m_H(r) = \begin{cases} 1: & 6 \leq r \leq 8 \\ 0: & \text{otherwise} \end{cases} \]

The crisp set H and the graph of m H are shown in the left half of Fig. 1. Every real number (r) either is in H or is not. Since m H maps all real numbers r ∈ R onto the two points (0,1), crisp sets correspond to two-valued logic: is or isn’t, on or off, black or white, 1 or 0. In logic, values of m H are called truth values with reference to the question, “Is r in H?” The answer is yes if and only if m H(r) = 1; otherwise, no.

Consider next the set F of real numbers that are close to 7. Since the property “close to 7” is fuzzy, there is not a unique membership function for F. Rather, the modeler must decide, based on the potential application and properties desired for F, what m F should be. Properties that might seem plausible for this F include (i) normality (m F(7) = 1), (ii) monotonicity (the closer r is to 7, the closer m F(r) is to 1, and conversely), and (iii) symmetry (numbers equally far left and right of 7 should have equal memberships).

Given these intuitive constraints, either of the functions shown in the right half of Fig. 1 might be a useful representation of F. m F1 is discrete (the staircase graph), while m F2 is continuous but not smooth (the triangle graph). One can easily construct a MF for F so that every number has some positive membership in F, but we would not expect numbers “far from 7,” 20 000 987 for example, to have much! One of the biggest differences between crisp and fuzzy sets is that the former always have unique MF’s, whereas every fuzzy set has an infinite number of MF’s that may represent it. This is at once both a weakness and a strength; uniqueness is sacrificed, but this gives a concomitant gain in terms of flexibility, enabling fuzzy models to be “adjusted” for maximum utility in a given situation.

In conventional set theory, sets of real objects, such as the numbers in H, are equivalent to, and isomorphically described by, a unique membership function such as m H. However, there is no set-theory equivalent of “real objects” corresponding to m F. Fuzzy sets are always (and only) functions, from a “universe of objects,” say X, into [0,1]. This is depicted in Fig. 2, which illustrates that the fuzzy set is the function m F that carries X into [0,1].

As defined, every function m : X → [0,1] is a fuzzy set. While this is true in a formal mathematical sense, many functions that qualify on this ground cannot be suitably interpreted as realizations of a conceptual fuzzy set. In other words, functions that map X into the unit interval may be fuzzy sets, but become fuzzy sets when, and only when, they match some intuitively plausible semantic description of imprecise properties of the objects in X. There are many good texts and monographs that describe various aspects of fuzzy sets and models; for example, interested readers may consult [2]-[14].

One of the first questions asked about this scheme, and the one that is still asked most often, concerns the relationship of fuzziness to probability. Are fuzzy sets just a clever disguise
Fig. 1. Membership functions for hard and fuzzy subsets of $\mathbb{R}$.

Confronted with this pair of bottles and given that you must drink from the one that you choose, which would you choose to drink from first? Most readers familiar with the basic ideas of fuzzy sets, when presented with this experiment, immediately see that while A could contain, say, swamp water, it would not (discounting the possibility of a Machiavellian fuzzy modeler) contain liquids such as hydrochloric acid. That is, a membership of 0.91 means that the contents of A are "fairly similar" to perfectly potable liquids (pure water). On the other hand, the probability that B is potable = 0.91 means that over a long run of experiments, the contents of B are expected to be potable about 91% of the trials; and the other 9%? In these cases the contents will be unsavory (indeed, possibly deadly)—about one chance in ten. Thus most subjects will opt for a chance to drink swamp water, and will choose bottle A.

Another facet of this example concerns the idea of observation. Continuing then, suppose that we examine the contents of A and B, and discover them to be as shown in the right half of Fig. 3; that is, A contains beer, while B contains hydrochloric acid. After observation then, the membership value for A will not be changed, whilst the probability value for B clearly drops from 0.91 to 0.0.

Finally what would be the effect of changing the numerical information in this example? Suppose that the membership and probability values were both 0.5—would this influence your choice? Almost certainly it would. In this case many observers would switch to bottle B, since it offers a 50% chance of being drinkable, whereas a membership value this low would presumably indicate a liquid unsuitable for drinking (this depends, of course, entirely on the MF of the fuzzy set $L$).

In summary, Example 1 shows that these two types of models possess philosophically different kinds of information: fuzzy memberships, which represent similarities of objects to imprecisely defined properties, and probabilities, which convey information about relative frequencies. Moreover, in-
interpretations about and decisions based on these values also depend on the actual numerical magnitudes assigned to particular objects and events. See [15] for an amusing contrary view.

Another common misunderstanding about fuzzy models over the years has been that they were offered as replacements for crisp (or probabilistic) models. To expand on this, first note from Figs. 1 and 2 that every crisp set is fuzzy, but not conversely. Most schemes that use the idea of fuzziness use it in this sense of embedding; that is, we work at preserving the conventional structure, and letting it dominate the output whenever it can, or whenever it must. Another example will illustrate this idea.

**Example 2:** Consider the plight of early mathematicians, who knew that the Taylor series for the real (bell-shaped) function \( f(x) = 1/(1 + x^2) \) was divergent at \( x = \pm 1 \) but could not understand why, especially since \( f \) is differentiable infinitely often at these two points. As is common knowledge for any student of complex variables nowadays, the complex function \( f(z) = 1/(1 + z^2) \) has poles at \( z = \pm i \), two purely imaginary numbers. Thus, the complex function, which is an embedding of its real antecedent, cannot have a convergent power series anywhere on the boundary of the unit disk in the plane; in particular at \( z = \pm 0.5 \pm 1 \), i.e., at the real numbers \( x = \pm 1 \). This exemplifies a general principle in mathematical modeling: given a real (seemingly insoluble) problem; enlarge the space, and look for a solution in some "imaginary" superset of the real problem; finally, specialize the "imaginary" solution to the original real constraints.

In Example 2 we spoke of "complexifying" the function \( f \) by embedding the real numbers in the complex plane, followed by "decomplexification" of the more general result to solve the original problem. Most fuzzy models follow a very similar pattern. Real problems that exhibit nonstatistical uncertainty are first "fuzzified," some type of analysis is done on the larger problem, and then the results are specialized back to the original problem. In Example 2 we might call the return to the real line decomplexifying the function; in fuzzy models, this part of the procedure has come to be known as defuzzification. Defuzzification is usually necessary, of course, because even though we instruct a student to "apply the brakes pretty soon," in fact, the brake pedal must be operated crisply, at some real time. In other words, we cannot admonish a motor to "speed up a little," even if this instruction comes from a fuzzy controller—we must alter its voltage by a specific amount. Thus defuzzification is both natural and necessary. Example 2 illustrates that this is hardly an idea that is novel; instead, we should regard it as a device that is useful.

**Example 3:** As a last, and perhaps more concrete, example about the use of fuzzy models, consider the system shown in Fig. 4, which depicts a simple inverted pendulum free to rotate in the plane of the figure on a pivot attached to the cart. The control problem is to keep the pendulum vertical at all times by applying a restoring force (control signal) \( F(t) \) to the cart at some discrete times \( t \) in response to changes in both the linear and angular position \((x(t), \theta(t))\) and velocity \((\dot{x}(t), \dot{\theta}(t))\) of the pendulum. This problem can be formulated many ways. In one of the simpler versions used in conventional control theory, linearization of the equations of motion results in a model of the system whose stability characteristics are determined by examination of the real parts of the eigenvalues \( \lambda \) of a \( 4 \times 4 \) matrix of system constants. The lower track in Fig. 4 represents this case. It is well known that the pendulum can be stabilized by requiring \( \text{Re}(\lambda) < 0 \), as shown in the middle of the lower track. This procedure is so commonplace in control engineering that most designers don't even think about the use of imaginary numbers to solve real problems, but it is clear that this process is exactly the same as was illustrated in Example 2—a real problem is solved by temporarily passing to a larger, imaginary setting, analyzing the situation in the superset, and then specializing the result to get the desired answer.

The upper track in Fig. 4 depicts an alternative solution to this control problem that is based on fuzzy sets. This approach to stabilization of the pendulum is also well known, and yields a solution that in some ways is much better; e.g., the fuzzy controller is much less sensitive to changes in parameters such as the length and mass of the pendulum [16]. Note again the embedding principle: fuzzify, solve, defuzzify, control.

The point of Example 3? Fuzzy models aren't really that different from more familiar ones. Sometimes they work better, and sometimes not. This is really the only criterion that should be used to judge any model, and there is much evidence nowadays that fuzzy approaches to real problems are often a good alternative to more familiar schemes. This is the point to which our discussion now turns.

II. NOTES ON THE EVOLUTION OF FUZZY MODELS AND THE IEEE TRANSACTIONS ON FUZZY SYSTEMS

Why an IEEE TRANSACTIONS ON FUZZY SYSTEMS? And further, why done under the aegis of the IEEE Neural Networks Council (NNC)? There are several compelling answers. First, the enormous success of commercial applications which are at least partially dependent on fuzzy technologies fielded (in the main) by Japanese companies has led to a surge of curiosity about the utility of fuzzy logic for scientific and engineering applications. Over the last five or ten years fuzzy models have supplanted more conventional technologies in many scientific applications and engineering systems, especially in control systems and pattern recognition. A recent *Newsweek* article indicates that the Japanese now hold thousands of patents on fuzzy devices used in applications as...
diverse as washing machines, TV camcorders, air conditioners, palm-top computers, vacuum cleaners, ship navigators, subway train controllers, and automobile transmissions [17]. It is this wealth of deployed, successful applications of fuzzy technology that is, in the main, responsible for current interest in the subject area.

Since 1965, many authors have generalized various parts of subdisciplines in mathematics, science, and engineering to include fuzzy cases. However, interest in fuzzy models was not really very widespread until their utility in fielded applications became apparent. The reasons for this delay in interest are many, but perhaps the most accurate explanation lies with the salient facts underlying the development of any new technology, which is succinctly captured in Fig. 5.

The horizontal axis if Fig. 5 is time, and the vertical axis is expectation—whose expectation? Well, usually, of the people who pay for development of the technology; but here I encourage you to interpret this axis in a much broader sense, for utility is, of course, in the eye of the user. The crucial part of Fig. 5 is the asymptote of reality, which bounds the delivery of the technology to a much lower expected value than early users project for it. The years shown along the time axis pertain to fuzzy models, and are, of course, approximate at best (with the exception of the initial one). When you look at this figure, you may enjoy deleting these years, and substituting your favorite new technology for the one illustrated. Each technology has its own evolution, and not all of them follow the pattern suggested by Fig. 5 (but you may be surprised to see how many do!). For example, try putting dates and identifying the people and events associated with, say, computational neural networks (CNN’s). There are several reasons for this. The marriage of fuzzy logic with CNN’s has a sound technical basis, because these two approaches generally attack the design of “intelligent” systems from quite different angles. CNN’s are essentially low-level, computational algorithms that (sometimes) offer good performance in dealing with sensor data used in pattern recognition and control. On the other hand, fuzzy logic is a means for representing, manipulating, and utilizing data and information that possess nonstatistical uncertainty. Thus, fuzzy methods often deal with issues such as reasoning on a higher (semantic or linguistic) level than CNN’s. Consequently, the two technologies often complement each other, CNN’s supplying the brute force necessary to accommodate and interpret large amounts of sensor data; and fuzzy logic providing a structural framework that utilizes and exploits these low-level results. There are also many ways to use either technology as a “tool” within the framework.
of a model based on the other. For example, the CNN is well known for its ability to represent functions. The basis of every fuzzy model is the membership function. So, a natural application of CNN’s in fuzzy models is to provide good approximations to the membership functions that are essential to the success of any fuzzy approach. Broadly speaking, then we may characterize efforts at merging these two technologies as (i) fuzzification of conventional CNN architectures and models and (ii) the use of CNN’s as tools in fuzzy models. One need look no further than the September 1992 issue of the IEEE TRANSACTIONS ON NEURAL NETWORKS to see evidence of this marriage, which is a special issue of TNN containing 19 papers on precisely this topic. References [18]-[24] are a sampler of books and articles that articulate or illustrate various aspects of this evolving relationship.

There are several major journals devoted to fuzzy systems: the SOFT Journal (Japan), Fuzzy Sets and Systems (North Holland), and the International Journal of Approximate Reasoning (Elsevier). It is appropriate that the IEEE create a flagship publication in the area of fuzzy systems, to collect and publish the best research on this important new technology. The IEEE TRANSACTIONS ON FUZZY SYSTEMS will publish only the highest quality technical papers in the theory, design, and application of fuzzy sets and systems that use them. Readers are encouraged to submit papers which disclose significant technical knowledge, exploratory developments, and applications of fuzzy systems. Emphasis will be given to engineering systems and scientific applications. The TRANSACTIONS will also contain a letters section, which will include information of current interest, as well as comments and rebuttals submitted in connection with published papers. Representative applications areas include, but are not limited to, the following aspects of fuzzy systems:

1) Fuzzy estimation, prediction, and control
2) Approximate reasoning
3) Intelligent systems design
4) Machine learning
5) Image processing and machine vision
6) Pattern recognition
7) Fuzzy neurocomputing
8) Electronic and photonic implementations
9) Medical computing applications
10) Robotics and motion control
11) Constraint propagation and optimization
12) Civil, chemical, and industrial engineering applications

Well, this has been a much longer preface than most journals ever contain! All that is left to do is thank everyone who has helped make the TRANSACTIONS a reality. First and foremost, that includes the thousands of researchers who have developed the field to a point where this journal is well justified. There are, of course, far too many people who had an active hand in starting TFS for me to recognize each one individually. However, it is appropriate to say that what has been done would have been quite impossible without the able and professional help of the people on (and behind) the various publication boards and NNC committees that helped mold the journal into a reality. A few persons should be specifically mentioned. First, Bob Marks, Pat Simpson, Russ Eberhart, and Toshio Fukuda, who had the vision to lead the IEEE Neural Networks Council toward its decision to sponsor this journal. Second, the associate editors and advisory board, who really did almost all of the hard work in getting this first issue to press, Without them, none of this would have been possible. And, finally, Chris Ralston and the staff at IEEE Publishing Services should be credited for accounting for many of the tedious details that go unnoticed when things work.

I hope you enjoy reading this inaugural issue, and that you find its contents useful and illuminating. Your suggestions on how to make this journal more valuable for the academic, industrial, and governmental communities are both welcome and appreciated—please let me know how we can improve it.

Jim Bezdek, Founding Editor
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REFERENCES

James C. Bezdek (M'80--SM'90--F'92) received a B.S. degree from the University of Nevada, Reno, in 1969 and a Ph.D. from Cornell University in 1973.

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