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A spherical dose model for radiosurgery plan optimization

Paul S Cho†, H Gorkem Kuterdem and Robert J. Marks II‡

† Department of Radiation Oncology, University of Washington, Seattle, WA 98 1956043, USA
‡ Department of Electrical Engineering, University of Washington, Seattle, WA 98 195-2500, USA

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Abstract. Conventional 3D dose calculations for stereotactic radiosurgery involve integration of individual static beams comprising a set of arcs. For iterative optimization of multiple isocentre treatment, which requires repetitive dose calculations at a large number of sample points, the conventional method is too slow. To overcome this problem spherically symmetric dose distributions are assumed. The authors describe a spherical dose model derived from a parametrized convolution of the collimator width and a dose spread kernel. The method is fast and easy to implement requiring just a single empirically derived value. Furthermore, the model is differentiable with respect to the parameters to be optimized. This property is useful when the optimization strategies rely on gradient information.

1. Introduction

Spherical dose approximation is often used as the basic building block in radiosurgery plan optimization involving multiple isocentres. In practice, spherically symmetric dose distributions can be generated by a set of non-coplanar arcs. The resultant dose distribution is relatively insensitive to the location of the isocentre and the number of arcs, if more than four arcs are used (Pike et *al* 1990, Schell *et al* 1990, Suh *et al* 1996). For irregularly shaped target volumes, multiple dose spheres must be superimposed in such a way as to achieve an integral dose distribution that meets the prescription. The variables to be optimized are the location and the size of the individual dose spheres.

To speed up the iterative optimization process which requires repetitive dose calculations, spherical dose models have been proposed. Suh *et al* (1996) used a five-coefficient fitting function based on the Cunningham model (Johns and Cunningham 1983). Treuer *et al* (1998) suggested a simplified empirical method based on off-axis ratios. In this paper we describe a fast, spherical dose model that utilizes a collimator-invariant dose spread parameter and is continuously differentiable with respect to the variables to be optimized. The latter property is useful when gradient information is required by the optimization algorithm. The present work is concerned only with the mathematical model of the spherical dose distribution. The model can be applied to the general problem of radiosurgery optimization utilizing various strategies and objective functions.

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2. Theory

The radial dose distribution at distance x from the isocentre of a dose sphere can be modelled by a parametrized convolution

$$g(x) = \Pi\left(\frac{x}{2r}\right) * h(x) \tag{1}$$

where

- the parameter r is a measure of the radius of the sphere
- $\Pi(z)$ is equal to one for $|z| \leqslant 1/2$ and is otherwise zero
- *h(x)* is a dose spread convolution kernel and
- the asterisk denotes convolution

$$g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\xi)h(x-\xi)\,\mathrm{d}\xi. \tag{2}$$

Since

$$\frac{\mathrm{d}g(x)}{\mathrm{d}x} = \frac{\mathrm{d}f(x)}{\mathrm{d}x} * h(x) \tag{3}$$

it follows from equation (1) that

$$\frac{\mathrm{d}g(x)}{\mathrm{d}x} = \left[\frac{d}{\mathrm{d}x}\Pi\left(\frac{x}{2r}\right)\right] * h(x) = [\delta(x+r) - \delta(x-r)] * h(x) = h(x+r) - h(x-r) \tag{4}$$

where 6(x) is the Dirac delta impulse. Therefore, the convolution kernel *h(x)* can be readily determined by simply taking the derivatives of the measured dose profile.

It is convenient to express the dose spread kernel in an analytic form. Assuming a Gaussian fit we write

$$h(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right). \tag{5}$$

Integrating equation (4) the radial dose distribution is given by

$$g(x) = \frac{1}{2}\left[\mathrm{erf}\left(\frac{x+r}{\sigma}\right) - \mathrm{erf}\left(\frac{x-r}{\sigma}\right)\right] \tag{6}$$

where erf denotes error function.

Let $\boldsymbol{x}$ with components (xi, $x_2, x_3\}$ define the three dimensional space and let

$$||\boldsymbol{x}|| = \sqrt{x_1^2 + x_2^2 + x_3^2}. \tag{7}$$

The equation for a dose sphere of radius r centred at the origin follows as

$$g(||\boldsymbol{x}||) = \frac{1}{2}\left[\mathrm{erf}\left(\frac{||\boldsymbol{x}|| + r}{\sigma}\right) - \mathrm{erf}\left(\frac{||\boldsymbol{x}|| - r}{\sigma}\right)\right]. \tag{8}$$

Similarly, a dose sphere centred at an isocentre c with radius r is

$$b(||\boldsymbol{s}||, r) = \frac{1}{2}\left[\mathrm{erf}\left(\frac{||\boldsymbol{s}|| + r}{\sigma}\right) - \mathrm{erf}\left(\frac{||\boldsymbol{s}|| - r}{\sigma}\right)\right] \tag{9}$$

where

$$\boldsymbol{s} = \boldsymbol{x} - \boldsymbol{c}. \tag{10}$$

If N dose spheres are superimposed the nth of which has weight w_n, radius r_n, and is centred at $\boldsymbol{c}_n$, the total delivered dose is

$$d(\boldsymbol{x}) = \sum_{n=1}^{N} w_n b(||\boldsymbol{c}||_n, r_n) \tag{11}$$

3. Results and discussion

For the purpose of iterative optimization, the dose distribution from multiple arcs can be estimated with sufficient accuracy by assuming radial symmetry of the dose profile of a stationary beam. This approximation permits the use of a collimator-independent σ, as shown below. The idea is to perform optimization of beam parameters using the approximate model and then calculate the final dose distribution with a more precise dose computation algorithm.

The convolution kernel $h(x)$ was calculated by equation (4) using a measured dataset $g(x)$ for a static 6 MV x-ray beam with a 10 mm radius collimator. The dose spread parameter σ was determined by fitting $h(x)$ to a Gaussian. Using this σ value of 2.872 mm and equation (9), the relative radial dose was calculated for collimator radii of 5, 10, and 15 mm. The results are plotted in figure 1 along with the measured data. The agreement with the measurements over the clinical range of collimator sizes is good. No appreciable improvement in accuracy was made when collimator-size-dependent σ values were used. If, however, it is desirable to use collimator-dependent σ, equation (9) can still be used by exchanging σ with σ_r.

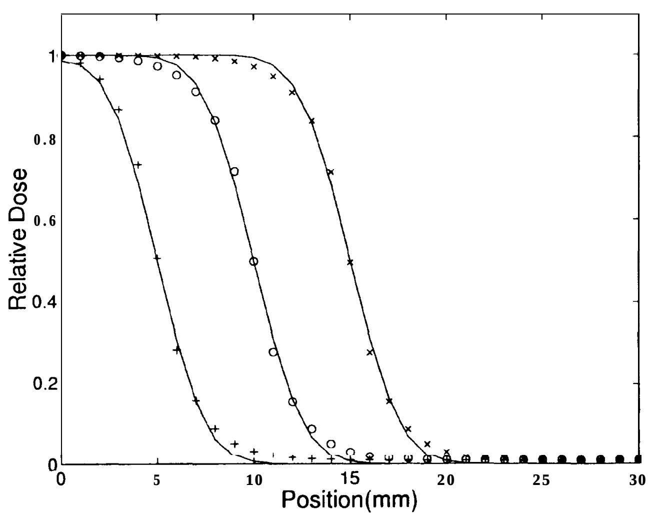

Figure 1. Relative dose as a function of radial distance for collimator radii of 5 (+), 10 (○) and 15 (x) mm. The calculated values are plotted as full curves and the measurements as data points.

The spherical dose model is simple to implement requiring only several lines of computer code. Evaluation of error function can be performed by using a published algorithm such as ACM Algorithm 209 (ACM 1980) which computes

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \mathrm{e}^{-u^2/2}\,\mathrm{d}u = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \int_{0}^{z} \mathrm{e}^{-u^2/2}\,\mathrm{d}u = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{z}{\sqrt{2}}\right)\right]. \tag{12}$$

Therefore, using Algorithm 209, the error function is obtained by

$$\mathrm{erf}(z) = 2F(\sqrt{2}z) - 1. \tag{13}$$

In our numerical implementation, the error functions were precomputed and stored in an array. Calculation time for 10^6 dose points was 0.306 s on a DEC Alpha Station 500/266MHz (Digital Equipment Corporation, Maynard, MA).

For optimization techniques that require gradient information, the partial derivatives of the spherical dose model are straightforwardly calculated as

$$\frac{\partial d(\boldsymbol{x})}{\partial w_n} = b(||\boldsymbol{c}||_n, r_n) \tag{14}$$

$$\frac{\partial d(\boldsymbol{x})}{\partial r_n} = \frac{w_n}{\sqrt{2\pi}\sigma}\left[\exp\left(-\frac{||\boldsymbol{s}||_n + r_n}{2\sigma^2}\right) + \exp\left(-\frac{||\boldsymbol{s}||_n - r_n}{2\sigma^2}\right)\right] \tag{15}$$

and

$$\frac{\partial d(\boldsymbol{x})}{\partial c_n[m]} = \frac{w_n}{\sqrt{2\pi}\sigma}\frac{\partial ||\boldsymbol{s}||_n}{\partial c_n[m]}\left[\exp\left(-\frac{||\boldsymbol{s}||_n + r_n}{2\sigma^2}\right) - \exp\left(-\frac{||\boldsymbol{s}||_n - r_n}{2\sigma^2}\right)\right] \quad m = 1,2,3 \tag{16}$$

where

$$\frac{\partial ||\boldsymbol{s}||_n}{\partial c_n[m]} = -\frac{x_m - c_n[m]}{||\boldsymbol{s}||}. \tag{17}$$

In summary, a spherical dose model for radiosurgery application was presented. The model requires just a single empirically derived value, the dose spread parameter σ. Its computational simplicity and speed make it a useful tool for iterative optimization routines. The model is differentiable with respect to optimization variables and can benefit optimization strategies that rely on gradient information.

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