Closed form bandlimited image extrapolation

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Digital implementations of a closed form 2-D bandlimited image extrapolation algorithm are presented for a number of elementary target images. The resulting figures of merit empirically verify the assumption that extrapolation is better near to where the image is known. Results of extrapolating a truncated image perturbed by white Gaussian noise suggest a required SNR of the order of 10^{10}.

I. Introduction

In this paper, we present empirical results of the closed form 2-D extrapolation theory developed in Ref. 1, Eq. (15). The algorithm is based on the 1-D extrapolation scheme proposed by Gerchberg,[2] and the corresponding closed form discrete algorithm developed by Sabri and Steenaart.[3] Stated simply, the extrapolation problem is to determine an image everywhere given information about the image over only a finite area. *A priori* knowledge of the image's bandwidth is also assumed.

A number of bandlimited target images were created, and a square $(N \times N)$, $N = 34$ matrix of sample values $\hat{u}$ was formed. A smaller $(m \times m)$ square matrix $\hat{u}_T$ was then used as the truncated image. The extrapolation matrices were formed by the matrix inversion technique described in Sec. 7 of Ref. 1 except that the Hilbert transform low pass matrix[4] was utilized in place of $D^{-1}\hat{G}_{\Omega_\eta}D$, $\eta = x,y$.

Each target function was chosen to explore a particular aspect of the algorithm performance.

II. Figures of Merit

It is desirable to have a figure of merit to quantify the goodness of the extrapolation results. Let $\hat{u}_E$ denote the extrapolation result. One obvious merit comparison is

$$\phi = \frac{(\hat{u}|\hat{u}_E)}{\|\hat{u}\|\,\|\hat{u}_E\|}, \tag{1}$$

where for real valued matrices

$$(\hat{u}|\hat{u}_E) \equiv \sum_{i,j=0}^{N} \hat{u}(i,j)\hat{u}_E(i,j), \tag{2}$$

and $\|A\|^2 = (A|A)$. From the Cauchy-Schwartz inequality $|\phi| \leq 1$.

This figure of merit, however, is insensitive to the goodness of the extrapolation near the truncated signal. An obvious alteration is to run the inner product summation from the center of the matrix to a $d \times d$ square, where $d \leq N$. The matrices $\hat{u}$ and $\hat{u}_E$, however, are equivalent to u_T within the $m \times m$ square. We remove this bias and write our final figure of merit as

$$\phi(d) = \frac{(\hat{u}|\hat{u}_E)_{m,d}}{\|\hat{u}\|_{m,d}\,\|\hat{u}_E\|_{m,d}}, \tag{3}$$

where

$$(\hat{u}|\hat{u}_E)_{m,d} = \sum_{(m+1)\times(m+1)\text{square}}^{d\times d\text{ square}} \hat{u}(i,j)\hat{u}_E(i,j),$$

and each square is centered. A value of $\phi(d)$ near unity then dictates a good result. Under the assumption that the extrapolation is better near where the image is known, $\phi(d)$ should be a monotonically decreasing function of d.

III. Implementations

The example implementations to follow are presented in pseudo 3-D plots. The bottom figure in each case corresponds to the target function. The center plot shows the truncated image, and the top plot is the extrapolation result.

A. Example 1

In Fig. 1, the target function is a 2-D sinc function,

$$g(x,y) = \{\sin[2(0.125)(x-17)]\}\{\text{sinc}[2(0.125)(y-17)]\},$$

where $\text{sinc}x = \sin\pi x/\pi x$. The sinc function was found to produce the most accurate reconstruction of any of the functions tried. In this example, the truncation aperture with width $m = 5$ passes less than half of the main lobe of the target sinc. From this the algorithm was able to recreate the target function with very high

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