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Linear coherent optical removal of multiplicative periodic degradations: processor theory

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Abstract. A linear coherent optical technique for restoration of images altered by a multiplicative periodic degradation is considered. By proper choice of a multiplicative grating, restoration can be performed using conventional linear coherent optics techniques. Specific attention is given to the case of continuous sampling in which the image is periodically set to zero in strips.

Keywords: optical pattern recognition; image restoration; optical processing; filters; periodic degradation.

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1. INTRODUCTION

In this paper, we present a linear coherent optical processor capable of restoring images degraded by multiplicative periodic degradations. Nonlinear coherent processors have been used to perform such restorations using homomorphic filtering.[1,2] Here, the degradation is processed after a logarithm has been taken. This transforms the noise from multiplicative to additive.

The coherent processor described herein is applicable to a large number of periodic degradations—even when identically zero over some subinterval of the period. This latter case will be considered as a specific example. The noise sensitivity of the restoration also will be addressed.

2. PRELIMINARIES

Our development will be for one-dimensional periodic degradations, although the restoration algorithm and its corresponding optical implementation are straightforwardly generalized to two dimensions. Let p(x) denote a periodic function with unit period and Fourier series,

$$p(x) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi nx) \ , \qquad (1)$$

where

$$c_n = \int_{-\infty}^{\infty} p(x)\,\mathrm{rect}(x)\exp(-j2\pi nx)\,dx \ , \qquad (2)$$

and rect(y) is unity for $|y| < ½$ and zero otherwise.

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Let f(x,y) denote an image. For a given period T, we define the degradation

$$g(x,y) = f(x,y)\,p(x/T) \ . \qquad (3)$$

The image is assumed to be band limited in x with maximum spatial frequency W. That is,

$$f(x,y) = \int_{-W}^{W} \hat{F}(u,y)\exp(j2\pi ux)\,du \ , \qquad (4)$$

where

$$\hat{F}(u,y) = \mathscr{F}_x f(x,y)$$

$$= \int_{-\infty}^{\infty} f(x,y)\exp(-j2\pi ux)\,dx \ , \qquad (5)$$

with $\mathscr{F}_x$ denoting the (one-dimensional) Fourier transform operator in x.

The spectrum of g(x,y) is

$$G(u,v) = F(u,v) * TP(Tu) \ , \qquad (6)$$

where $*$ denotes convolution with respect to u, $P(u) = \mathscr{F}_x p(x)$, and $F(u,v) = \mathscr{F}_y \hat{F}(u,y)$. From Eq. (1), we have

$$P(u) = \sum_{n=-\infty}^{\infty} c_n \delta(u-n) \ . \qquad (7)$$

Thus, Eq. (6) can be written as

$$G(u,v) = \sum_{n=-\infty}^{\infty} c_n F\left(u - \frac{n}{T}, v\right) \ . \qquad (8)$$

Thus, the net effect of a multiplicative periodic degradation is replication of spectra in the frequency domain. The weight of the *n*th replicated spectrum is equal to the *n*th coefficient of the Fourier series of p(x).

3. RESTORATION METHODOLOGY

The coherent optical generation of the spectrum in Eq. (8) is shown in Fig. 1. In Fig. 1(a), an amplitude transmittance f(x, y) is Fourier transformed by a thin lens with focal length f. The field amplitude in the back focal plane is then proportional to F(u, v), where

$$(u,v) = \left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right), \tag{9}$$

with λ being the wavelength of the unit amplitude coherent plane wave illumination.[3]

We can conceptually construct g(x, y) in Eq. (3) by placing amplitude transmittances of f(x, y) and p(x/T) back to back. As shown in Fig. 1(b), the corresponding spectrum is replicated in accordance with Eq. (8). The cross section of this spectrum is shown in Fig. 2. The component spectra are separated by 1/T. Note that if $1/T > 2W$, the spectra do not overlap. Thus, f(x, y) can be regained by isolating the middle spectra and inverse transforming. This can be done optically, as shown in Fig. 1(b), by masking out all but the zero*th*-order spectrum in the (u, v) plane, followed by Fourier transforming with a second lens.

We are interested in the case in which the spectra overlap, i.e., where the data are aliased. In Fig. 2, we have first-order aliasing since only the spectra labeled c_1 and c_{-1} overlap the c_0 spectrum. If T is large enough so that, in addition, the c_2 and c_{-2} labeled spectra also overlap, we have second-order aliasing, etc. In general, the order of aliasing is

$$M = \langle 2WT \rangle , \tag{10}$$

where $\langle a \rangle$ denotes the greatest integer not exceeding a.

How can we extract the zero*th*-order spectrum from the overlapping spectra? In many cases, this can be done by appropriately replicating the replications. If the coefficients of the superimposed spectra are chosen correctly, the spectra overlapping the zero*th*-order spectrum can be totally canceled.

The replicated spectrum can be replicated as shown in Fig. 1(c), where, back to back with our degradation g(x, y), we have placed a 1-D grating, with period T and amplitude $\theta_M(x/T)$. The Fourier series of the grating is

$$\theta_M(x) = \sum_{m=-M}^{M} b_m \exp(j2\pi mx) , \tag{11}$$

where the coefficients have yet to be specified and, as we shall see shortly, 2M + 1 terms suffice. Since the processor is linear, a grating transmittance of $C\theta_M(x)$ is also acceptable. The processor output is then simply multiplied by the constant C. The constant should be chosen such that the grating is passive, i.e., has a transmittance that everywhere lies within the unit circle on the complex plane. This can always be done if all the b_m coefficients are finite. C can also be chosen to assure maximum dynamic range for the passive grating.

Proceeding in a manner similar to that used before, the product

$$h(x,y) = g(x,y)\,\theta_M(x/T) \tag{12}$$

has a 2-D spectrum of

$$H(u,v) = \sum_{m=-M}^{M} b_m G\left(u - \frac{m}{T}, v\right)$$

$$= \sum_{m=-M}^{M} b_m \sum_{n=-\infty}^{\infty} c_n F\left(u - \frac{n+m}{T}, v\right) , \tag{13}$$

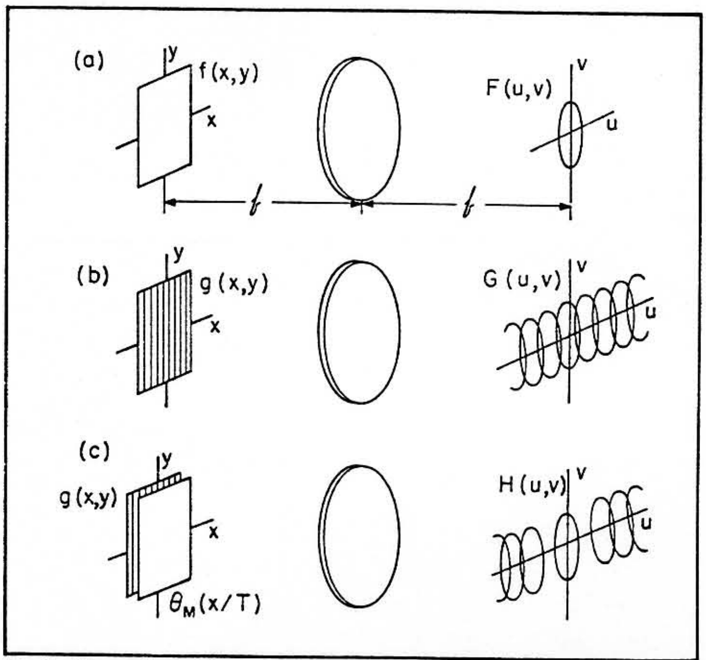

Fig. 1. Illustration of the restoration methodology: (a) an amplitude transmittance and its spectrum, (b) the duplicated spectrum formed by multiplying the amplitude transmittance by p(x/T) to form the degradation g(x, y) = f(x, y) p(x/T), and (c) isolation of the zero*th*-order spectrum by replicating the replications with the grating $\theta_M(x/T)$.

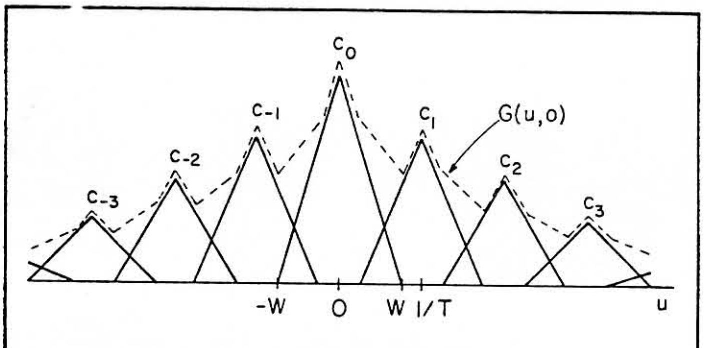

Fig. 2. Cross section of G(u, v) [from Fig. 1(b)]. The spectrum F(u, 0) is triangular. The replicated spectra overlap in first-order aliasing.

where, in the second step, we have used Eq. (8) and reversed summation orders. Setting p = n + m in the n sum gives

$$H(u,v) = \sum_{p=-\infty}^{\infty} \left(\sum_{m=-M}^{M} b_m c_{p-m} \right) F\left(u - \frac{p}{T}, v\right) . \tag{14}$$

We desire to isolate the zero*th*-order spectrum. Thus, we require that

$$H(u,v)\,\mathrm{rect}(u/2W) = F(u,v) . \tag{15}$$

This occurs in Eq. (14) when

$$\sum_{m=-M}^{M} b_m c_{p-m} = \delta_p , \qquad -M \le p \le M , \tag{16}$$

where δ_p, the Kronecker delta, is one for p = 0 and zero otherwise. The b_m are found through solution of these 2M + 1 linear equations

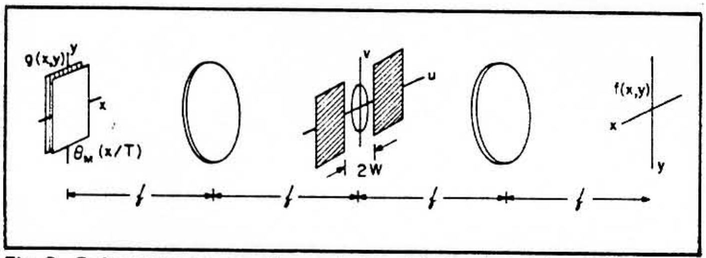

Fig. 3. Coherent processor for restoring objects altered by multiplicative periodic degradations.

[assuming that Eq. (16) is not singular]. Once isolated, we can block the nonoverlapping spectra with a 1-D low-pass filter slit and re-image, as shown in Fig. 3.

4. EXAMPLE: CONTINUOUS SAMPLING

As an example application, consider the case in which the periodic degradation is of the form of a Ronchi ruling transmittance with duty cycle α:

$$p(x) = \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{x-n}{\alpha}\right) . \tag{17}$$

The image is thus known within αT-width strips placed T units apart and otherwise is set to zero. Restoration here is thus an interpolation problem. This specific case of the restoration algorithm has been considered elsewhere in more depth.[4,5] Consider first-order aliasing for which Eq. (11) becomes

$$\theta_1(x) = b_0\left[1 + a(\alpha)\cos(2\pi x)\right] , \tag{18}$$

where we have recognized that $b_1 = b_{-1}$ and

$$a(\alpha) = \frac{2b_1}{b_0} . \tag{19}$$

If $\theta_1(x)$ has the same sign for all x, the grating is not bipolar. This is equivalent to requiring that

$$|a(\alpha)| \leq 1 . \tag{20}$$

Solving Eq. (16), we find that

$$a(\alpha) = \frac{-2c_1}{c_0 + c_2} , \tag{21}$$

where $c_n = \alpha \operatorname{sinc} \alpha n$. A plot of $|a(\alpha)|$ is shown in Fig. 4. Equation (20) is satisfied only for duty cycles in excess of $\alpha = 0.68$. Otherwise, a bipolar grating is needed, i.e., one that is holographically recorded.

5. NOTES

Of paramount importance is the sensitivity of our restoration technique to data perturbations. A detailed analysis has been made for

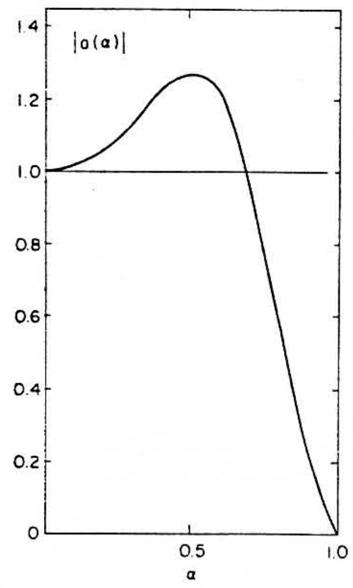

Fig. 4. Function used for determining the polarity status for restoring continuously sampled objects. For $\alpha > 0.68$, the sinusoidal amplitude grating is positive and real. Otherwise, a bipolar grating is required.

the case of continuous sampling.[5] The performance of the algorithm, in general, will improve with decreased order of aliasing and degrade as the condition of the $M \times M$ matrix of c coefficients corresponding to Eq. (16) increases.[6] For example, if

$$p(x) = \sum_{n=-\infty}^{\infty} \delta(x-n) , \tag{22}$$

then, for $M > 0$, Eq. (16) has no solution. Indeed, here we would be trying to recover aliased spectra in the classical Whittaker-Shannon sampling theorem.[3,7] The corresponding c_p are all equal. Thus, the matrix of c's in Eq. (16) is singular, and restoration is not possible.

6. REFERENCES

1. S. H. Lee, *Optical Processing Fundamentals*, p. 263, Springer-Verlag, Berlin (1981).
2. F. T. S. Yu, *Optical Information Processing*, p. 235, Wiley, New York (1983).
3. J. Gaskill, *Linear Systems, Fourier Transforms and Optics*, Wiley, New York (1978).
4. R. J. Marks II, IEEE Trans. Acoust., Speech and Signal Processing ASSP-30, 937 (1982).
5. R. J. Marks II and D. Kaplan, J. Opt. Soc. Am. 73, 1518 (1983).
6. T. K. Sarker, D. D. Wiener, and V. K. Jain, IEEE Trans. Antennas Propag. AP-29, 373 (1981).
7. A. Jerri, Proc. IEEE 65, 1565 (1977).