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Title: **Image Sampling Density Reduction Below That of Nyquist**

Author: **Kwan Fai Cheung**
Advisor: **Robert J. Marks, II**
Granting Institution: University of Washington

The Nyquist sampling density of a bandlimited function is that density corresponding to maximally dense packed spectral replications. The minimum sampling density, on the other hand, is the sampling density where the samples are linearly independent. If there exist gaps among spectral replications, the samples are linearly dependent and the function is oversampled. For 1-D lowpass bandlimited functions, the Nyquist density is the minimum sampling density. This, however, may not be the case for M-D lowpass bandlimited functions. In many cases, the Nyquist density is higher than the minimum density. Whether the two are the same depends upon the shape of the support of the function's spectrum. For example, a 2-D function whose support is circular always has gaps among its spectral replications. Thus, the Nyquist density is higher than the minimum density. However, if the support is rectangular, then the Nyquist density is also the minimum density. Thus, to sample a M-D bandlimited function at the minimum density is not as trivial as its 1-D counterpart. By using a sampling decimation technique, all M-D bandlimited functions can be sampled directly and arbitrarily close to the minimum density. This technique is applicable to any periodic sampling geometry and any spectral support. Indeed, the minimum sampling density is shown to be equal to the hyperarea of the support of the function's spectrum, regardless of the shape of that support. The restoration of the decimated samples and ultimately the whole signal can be performed by linear interpolation. The resulting interpolation noise level varies with the decimation geometry. The greater the clustering of the decimated samples, the higher the noise level. The optimal decimation geometry, which yields the smallest INL, can be located by a Gram-Schmidt type algorithm.

Kwang Cheung, "Image Sampling Density Reduction Below That of Nyquist" (Ph.D. Dissertation Abstract).
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