Minimum Power Broadcast Trees for Wireless Networks: Integer Programming Formulations

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Abstract— Wireless multicast/broadcast sessions, unlike wired networks, inherently reaches several nodes with a single transmission. For omnidirectional wireless broadcast to a node, all nodes closer will also be reached. Heuristic algorithms for constructing the minimum power tree in wireless networks have been proposed by Wieselthier *et al.* and Stojmenovic *et al.* Recently, an evolutionary search procedure has been proposed by Marks *et al.* In this paper, we present three different integer programming models which can be used for an optimal solution of the minimum power broadcast/multicast problem in wireless networks. The models assume complete knowledge of the distance matrix and is therefore most suited for networks where the locations of the nodes are fixed.

I. INTRODUCTION

For a given node constellation with an identified source node, the minimum power broadcast (MPB) problem is to communicate to all remaining nodes, either directly or hopping, such that the overall transmission power is minimized. We assume that no power expenditure is involved in signal reception/processing activities. Unlike wired networks, where a transmission $i \rightarrow j$ reaches only node j, it is possible to reach several nodes by a single transmission in wireless networks. If all nodes have omnidirectional antennas, nodes which are closer to i than j will also receive the transmission directed to j. This is the wireless advantage property [1].

To the best of our knowledge, a couple of heuristic procedures have been suggested so far for solving the MPB problem in wireless networks. Wieselthier, Nguyen and Ephremides [1] proposed the broadcast incremental power (BIP) algorithm for constructing the minimum-power tree for wireless networks. In this algorithm, new nodes are added to the tree on a minimum incremental cost basis, until all intended destination nodes are included. An internal nodes based broadcasting procedure was suggested by Stojmenovic, Seddigh and Zunic [5]. Recently, an evolutionary approach using genetic algorithms has been proposed by Marks, Das, El-Sharkawi, Arabshahi and Gray [3]. Methods for generating initial solutions and checking the viability of evolved solutions are described in [3].

While the performances of the above procedures can certainly be compared among themselves, in the absence of any optimal solution procedure, it has not been possible to judge the quality of the solutions with respect to the optimal. This paper attempts to fill that void by proposing three different integer programming (IP) models that can be solved by any standard IP technique, e.g., linear programming (LP) based branch-and-bound. All the models discussed in this paper assume complete knowledge of pairwise distances between the nodes.

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II. NETWORK MODEL

We assume a fixed N-node network with a specified source node which has to broadcast a message to all other nodes in the network. Any node can be used as a relay node to reach other nodes in the network. Nodes that receive a transmission but do not retransmit it are classified as leaf nodes. Nodes that transmit, including the source node, are called hop nodes. The remaining nodes are unconnected. Clearly, in a broadcast application, there cannot be any unconnected nodes in a connection tree. In a multicast (source-to-many) application, we assume that it is possible to use non-destination nodes as a hop node to relay information to a destination node(s).

For a transmission from node i to j, separated by a distance r_{ij}, the transmitter power at node i is modeled to be proportional to r_{ij}^{α} where α is the channel loss exponent (typically between 2 and 4, depending on the channel medium). Without any loss of generality, we can set the proportionality constant to one, so that the transmitter power, p_T, at node i is given by:

$$p_T = r_{ij}^{\alpha} \tag{1}$$

III. MPB vs. TSP

In this section, we explain the similarities and the differences between the MPB problem in wireless networks and the traveling salesman problem (TSP). Given a set of N cities and a cost c_{ij} of moving from city i to city j $(1 \leq i \neq j \leq N)$, the TSP attempts to find a minimum cost tour of the cities, subject to the following constraints:

- Constraint (1): Departing from his home base, the salesman must visit each city exactly once.
- Constraint (2): After visiting a city, the salesman must leave for another city.
- Constraint (3): The salesman must return to his home base.

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- Constraint (4): No subtours (i.e., cycles not including the home base) are allowed.

A variation of this problem is the open tour (OT) case, where the salesman need not return to his home base (constraint (3) relaxed) after visiting all cities. Also, constraint (4) is modified so that "there are no cycles in the optimal solution". We will refer to this variation of the TSP as the OT-TSP.

The OT-TSP is closely related to the minimum spanning tree (MST) problem. Given an undirected graph $G = (V, E)$, where V is the set of vertices and E is the set of edges, the MST problem seeks to find the tree spanning G such that the total edge weight is minimum. Explained in the context of the traveling salesman, solving the TSP without constraints (2) and (3) and with a modified constraint (4) as explained above yields the MST. Note that relaxing constraint (2) implies:

- It is not necessary for the salesman to make a trip out of every city.
- Multiple trips may be made out of a city.

The MPB problem in wireless networks can be viewed from the perspective of the OT-TSP as well as the MST. We first examine the MPB problem from the context of the OT-TSP. Section IV details the similarities and the differences between the MPB and the MST problems.

If the rules of the OT-TSP are modified such that:

- if the salesman actually visits city j from city i, he can claim to have also implicitly visited all cities within the circle centered at i and radius r_{ij}. Note that while actual visitations incur a cost, implicit visitations are free. Figure 1 illustrates the actually and implicitly visited nodes associated with the transmission $i \rightarrow j$. The solid line indicates an actual transmission while the dashed lines indicate implicit transmissions.
- the salesman must have actually or implicitly visited city i before he can make a trip out of city i.
- the salesman can make at most one trip out of city i.
- departing from his home base, the salesman has to visit all cities, actually or implicitly.

we have the wireless MPB problem in wireless networks. This interpretation of the MPB problem will be used to develop an IP model of the MPB problem in Section VI. In a network context, the salesmans implicit visitation privileges are a consequence of the wireless nature of the network.

An important difference between OT-TSP and the above interpretation of MPB is that, while each city in OT-TSP has to be visited exactly once, the optimal solution in MPB can involve multiple implicit visitations to a node since no cost is incurred due to such visitations. Referring to the 5-node network in Figure 2, suppose the optimal MPB tree is $\{4 \rightarrow 2,\ 3 \rightarrow 5\}$. Note that nodes 1 and 3 are closer to node 4 than node 2 and nodes 1, 2 and 4 are closer to node 3 than node 5. If the source (node 4) uses this tree to communicate with other nodes in the network, nodes 1 and 2 will receive the transmission twice; implicitly in both cases for node 1 but once actually and once implicitly for node 2.

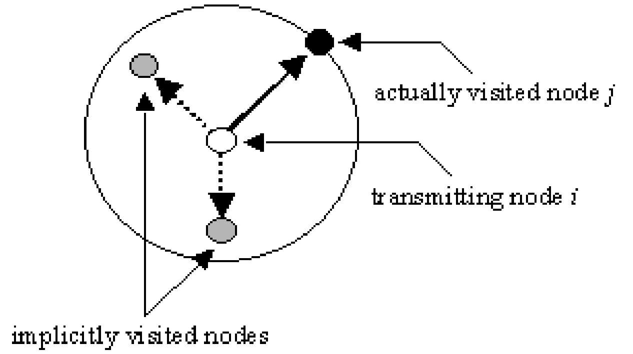

Fig. 1. Illustration of actually visited nodes and implicitly visited nodes in wireless networks.

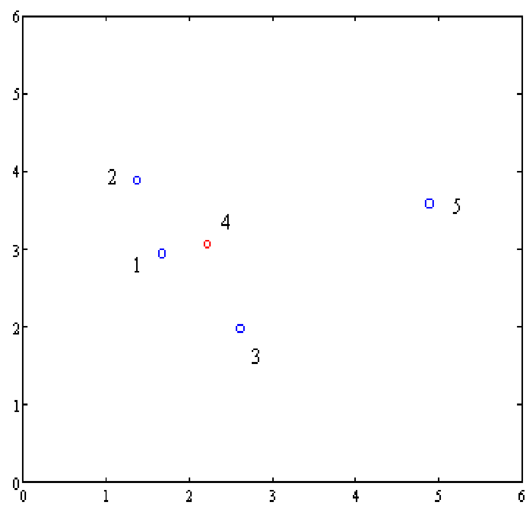

Fig. 2. An example 5-node network.

IV. ALTERNATE VIEW OF IMPLICIT VISITATION

From the traveling salesman aspect, implicit visitations can be alternately interpreted as the salesman being allowed to make any number of actual trips (note the similarity here with the MST problem) out of a city, with the condition that the cost he incurs is the maximum of the individual costs of the trips he makes out of the city. We will illustrate with an example.

In Figure 3, the solid lines indicate the costliest paths out of any city. Suppose the optimal MPB solution for the above network is: $\{3 \rightarrow 4, 4 \rightarrow 6, 6 \rightarrow 8, 5 \rightarrow 7\}$. This solution is interpreted as follows:

1) the salesman makes three actual trips out of city 3, to cities 1, 2 and 4. Charged only for the trip to 4.
2) makes two actual trips out of city 4, to cities 5 and 6. Charged only for the trip to 6.
3) makes one actual trip out of city 5, to city 7. Charged for the trip.
4) makes one actual trip out of city 6, to city 8. Charged for the trip.
5) makes no trips out of cities 1, 2, 7 and 8.

The difference between the MST problem and the MPB problem in wireless networks is now evident. Let C_{ij} be the cost of the arc (i, j) and X_{ij} be a binary variable such

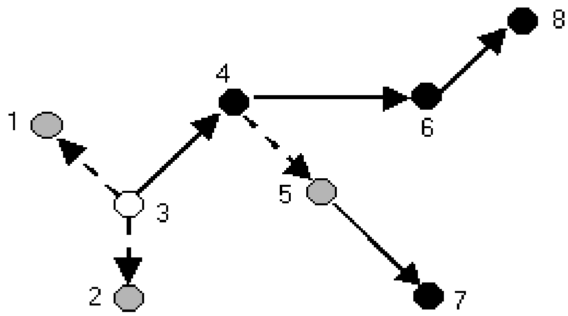

Fig. 3. An example 8-node network to illustrate alternate view of implicit visitation

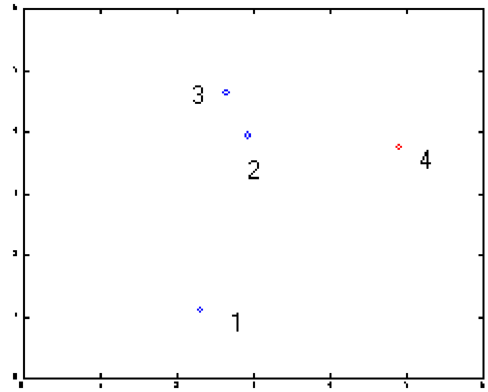

Fig. 4. Example 4-node network: node 4 is the source.

that it is equal to 1 if the edge (i, j) is used in the final solution and 0 otherwise. The objective functions for the MST and the MPB can then be written as follows:

$$\textbf{MST}: \ minimize \sum_i \sum_j C_{ij} X_{ij}; \ i \neq j \tag{2}$$

$$\textbf{MPB}: \ minimize \sum_i max_j \left(C_{ij} X_{ij} \right); \ i \neq j \tag{3}$$

It follows from (2) and (3) that the MST of a wired network is not necessarily the MPB solution if the same network is assumed to be wireless. Equations (2) and (3) also imply that the cost of the MPB solution for wireless networks can be no worse than the cost of the MST solution. The ideas discussed in this section will be used to develop an alternate IP model of the MPB problem in Section VII.

V. TERMINOLOGY

Before discussing the IP models for the MPB problem, we offer the following definitions.

A. *Power Matrix*

For an N-node network, the power matrix, $\mathbf{P}$, is an $N \times N$ matrix. The $(i, j)th$ element of the power matrix defines the power required for node i to transmit to node j and is given by:

$$\mathbf{P}_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{\alpha/2} = r_{ij}^{\alpha} \tag{4}$$

where $\{(x_i, y_i) : 1 \leq i \leq N\}$ are the coordinates of the nodes in the network, α is the channel loss exponent and r_{ij} is the Euclidean distance between nodes i and j. For example, the power matrix of the network in Figure 4, assuming $\alpha = 2$, is:

$$\mathbf{P} = \begin{bmatrix} 0 & 8.4645 & 12.5538 & 13.6351 \\ 8.4645 & 0 & 0.5470 & 3.8732 \\ 12.5538 & 0.5470 & 0 & 5.7910 \\ 13.6351 & 3.8732 & 5.7910 & 0 \end{bmatrix} \tag{5}$$

B. *Reward Matrix*

Each transmission in a wireless network will result in one or more nodes being reached. The reward matrix, $\mathbf{R}$, of a network is an N-element binary encoding of all the nodes covered (or not covered) by all possible transmissions in the network. In MATLAB© notation, $\mathbf{R}$ is a cell array, each cell being an N-element vector. We will use the notation $\mathbf{R}_{mn}(p)$ to index the pth element of the (m, n) cell in $\mathbf{R}$.

The reward matrix is computed as follows:

$$\mathbf{R}_{mn}(p) = \begin{cases} 1, & \text{if } \mathbf{P}_{mp} \leq \mathbf{P}_{mn} \\ 0, & \text{otherwise} \end{cases}$$

For example, referring to Figure 4, the transmission $2 \rightarrow 4$ will result in nodes 3 and 4 being covered. This information is encoded in the (2,4) cell of the reward matrix as: $\mathbf{R}_{24} = [0\ 0\ 1\ 1]$. The reward matrix of the wireless network in Figure 4 is:

$$\mathbf{R} = \begin{bmatrix} [0\ 0\ 0\ 0] & [0\ 1\ 0\ 0] & [0\ 1\ 1\ 0] & [0\ 1\ 1\ 1] \\ [1\ 0\ 1\ 1] & [0\ 0\ 0\ 0] & [0\ 0\ 1\ 0] & [0\ 0\ 1\ 1] \\ [1\ 1\ 0\ 1] & [0\ 1\ 0\ 0] & [0\ 0\ 0\ 0] & [0\ 1\ 0\ 1] \\ [1\ 1\ 1\ 0] & [0\ 1\ 0\ 0] & [0\ 1\ 1\ 0] & [0\ 0\ 0\ 0] \end{bmatrix} \tag{6}$$

Note that the reward matrix is not necessarily "symmetric"; *i.e.*, the vector $\mathbf{R}_{mn}$ is not necessarily equal to $\mathbf{R}_{nm}$. For example, referring to (6), while the transmission $1 \rightarrow 3$ reaches nodes 2 and 3 ($\mathbf{R}_{13}$), the transmission $3 \rightarrow 1$ reaches nodes 1, 2 and 4 ($\mathbf{R}_{31}$).

VI. IP FORMULATION 'A'

Referring to the 4-node network in Figure 4, let $\{Y_i : 1 \leq i \leq 4\}$ be the transmitter power levels at the 4 nodes (continuous variables) and $\{X_{ij} : 1 \leq i \neq j \leq 4\}$ be binary variables such that $X_{ij} = 1$ if the transmission $i \rightarrow j$ is used in the final solution and 0 otherwise.

The objective function is therefore:

$$minimize \sum_{i=1}^{4} Y_i \tag{7}$$

The first set of constraints defines the relations between the continuous variables Y_i and the binary variables X_{ij}. These are:

$$Y_i - \sum_{j=1}^{4} \mathbf{P}_{ij} X_{ij} = 0; \quad i \neq j, 1 \leq i \leq 4 \tag{8}$$

where $\mathbf{P}_{ij}$ is the $(i,j)th$ element of the power matrix $\mathbf{P}$.

In the wireless MPB problem, only the source node is required to transmit exactly once. Other nodes may or may not transmit. However, if a node does transmit, it can do so once. These conditions are expressed using the following constraints.

$$\begin{aligned} X_{12} + X_{13} + X_{14} &\leq 1 \\ X_{21} + X_{23} + X_{24} &\leq 1 \\ X_{31} + X_{32} + X_{34} &\leq 1 \\ X_{41} + X_{42} + X_{43} &= 1 \end{aligned} \tag{9}$$

Next, we introduce integer auxiliary variables X_{ijk}[1] which are equal to 1 if the kth transmission in the final solution is $i \rightarrow j$ and 0 otherwise. Note that, for an N-node network, there can be at most $N-1$ steps (transmissions) in the solution ($1 \leq k \leq N-1$). These auxiliary variables are necessary to ensure proper sequentiality[2] of the final solution. The set of constraints in (10) defines the relation between the variables X_{ijk} and X_{ij}.

$$X_{ij} = \sum_{k=1}^{3} X_{ijk}, \quad 1 \leq i \neq j \leq 4 \tag{10}$$

Since the first transmission must be from the source (node 4 in our example), we can write:

$$X_{41(1)} + X_{42(1)} + X_{43(1)} = 1; \tag{11}$$

$$\begin{aligned} & X_{12(1)} + X_{13(1)} + X_{14(1)} \\ + \; & X_{21(1)} + X_{23(1)} + X_{24(1)} \\ + \; & X_{31(1)} + X_{32(1)} + X_{34(1)} = 0; \end{aligned} \tag{12}$$

where the k indices have been put in parentheses for clarity.

The set of nodes that can transmit in step 2 is restricted by the choice of transmission in step 1. For example, if node 3 is to transmit in step 2, it has to be reached by the transmission in step 1; *i.e.*, we must have either $X_{41(1)} = 1$ or $X_{43(1)} = 1$. Note that the possible transmissions in step 1 are $4 \rightarrow 1$, $4 \rightarrow 2$ and $4 \rightarrow 3$. Of these possible transmissions, node 3 can be reached only if the transmission chosen is either $4 \rightarrow 1$ or $4 \rightarrow 3$. This information is contained in cell $\mathbf{R}_{43}$ of the reward matrix. Similarly, node 1 can transmit in step 2 if the transmission chosen in step 1 is $4 \rightarrow 1$ and node 2 can transmit in step 2 if the transmission chosen in step 1 is either $4 \rightarrow 1$ or $4 \rightarrow 2$ or $4 \rightarrow 3$. We can thus set up the *node transmission blocking constraints* for step 2 as follows:

$$\begin{aligned} X_{12(2)} + X_{13(2)} + X_{14(2)} - X_{41(1)} &\leq 0; \\ X_{21(2)} + X_{23(2)} + X_{24(2)} - X_{41(1)} - X_{42(1)} - X_{43(1)} &\leq 0; \\ X_{31(2)} + X_{32(2)} + X_{34(2)} - X_{41(1)} - X_{43(1)} &\leq 0; \end{aligned} \tag{13}$$

Note that, for example, if $X_{41(1)} + X_{43(1)} = 1$ ($\Rightarrow$ node 3 has been reached in step 1), the expression $X_{31(2)} + X_{32(2)} + X_{34(2)}$ can be either 0 or 1. This implies that node 3 is free to transmit in step 2; whether it does so or not is to be decided by the optimization process. However, if $X_{41(1)} + X_{43(1)} = 0$ ($\Rightarrow$ node 3 has not been reached in step 1), the expression $X_{31(2)} + X_{32(2)} + X_{34(2)}$ is forced to be 0.

In general, the condition that node i ($i \neq$ source) can transmit in step k ($k \geq 2$) only if it has been reached by any of the transmissions *upto* step $k-1$ can be expressed as:

$$\sum_{j=1}^{N} X_{ijk} - \sum_{p=1}^{k-1} \sum_{\substack{m,n=1 \\ m \neq n}}^{N} \mathbf{R}_{mn}(i) X_{mnp} \leq 0; \; i \neq j \tag{14}$$

If node i has not been reached by step $k-1$,

$$\sum_{p=1}^{k-1} \sum_{\substack{m,n=1 \\ m \neq n}}^{N} \mathbf{R}_{mn}(i) X_{mnp} = 0; \; i \neq \mathit{source} \tag{15}$$

and hence the term $\sum_{j=1}^{N} X_{ijk}$ in (14) is forced to be zero, implying no transmission from node i in step k. The expression on the left hand side of (15) is therefore an indicator of whether node i has been reached or not by step $k-1$. Also, for $k = N$, the term

$$\sum_{p=1}^{N-1} \sum_{\substack{m,n=1 \\ m \neq n}}^{N} \mathbf{R}_{mn}(i) X_{mnp} \equiv \sum_{\substack{m,n=1 \\ m \neq n}}^{N} \mathbf{R}_{mn}(i) X_{mn}$$

equals the number of times node i is reached in the final solution. Consequently, if we assume that there is a fixed cost associated with signal reception (say α), which we have ignored so far, adding the term

$$\alpha \left(\sum_{i=1}^{N} \sum_{\substack{m,n=1 \\ m \neq n}}^{N} \mathbf{R}_{mn}(i) X_{mn} \right)$$

to the objective function will ensure that the final solution is optimal with respect to the sum of total transmission and reception powers.

The next set of constraints are *step transmission forcing constraints* and ensures that there is a transmission for step 2 if there is at least one node which has not

[1] A similar auxiliary variable formulation for the TSP was suggested by Flood [6].

[2] Sequentiality here means that if node i is the transmitting node in the kth step of the solution, it must have been reached by any of the transmissions upto step $k-1$

been reached upto step 1.

$$\sum_{\substack{m,n=1\\m\neq n}}^{4} X_{mn(2)} \leq 1; \quad (16)$$

$$\sum_{p=1}^{1}\sum_{\substack{m,n=1\\m\neq n}}^{4} \mathbf{R}_{mn}(i)X_{mnp} + \sum_{\substack{m,n=1\\m\neq n}}^{4} X_{mn(2)} \geq 1; \forall i \neq source \quad (17)$$

Constraint (16) ensures that there is at most one transmission in step 2. Constraint (17) forces the term on the left hand side of (16) to be equal to 1 (thereby forcing a transmission) if at least one of the terms in

$$\sum_{p=1}^{1}\sum_{\substack{m,n=1\\m\neq n}}^{4} \mathbf{R}_{mn}(i)X_{mnp},\ \forall i \neq source$$

is 0 (*i.e.*, there is at least one node which has not been reached after step 1).

The node transmission blocking constraints and step transmission forcing constraints need to be repeated for all steps $2 \leq k \leq N-1$. For our 4-node example, we therefore have:

$$\sum_{\substack{j=1\\j\neq i}}^{4} X_{ij(3)} - \sum_{p=1}^{2}\sum_{\substack{m,n=1\\m\neq n}}^{4} \mathbf{R}_{mn}(i)X_{mnp} \leq 0; \forall i \neq source \quad (18)$$

$$\sum_{\substack{m,n=1\\m\neq n}}^{4} X_{mn(3)} \leq 1; \quad (19)$$

$$\sum_{p=1}^{2}\sum_{\substack{m,n=1\\m\neq n}}^{4} \mathbf{R}_{mn}(i)X_{mnp} + \sum_{\substack{m,n=1\\m\neq n}}^{4} X_{mn(3)} \geq 1; \forall i \neq source \quad (20)$$

Finally, in a broadcast application, all nodes must be reached after the last step. As noted before in Section (III), in a wireless network, it is possible for one or more nodes to be reached more than once. The set of constraints (*node reachability constraints*) which ensures that all nodes (other than the source) are reached at least once in the solution is:

$$\sum_{\substack{m,n=1\\m\neq n}}^{4} \mathbf{R}_{mn}(i)X_{mn} \geq 1; \quad \forall i \neq source \quad (21)$$

A. *IP Formulation* 'A': *Generalized Model*

Let V be the set of all nodes in the network and D the set of intended destination nodes. For broadcast applications, the set D is the set of all nodes in V except the source and for multicast applications, the set D is the set of some nodes in V except the source. The IP formulation explained above for the example 4-node network can be easily generalized for broadcast/multicast applications in an N-node wireless network, as shown in Figure 5. Note that no upper bound is required to be declared for the integer variables X_{ijk} as it is set implicitly by equations (23), (24) and (25). The number of variables and constraints in this formulation are both of the order $O(N^3)$ (assuming $k^{MAX} = N-1$), similar to Flood's IP formulation of the TSP [6].

B. *Obtaining sub − optimal solutions by limiting k*

We mentioned in Section VI that the node transmission blocking constraints and step transmission forcing constraints need to be repeated for all steps $2 \leq k \leq N-1$ in a broadcast application. This is necessary to obtain the optimal solution. A sub-optimal solution can however be obtained using the same model by limiting k such that $k \leq k^{MAX} \leq N-1$. Doing so would not render the problem infeasible[3] because the 1-step solution $\{source \rightarrow node\ farthest\ from\ source\}$ covers all the nodes and hence is always feasible. In fact, it can be argued that a feasible solution exists for all choices of k. To see why, let $\pi_N = \{i_1,\ i_2\ \cdots\ i_{N-1},\ i_N\}$ be an ordering of the nodes in an N-node network such that i_1 is the source, i_2 is the node closest to the source, $\cdots$ and i_N is the node farthest from the source. For any k, it can be easily verified that the transmission sequence $\{i_1 \rightarrow i_k \rightarrow i_{k+1} \rightarrow i_{k+2} \cdots i_{N-2} \rightarrow i_{N-1} \rightarrow i_N\}$ is always a feasible broadcast tree. For example, in a 5-node network with node 1 being the source, suppose $\pi_5 = \{1, 5, 2, 4, 3\}$. For $k = 3$, the transmission sequence $\{1 \rightarrow 2, 2 \rightarrow 4, 4 \rightarrow 3\}$ constitutes a valid broadcast tree.

VII. IP FORMULATION 'B'

This formulation utilizes the alternate view of implicit visitation discussed in Section IV. Let $\{Y_i : 1 \leq i \leq N\}$ be the transmitter power levels at the nodes (continuous variables) and $\{X_{ij} : 1 \leq i \neq j \leq N\}$ be binary variables such that $X_{ij} = 1$ if the transmission $i \rightarrow j$ is used in the final solution and 0 otherwise. V is the set of all nodes in the network and D is the set of intended destination nodes.

As in Section VI, the objective function is:

$$minimize \sum_{i=1}^{N} Y_i \quad (33)$$

The first set of constraints are used for proper cost accounting and reflects the condition that the cost incurred at node i is the maximum of the individual costs of the transmissions out of node i (Section IV).

$$Y_i - \mathbf{P}_{ij}X_{ij} \geq 0; \quad \forall (i,j) \in V,\ i \neq j \quad (34)$$

where $\mathbf{P}_{ij}$ is the $(i,j)th$ element of the power matrix $\mathbf{P}$.

The next set of constraints expresses the condition that the source node must transmit at least once. No constraints are required for the other nodes since they are free to transmit to any number of nodes, or not to transmit at all.

$$\sum_{j=1}^{N} X_{ij} \geq 1; \quad i = source,\ i \neq j \quad (35)$$

[3]Feasibility implies that all destination nodes are reached

$$minimize \sum_{i=1}^{N} Y_i$$

subject to:

$$Y_i - \sum_{j=1}^{N} \mathbf{P}_{ij} X_{ij} = 0; \;\; \forall i \in V, \; i \neq j \tag{22}$$

$$\sum_{j=1}^{N} X_{ij} = 1; \;\; i = source, \; i \neq j \tag{23}$$

$$\sum_{j=1}^{N} X_{ij} \leq 1; \;\; \forall i \in \{V \setminus source\}, \; i \neq j \tag{24}$$

$$X_{ij} - \sum_{k=1}^{N-1} X_{ijk} = 0; \;\; \forall (i,j) \in V, \; i \neq j \tag{25}$$

$$\sum_{j=1}^{N} X_{ijk} = 1; \;\; i = source, \; i \neq j, \; k = 1 \tag{26}$$

$$\sum_{i=1}^{N} \sum_{j=1}^{N} X_{ijk} = 0; \;\; i \neq source, \; i \neq j, \; k = 1 \tag{27}$$

$$\sum_{j=1}^{N} X_{ijk} - \sum_{p=1}^{k-1} \sum_{m=1}^{N} \sum_{n=1}^{N} \mathbf{R}_{mn}(i) X_{mnp} \leq 0; \;\; \forall i \in \{V \setminus source\}, \; i \neq j, \; m \neq n, \; 2 \leq k \leq k^{MAX} \tag{28}$$

$$\sum_{m=1}^{N} \sum_{n=1}^{N} X_{mnk} \leq 1; \;\; m \neq n, \; 2 \leq k \leq k^{MAX} \tag{29}$$

$$\sum_{p=1}^{k-1} \sum_{m=1}^{N} \sum_{n=1}^{N} \mathbf{R}_{mn}(i) X_{mnp} + \sum_{m=1}^{N} \sum_{n=1}^{N} X_{mnk} \geq 1; \;\; \forall i \in \{V \setminus source\}, \; i \neq j, \; m \neq n, \; 2 \leq k \leq k^{MAX} \tag{30}$$

$$\sum_{m=1}^{N} \sum_{n=1}^{N} \mathbf{R}_{mn}(i) X_{mn} \geq 1; \;\; \forall i \in D, \; m \neq n \tag{31}$$

$$X_{ijk} \geq 0, \text{ integers}; \;\; \forall (i,j) \in V, \; i \neq j, \; 1 \leq k \leq k^{MAX} \tag{32}$$

Fig. 5. IP formulation 'A' for the minimum power broadcast problem

Since any number of transmissions can be made out of a node i (only one of which adds to the overall cost), the *node reachability constraints* in this formulation can be simply written as[4]:

$$\sum_{i=1}^{N} X_{ij} = 1; \;\; \forall j \in D, \; i \neq j \tag{36}$$

Note the equality relationship in (36), as opposed to the '$\geq$' relationship in the node reachability constraints in Formulation 'A' (31). An equality relationship works for this formulation because, if a node j is reached from node i in the optimal solution, either by a free transmission or a cost-incurring one, it is not necessary for any other node in the network to reach node j, using a free transmission or otherwise (from a modeling aspect)[5]. Also, (36) effectively constrains the number of $\{X_{ij}; \; i \neq j\}$ variables which can have a value of 1 in the optimal solution to $\mathcal{C}_D$, where $\mathcal{C}_D$ is the cardinality of set D.

The constraints we have thus far can however lead to loops and disjoint sets in the final solution, as illustrated in Figure 6. The solid lines in the figure indicate cost-incurring (actual) transmissions and the dashed lines indicate free (implicit) transmissions. Nodes 4, 5 and 6 form a loop in Figure 6. The sets of nodes {1,2,3} and {4,5,6} are disjoint. Disjoint sets and loops will generally be present

[4]The column sums corresponding to non-destination nodes should not be set to zero. In a multicast application, such nodes can be used as relay nodes to reach intended destination nodes. Forcing the column sums of non-destination nodes to zero will preclude this possibility.

[5]In a physical system, a node may have no control on the number of implicit transmissions it receives from other nodes.

in the solution if there is a cluster of nodes (nodes 4, 5 and 6 in our example) in the network which are far removed from the rest of the nodes. In such a situation, (36) will force a loop if the cost of the solution with the loop is less than the cost of the *true solution* requiring no loops and disjoint sets. It should be noted, however, that loops may not necessarily be formed by cost-incurring transmissions, as is the case in Figure 6. Figure 7 illustrates a case where a loop is formed by a combination of cost-incurring and free transmissions.

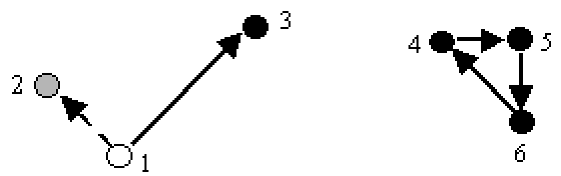

Fig. 6. Example to illustrate loops and disjoint sets. The sets {1,2,3} and {4,5,6} are disjoint. Nodes {4,5,6} form a loop.

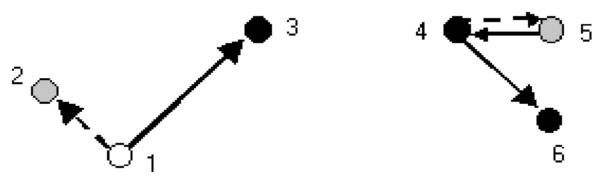

Fig. 7. The loop $\{4 \leftrightarrow 5\}$ is formed by a combination of cost-incurring and free transmissions. As in Figure 6, The sets {1,2,3} and {4,5,6} are disjoint.

What we need therefore are constraints to prevent any loops in the final solution. Referring to Figure 6, if we can prevent the loop $4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 4$, one of the nodes in the cluster {4,5,6} will be forced to receive a transmission from any of the nodes in the cluster {1,2,3} (thereby also solving the problem of disjoint sets), as otherwise it will violate constraint (36). This will also ensure that there are no disjoint sets in the final solution. For example, if the loop is broken at the edge $6 \rightarrow 4$, node 4 will violate (36) if it does not receive a transmission from any of the nodes in the cluster {1,2,3}. Similarly, if we can prevent the loop between nodes 4 and 5 in Figure 7 and ensure that there is no loop between nodes 4 and 6, node 4 will be forced to receive a transmission from any of the nodes in the cluster {1,2,3}.

The argument that preventing loops will prevent disjoint sets is valid only if a broadcast application is assumed and may not hold for a multicast application. For example, assume that nodes 2, 3, 5 and 6 in Figure 8 are the intended destination nodes. Since node 4 is not a destination node, there is no requirement that it be reached. However, it is free to transmit, as mentioned in footnote 4. Consequently, we have a situation where node 4 transmits to node 6 (covering node 5 in the process), but does not have to receive a transmission before it transmits. This can be avoided by adding constraints stipulating that a node (except the source) can transmit only if it receives a transmission from some other node.

$$\sum_{j=1}^{N} X_{ij} \leq (N-1) \sum_{j=1}^{N} X_{ji}; \forall (i,j) \in \{V \setminus source\}, i \neq j \tag{37}$$

With (37) in place, node 4 will be forced to receive a transmission from any of the nodes in the cluster {1,2,3} since receiving a transmission from either node 5 or 6 will result in a loop.

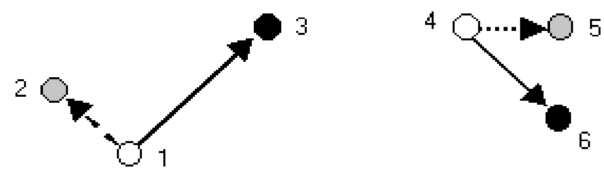

Fig. 8. Disjoint sets can be present in a multicast solution even when there are no loops.

Miller ([7]) suggested using the following constraint to prevent subtours in TSP solutions.

$$U_i - U_j + NX_{ij} \leq (N-1); \quad \forall (i,j) \in V, \ i \neq j \tag{38}$$

where the U_i's are sequencing variables and denote the order in which the nodes are covered in the final solution. Suppose nodes i, j and k form a loop. Using (38), we have the three inequalities

$$U_i - U_j + NX_{ij} \leq (N-1)$$
$$U_j - U_k + NX_{jk} \leq (N-1)$$
$$U_k - U_i + NX_{ki} \leq (N-1)$$

where $X_{ij} = X_{jk} = X_{ki} = 1$. Adding up the three inequalities will give $N \leq (N-1)$, a contradiction. Equation (38) can therefore be used in the IP formulation for MPB to prevent any loops (and thereby also ensuring that there are no disjoint sets) in the solution. Since the first transmission must be from the source, we will use (38) in conjunction with:

$$U_i = 1; \quad i = source \tag{39}$$
$$U_i \geq 2; \quad \forall i \in \{V \setminus source\} \tag{40}$$
$$U_i \leq N; \ \forall i \in \{V \setminus source\} \tag{41}$$

The objective function (33) subject to (34) to (41) and the integrality constraints

$$X_{ij} \in \{0,1\}; \quad \forall (i,j) \in V, \ i \neq j \tag{42}$$

solves the minimum power broadcast/multicast problems in wireless networks. While (37) is a required constraint for multicast, it is optional for a broadcast application. The number of variables and constraints in this formulation are both of the order $O(N^2)$.

Finally, it may be noted that while the $\{Y_i\}$ variables are required in this formulation for proper power accounting, it is possible to write IP formulation 'A' directly in terms of the $\{X_{ij}\}$ variables since the $\{Y_i\}$ variables are related to the $\{X_{ij}\}$ variables by equality relationships (22).

VIII. NOTE ON THE SOLUTIONS OBTAINED USING FORMULATIONS 'A' and 'B'

It is interesting to note that while the value of the objective function will be the same in the optimal solutions obtained using either of the two formulations, the $\{X_{ij}\}$ variables can be different in the two solutions. This is because of the manner in which power expenditures are accounted for at the nodes in the two formulations. In formulation 'A', a node is constrained to a maximum of one transmission, the power expenditure being simply the corresponding element from the power matrix. In formulation 'B', however, a node can send as many as $N-1$ transmissions, the power expenditure being defined as the maximum of the cost of the individual transmissions. We use Figure 9 to illustrate the above.

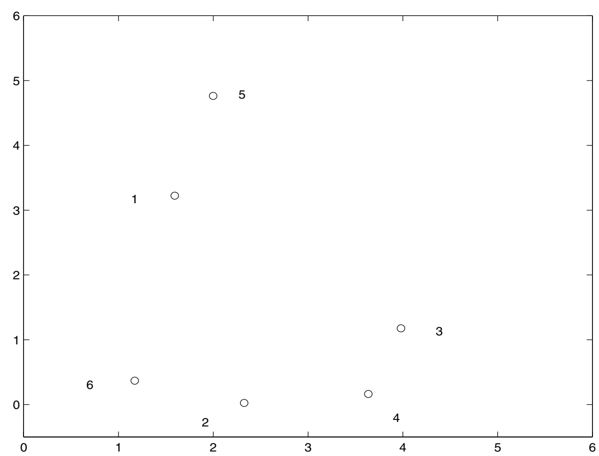

Fig. 9. Example 6-node network: node 5 is the source.

The power matrix for the above network is:

$$\mathbf{P} = \begin{bmatrix} 0 & 10.78 & 9.89 & 13.55 & 2.53 & 8.34 \\ 10.78 & 0 & 4.06 & 1.73 & 22.56 & 1.45 \\ 9.89 & 4.06 & 0 & 1.15 & 16.78 & 8.54 \\ 13.55 & 1.73 & 1.15 & 0 & 23.83 & 6.10 \\ 2.53 & 22.56 & 16.78 & 23.83 & 0 & 20.00 \\ 8.34 & 1.45 & 8.54 & 6.10 & 20.00 & 0 \end{bmatrix} \quad (43)$$

Using both formulations, the optimal node power settings, assuming a broadcast application, are:

$$\vec{Y}^{\ (opt)} = [10.78\ 0\ 1.15\ 0\ 2.53\ 0] \quad (44)$$

The optimal value of the objective function is therefore: $\sum_{i=1}^{8} \vec{Y}_i^{\ (opt)} = 14.46$.

However, the status of the $\{X_{ij}\}$ variables in the optimum solutions are different, as is evident from (45) and (46),

$$\mathbf{X}_A^{(opt)} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (45)$$

$$\mathbf{X}_B^{(opt)} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (46)$$

where $\mathbf{X}_A^{(opt)}$ and $\mathbf{X}_B^{(opt)}$ are matrices containing the optimal $\{X_{ij}\}$ values for formulations 'A' and 'B'. While the actual transmissions in the optimal connection tree (which is $[5 \to 1,\ 1 \to 2,\ 3 \to 4]$) are readily evident from (45), it is not so in (46). To be specific, the '1' entries in row 1 of $\mathbf{X}_B^{(opt)}$ need to be matched with the corresponding elements of the power matrix ($\mathbf{P}_{12}$, $\mathbf{P}_{13}$, $\mathbf{P}_{16}$) to establish which entry corresponds to the cost-incurring (actual) transmission, the rest being free (implicit) transmissions. Using the power matrix (43), we find that the transmission $1 \to 2$ is an actual transmission while the others, $1 \to 3$ and $1 \to 6$, are implicit.

We can also observe that while all column sums (except column 5, which is for the source) in (46) are 1, the row sums vary from 0 (row 2 for example) to 3 (row 1). In general, for a broadcast application, the column sums in $\mathbf{X}_B^{(opt)}$ will all be 1 (except for the source column), but the row sums can be any integer between 0 and $N-1$ (except for the source row for which it will be at least 1). For $\mathbf{X}_A^{(opt)}$, however, all row and column sums are either 0 or 1 (for the source, the row sum will be 1 and the column sum will be 0).

IX. IP FORMULATION 'C'

This formulation is built upon a network flow model and its interpretation follows from Formulation 'B'. A flow interpretation of the optimal $\{X_{ij}\}$ values in (46) is shown in Figure 10. Node numbers are in bold italics.

Since the solution in (46) is for a broadcast application, we can interpret it in terms of the following flow model:

1) Node 5 (the source) is the supply node, with 5 (sum of all elements in $\mathbf{X}_B^{(opt)}$) units of supply. In general, the number of units of supply is equal to the cardinality of D, where D is the set of all destination nodes.
   All other nodes (in general, the set of destination nodes, D) are *demand nodes*, with 1 unit of demand each.
2) The supply node routes all 5 units to node 1 ($F_{51} = 5$, where F_{ij} is the flow in arc ij), which keeps 1

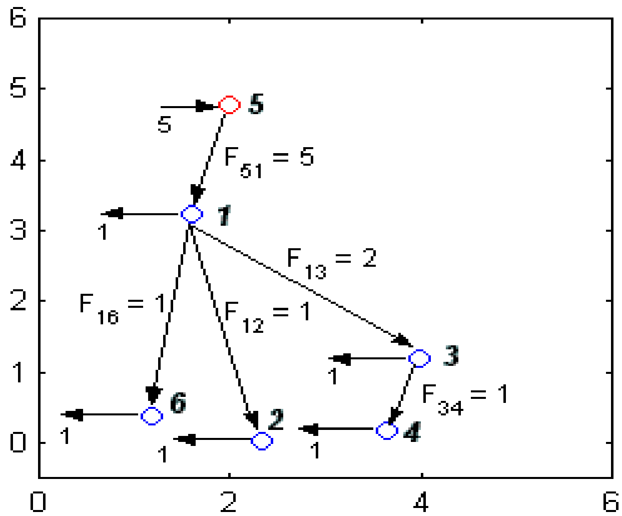

Fig. 10. Flow interpretation of the optimal $\{X_{ij}\}$ values in (46).

unit to satisfy its own demand, while forwarding the balance 4 units to other nodes. Specifically, it sends 1 unit each to nodes 2 and 6 ($F_{12} = 1$ and $F_{16} = 1$) and 2 units to node 3 ($F_{13} = 2$).

3) Nodes 2 and 6 keep the units they receive to satisfy their own demands. Node 3, on the other hand, keeps 1 unit for itself and sends the remaining 1 unit to node 4 ($F_{34} = 1$).
4) Node 4 keeps the unit it receives to satisfy its own demand.

Suppose we solve a network flow problem (we will show later how to) and come up with the flows in the arcs. How do we account for the costs involved with the flows? First, we note that the cost of using an arc is independent of the number of units (greater than zero) flowing through the arc; *i.e.*, no matter how many units are sent through the arc ij, as long as it is not zero, the cost is simply $\mathbf{P}_{ij}$, where $\mathbf{P}_{ij}$ is the $(i,j)th$ element of the power matrix $\mathbf{P}$. If there is no flow in an arc, the cost is zero. For example, the cost associated with the 5 units of flow in the arc $5 \rightarrow 1$ in Figure 10 is $P_{51} = 2.53$. This suggests that we should define additional variables (say X_{ij}) such that $X_{ij} = 1$ if $F_{ij} > 0$.

Assuming that we are able to write out constraints which satisfy the above relationship, we now have to account for the fact that, in a wireless network, there can be multiple flows out of a node but the net cost incurred is the maximum of the individual costs due to the positive flows in the arcs out of the node. For example, the cost incurred at node 1 in Figure 10 is simply $\mathbf{P}_{12}$ and not $\mathbf{P}_{16}+\mathbf{P}_{12}+\mathbf{P}_{13}$, since $\mathbf{P}_{12} > \mathbf{P}_{13}, \mathbf{P}_{16}$ (43). The tools to resolve this are already in place, as we saw in Section VII. Defining Y_1 to be the cost incurred at node 1, the set of constraints

$$Y_1 - \mathbf{P}_{1j} X_{1j} \geq 0; \quad 2 \leq j \leq 6 \tag{47}$$

will ensure that $Y_1 = \mathbf{P}_{12}$, if the objective function is to minimize $\sum_{i=1}^{6} Y_i$.

We will now generalize the above approach for an arbitrary N node network. Let V be the set of all nodes and D the set of all destination nodes. The objective function is:

$$minimize \sum_{i=1}^{N} Y_i \tag{48}$$

As in IP formulation 'B', the first set of constraints ensure proper power accounting at the nodes.

$$Y_i - \mathbf{P}_{ij} X_{ij} \geq 0; \quad \forall (i,j) \in V, \ i \neq j \tag{49}$$

The second set of constraints relates the X_{ij} variables to the flow variables F_{ij} and ensures that $X_{ij} = 1$ if $F_{ij} > 0$.

$$\mathcal{C}_D X_{ij} - F_{ij} \geq 0; \quad \forall (i,j) \in V, \ i \neq j \tag{50}$$

where $\mathcal{C}_D$ is the cardinality of set D. The coefficient of X_{ij} in (50) is due to the fact that the maximum flow out of a node is equal to the number of *demand nodes* (or destination nodes) in the network. Equation (50) leaves open the possibility of X_{ij} being equal to 1 for $F_{ij} = 0$. However, if there is no flow out of node i, *i.e.*, $F_{ij} = 0, \forall j$, setting $X_{ij} = 1$ would unnecessarily increase the cost of the optimal solution. On the other hand, if there are multiple flows out of node i, suppose j^* is the node such that $\hat{Y}_i = \mathbf{P}_{ij^*} X_{ij^*} = max_j \ (\mathbf{P}_{ij} X_{ij})$ is part of the optimal solution. In this case, setting $X_{ij} = 1, \ j \neq j^*$, would not affect the cost of the optimal solution if $\mathbf{P}_{ij} X_{ij} \leq \mathbf{P}_{ij^*} X_{ij^*}$ (49). If, however, $\mathbf{P}_{ij} X_{ij} > \mathbf{P}_{ij^*} X_{ij^*}$, this solution cannot be optimal since it can easily be improved by setting $X_{ij} = 0$.

Next, we write the flow control equations (see for example [9]):

$$\sum_{j=1}^{N} F_{ij} = \mathcal{C}_D; \quad i = source, \ i \neq j \tag{51}$$

$$\sum_{j=1}^{N} F_{ji} = 0; \quad i = source, \ i \neq j \tag{52}$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} = 1; \quad \forall i \in D, \ i \neq j \tag{53}$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} = 0; \quad \forall i \notin D, \ i \neq j \tag{54}$$

Note that (53) also serves as *node reachability constraints* in this formulation, allowing non-destination nodes to be used as hop nodes in a multicast application.

The final set of constraints express the integrality of the X_{ij} variables and non-negativity of the F_{ij} variables.

$$X_{ij} \in \{0,1\}; \quad \forall (i,j) \in V, \ i \neq j \tag{55}$$

$$F_{ij} \geq 0; \quad \forall (i,j) \in V, \ i \neq j \tag{56}$$

To summarize, the objective function (48) subject to (49) to (56) solves the minimum power broadcast/multicast problems in wireless networks..

The number of variables in this formulation is approximately $2N^2 + N$, roughly N^2 more than formulation 'B' due to the presence of the additional flow variables $\{F_{ij}\}$. The number of constraints is on the order of $O(N^2)$. The most important feature of this formulation is that, unlike formulation 'B', loop-breaking constraints are not required here.

X. A NOTE ON PREVENTION OF DISJOINT SETS AND LOOPS IN FORMULATIONS 'A', 'B' and 'C'

It is interesting to note that prevention of loops and disjoint sets is handled differently by each of the three IP models discussed for the MPB problem. Formulation 'A' does not have any loop prevention constraints but uses the *node transmission blocking constraints* (28) to prevent disjoint sets. It can be argued that if there are no disjoint sets in the MPB tree, there can be no loops either in the solution obtained using this formulation. To see why, let $i \rightarrow j$ be the mth transmission and $j \rightarrow i$ be the nth $(n > m)$ transmission in the MPB tree. First, we consider the case where no other node is reached by either of these two transmissions. In this case, clearly, one of them can be deleted from the solution, leading to a reduction in the overall tree power. The same is true if the set of nodes reached by the transmission $i \rightarrow j$ is exactly the same as the set of nodes reached by $j \rightarrow i$ (*i.e*, $\mathbf{R}_{ij} = \mathbf{R}_{ji}$). Next, we consider the case where there is at least one node which is reached by any of the two transmissions but not by the other. For example, suppose $i \rightarrow j$ also reaches nodes p_1 and p_2, while $j \rightarrow i$ also reaches nodes p_1, p_3 and p_4. Clearly, node j does not need to reach either i or p_1 since these have already been reached by prior transmissions. Assuming node p_3 is nearer to j than p_4 and p_4 is nearer to j than i, the MPB tree can be improved by choosing the transmission $j \rightarrow p_4$, which will ensure that node p_3 is covered as well.

Formulation 'B', on the other hand, uses the loop prevention constraints suggested by Miller [7] for solving the TSP to prevent loops in the MPB tree. As argued in Section VII, the loop-prevention constraints are sufficient to prevent any disjoint sets in the broadcast tree. However, for a multicast application, additional constraints preventing a node from transmitting unless it receives from some other node are required to prevent disjoint sets (37).

Finally, we note that Formulation 'C' solves the problem of loops and disjoint sets by using an underlying flow model, with the flow balance equations, (51) to (54), ensuring the validity of the solution returned by the model.

XI. CONCLUSION

We have proposed three integer programming models to solve the minimum power broadcast problem in wireless networks. Currently, we are using an LP-based branch-and-bound method for solving the models. Development of sophisticated and customized methods, using cutting planes or branch-and-cut techniques, for example, will be taken up in future. An analytical study of the tightness of the IP models as also the properties of their LP relaxations are also planned for the future.

While optimal solutions can now be obtained for fixed wireless networks (*i.e.*, networks where the nodes are not mobile), the IP models can also be used to assess the performance of heuristic algorithms for mobile networks by running them at discrete time instances. The possibility of using a heuristic patch-up procedure together with optimization at regular time intervals for mobile networks also needs to be explored.

XII. ACKNOWLEDGEMENT

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XIII. REFERENCES

1) J.E.Wieselthier, G.D. Nguyen and A. Ephremides, "On the construction of energy-efficient broadcast and multicast trees in wireless networks", IEEE INFOCOM 2000, pp. 585- 594.
2) D. Bertsekas and R. Gallager, Data Networks, Englewood Cliffs: Prentice Hall, 1992.
3) Robert J. Marks II, Arindam K. Das, Mohamed El-Sharkawi, Payman Arabshahi & Andrew Gray, "Minimum Power Broadcast Trees for Wireless Networks: Optimizing Using the Viability Lemma", Proceedings of the IEEE International Symposium on Circuits and Systems, Scottsdale, Arizona, 2002.
4) Robert J. Marks II, Arindam K. Das, Mohamed El-Sharkawi, Payman Arabshahi & Andrew Gray, "Maximizing Lifetime in an Energy Constrained Wireless Sensor Array Using Team Optimization of Cooperating Systems", Proceedings of the World Congress on Computational Intelligence, Honolulu, Hawaii, 2002.
5) Ivan Stojmenovic, Mahtab Seddigh and Jovisa Zunic, "Internal Nodes Based Broadcasting in Wireless Networks", Proceedings of the 34th Hawaii International Conference on System Sciences, 2001.
6) Flood Merrill M., 1956-1, "The Traveling Salesman Problem," in J. F. McCloskey and J. M. Coppinger (eds), Operations Research for Management, Vol. II, Johns Hopkins Press, Baltimore, Maryland, 1956, pp. 340-357.
7) Miller L. B., A. W. Tucker and R. A. Zemlin, "Integer Programming Formulations of Traveling Salesman Problems," J. Assoc. Comput. Mach., Vol. 7, No. 4, October 1960.
8) George B. Dantzig, Linear Programming and Extensions, Princeton University Press, Princeton, New Jersey, 1963.
9) Richard Kipp Martin, Large Scale Linear and Integer Optimization: A Unified Approach, Kluwer Academic Publishers, Massachusetts, 1999.