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PRESENTATION

Maximization of Time-to-first-failure for Multicasting in Wireless Networks: Optimal Solution

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Outline

- **Problem Statement:** Maximizing the time-to-first-failure (TTFF) – the time till the first node in the network runs out of battery energy, in energy constrained broadcast wireless networks. Is this enough?
- **Issues:**
  - TTFF metric, by itself, fails to provide the “ideally optimum” multicast tree.
  - Case of prioritized nodes.
- **Results:**
  - A composite weighted objective function which maximizes the TTFF and minimizes the sum of transmitter powers.
  - A mixed integer linear programming (MILP) model for solving the joint optimization problem optimally.

Problem Statement

- We consider energy constrained broadcast wireless networks where each node is powered by batteries.
- In applications where replacement/maintenance of batteries is difficult or infeasible, it is important to design routing protocols which maximize the *lifetime* of the network.
- A metric commonly used to define the lifetime of a network is the duration of time before any node in the network runs out of its battery energy.
- We wish to maximize this *time-to-first-failure* (TTFF), also known as *system lifetime* or *network lifetime*.

Past Research

- This problem was first addressed by Chang and Tassiulas for a unicast application [*Infocom 2000*].
- Subsequent research in this area for unicast and multicast applications includes [Marks et. al, *WCCI 2003*], [Misra and Banerjee, *WCNC 2002*], [Kang and Poovendran, *WCCI 2002*] and [Toh, *IEEE Comm. Mag.*, 2001].
- In [Das et. al, *Globecom 2003*], we show that maximization of the TTFF for a broadcast application can be solved optimally by a greedy algorithm in polynomial time.

Network Model

- Fixed N-node network with a specified source node which has to broadcast a message to all other nodes.
- Any node can be used as a relay node to reach other nodes.
- All nodes have omni-directional antennas – if node i transmits to node j, all nodes closer to i than j will also receive the transmission (given line-of-sight).
- For a transmission from node i to j, the received signal power at j varies as $d_{ij}^{-\alpha}$, where:

$$d_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{1/2}$$

is the Euclidean distance between nodes i and j, (x_i, y_i) are the coordinates of node i and α (around $2 \leq \alpha \leq 4$) is the channel loss exponent.

Network Model

- Transmitter power at i necessary to support the link $i \longrightarrow j$, $\mathbf{P}_{ij}$ is (accounting for fading and antenna gain factors):

$$\mathbf{P}_{ij} = d_{ij}^{\alpha} \tag{1}$$

- The power matrix $\mathbf{P}$, is an $N \times N$ symmetric matrix whose $(i, j)^{th}$ element, $\mathbf{P}_{ij}$, represents the power required to support the link $i \longrightarrow j$.
- Power expenditures due to signal reception and processing are negligible compared to signal transmission.
- Hence the lifetime is determined solely by the choice of transmitter powers and residual energy levels of the nodes.

Problem Statement

- $E(t)$ is a vector of **node residual energies** at time t, the i^{th} element of which represents the residual energy of node i at time t.
- Y is a vector of **node transmission powers** with elements Y_i representing the transmitter power level of node i.
- Each node has a constraint on maximum transmitter power:

$$Y_i \leq Y_i^{max} : \forall i \in \mathcal{N} \text{ (set of all network nodes, cardinality } N). \quad (2)$$

- s is the source; $\mathcal{D} \subseteq \{\mathcal{N} \setminus s\}$ is the set of destination nodes, cardinality D.
- $\mathcal{E}$ is the set of all directed edges, cardinality E.

$$\mathcal{E} = \{(i \rightarrow j) : (i,j) \in \mathcal{N}, i \neq j, \mathbf{P}_{ij} \leq Y_i^{max}, j \neq s\} \quad (3)$$

- $L_i(t) \triangleq E_i(t)/Y_i$ is the *lifetime of node i*.

Problem Statement

The problem of maximizing the (residual) TTFF can be written as:

$$\text{maximize } \{\min_{i \in \mathcal{N}} L_i(t)\} \tag{4}$$

subject to

1. All nodes, other than the source, must be reached, either actually or implicitly.
2. The source node must reach at least one other node.
3. The solution must be a *connected tree*; *i.e.*, there must be directed paths from the source to all destination nodes, possibly involving other intermediate nodes.

We implicitly recognize the dependence of TTFF on the time origin t and use notations E_i and L_i henceforth.

Problem Statement

Let S be the set of nodes that are geometrically closer to i than j ($\Rightarrow \mathbf{P}_{ij} > \mathbf{P}_{ik} : \forall k \in S$).

- Nodes that belong in S are said to receive the transmission from i **implicitly** (no additional cost is incurred to reach them) and the set of transmissions $\{i \longrightarrow k : \forall k \in S\}$ are referred to as **implicit transmissions**.

- The transmission $i \longrightarrow j$ is referred to as an **actual transmission**.

Let $\{X_{ij} : (i \longrightarrow j) \in \mathcal{E}\}$ be a set of binary variables such that $X_{ij} = 1$ if the transmission $i \longrightarrow j$ is used in the optimum tree and 0 otherwise.

$$Y_i = \text{max}_j\{X_{ij}\mathbf{P}_{ij} : j \neq i\} \quad (5)$$

where $X_{ij} = 1$ if node j is reached from node i (actually or implicitly) and 0 otherwise.

Problem Statement

Expressing the objective function in (4) as a minimax optimization problem:

$$\begin{aligned}
& \text{maximize}\left(\text{min}_i \; L_i\right) \\
= \quad & \text{maximize}\left(\text{min}_i \; E_i/Y_i\right) \\
= \quad & \text{minimize}\left(\text{max}_i \; Y_i/E_i\right) && (6) \\
= \quad & \text{minimize}\left(\text{max}_i \left[\text{max}_j \left(\mathbf{P}_{ij}X_{ij}\right)/E_i\right]\right) && (7) \\
= \quad & \text{minimize}\left(\text{max}_{i,j} \left[\mathbf{P}_{ij}X_{ij}/E_i\right]\right) && (8) \\
= \quad & \text{minimize} \;\; \sigma && (9)
\end{aligned}$$

where

$$\sigma = \text{max}_{i,j} \left(\mathbf{P}_{ij}X_{ij}/E_i\right) = 1/\tau \qquad (10)$$

and τ is the TTFF.

Problem Statement

We also define two other terms:

- For a given connection tree, T, we define its **critical node** to be the node whose residual lifetime is equal to the TTFF of the tree:

$$\text{Critical node } = \text{argmin}_i \left(E_i/Y_i\right) \tag{11}$$

  Note that for any non-transmitting node, $Y_i = 0$, and hence the residual lifetime of that node is ∞.

- A transmission ($i \longrightarrow j$) is defined to be the **critical transmission** in a tree if, given that

$$i = \text{argmin}_k(E_k/Y_k)$$
$$E_i/\mathbf{P}_{ij} = \text{TTFF} \tag{12}$$

Inadequacy of the TTFF criterion

Consider the 6-node network and the broadcast tree below. Assume $\alpha = 2$, and that the residual energy of all nodes is 10.

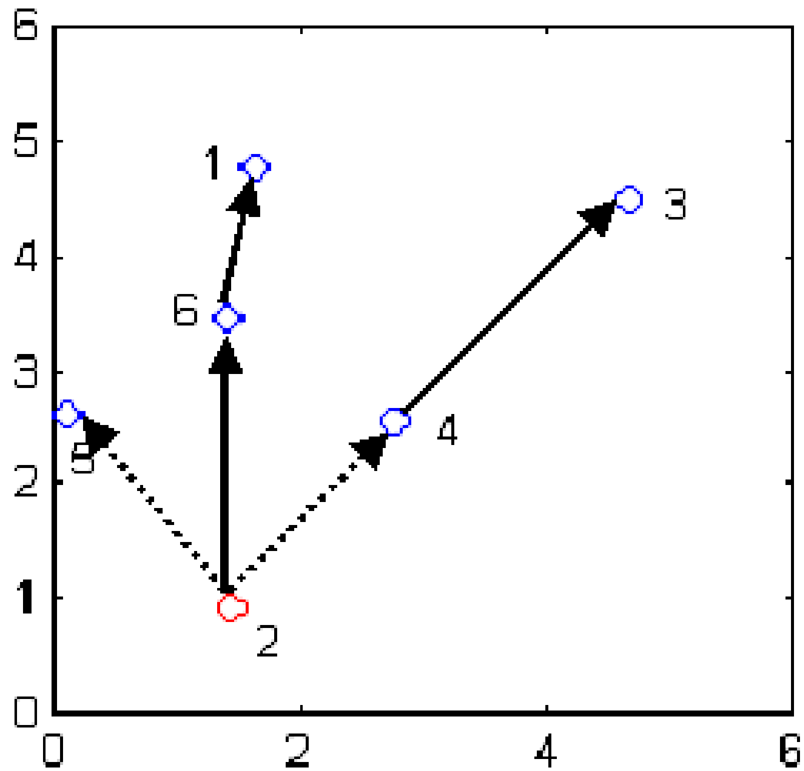

Figure 1: A 6-node network.

Inadequacy of the TTFF criterion

The power matrix of the network is:

$$\mathbf{P} = \begin{bmatrix} 0 & 14.86 & 9.31 & 6.33 & 7.01 & 1.76 \\ 14.86 & 0 & 23.18 & 4.39 & 4.58 & 6.46 \\ 9.31 & 23.18 & 0 & 7.41 & 24.32 & 11.65 \\ 6.33 & 4.39 & 7.41 & 0 & 7.11 & 2.73 \\ 7.01 & 4.58 & 24.32 & 7.11 & 0 & 2.43 \\ 1.76 & 6.46 & 11.65 & 2.73 & 2.43 & 0 \end{bmatrix} \tag{13}$$

- Residual lifetime vector of the nodes is: $\vec{L}_1 = [\infty, 1.55, \infty, 1.35, \infty, 5.69]$.
- Lifetimes of nodes 1, 3 and 5 are ∞ (non-transmitting nodes).
- Node 4 is the critical node in the tree and $4 \longrightarrow 3$ is the critical transmission.

Inadequacy of the TTFF Criterion

Now consider the broadcast tree below.

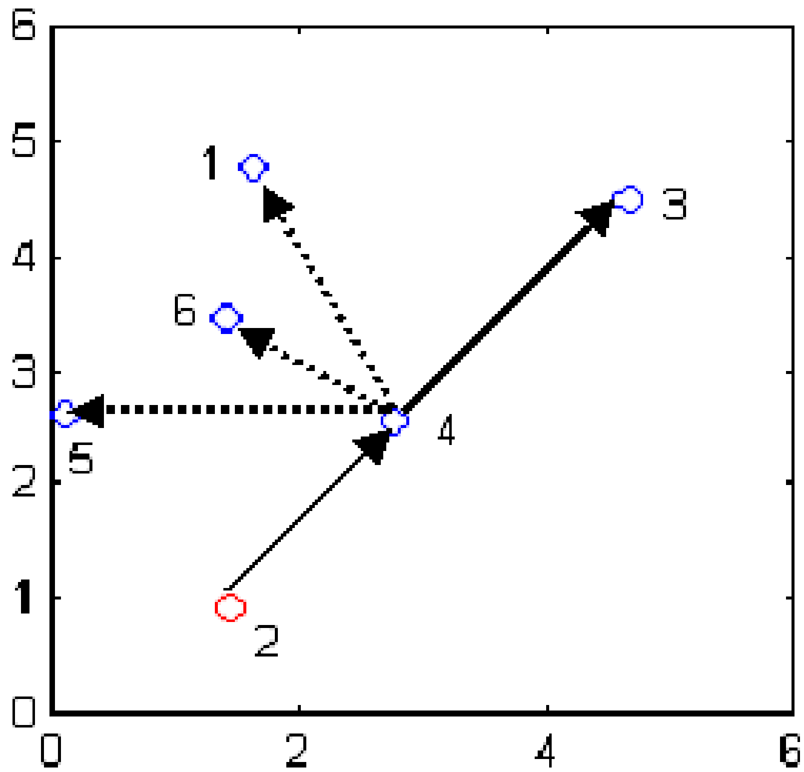

Figure 2: An alternate broadcast tree with the same TTFF, 1.35, as in Figure 1.

Inadequacy of the TTFF Criterion

- The residual lifetime vector here is: $\vec{L}_2 = [\infty, 2.28, \infty, 1.35, \infty, \infty]$.
- The TTFF of this tree is identical to that of Fig. 1.
- However, note that the lifetime of node 2 is higher (2.28, as compared to 1.55) than its lifetime in Fig. 1.
- Also, the lifetime of node 6 is now ∞, compared to 5.69 in Fig. 1, since it is a non-transmitting node.
- Clearly, for the same TTFF, this broadcast tree is better than that shown in Fig. 1.

Inadequacy of the TTFF Criterion

In general, given two trees T_m and T_n with the same TTFF, T_m is considered better ("leaner") than T_n if:

- There is at least one node in T_m whose residual lifetime is greater in T_m than in T_n, and,
- The residual lifetimes of all other nodes in T_m are at least as high as in T_n.

One way of obtaining a "lean" optimum solution is to consider a joint optimization function of the form:

$$\text{minimize} \left(w_1 \sigma + w_2 \sum_{i=1}^{N} Y_i \right) \tag{14}$$

where $\sum_{i=1}^{N} Y_i$ is the sum of transmitter powers, σ is the inverse of the TTFF in Eq. (10) and $\{w_1, w_2\}$ are suitably chosen non-negative penalty factors.

Inadequacy of the TTFF Criterion

- The tree in Fig. 2 is characterized by a smaller total transmitter power, 11.80 units ($P_{24} + P_{43}$), compared to Fig. 1 (15.63 units - $P_{26} + P_{61} + P_{43}$).
- In Eq. (14), σ, may be viewed as the global cost while $\sum_{i=1}^{N} Y_i$, may be viewed as the sum of local costs. Thus varying w_1 and w_2 represents a tradeoff between global and local costs.
- Trading off w_1 versus w_2 also affects the number of hops in the optimal solution, important by itself for certain military applications, since using a large number of hops increases the probability of detection/interception.
- In general, the optimal tree for $w_2 = 0$ uses a far more number of hops – thus incurring higher average path delay – than the optimal tree for $w_1 = 0$.

Inadequacy of the TTFF criterion

- For the special case of $w_2 = 0$, an optimal polynomial time algorithm exists [Das et. al, *Globecom 2003*].
- If the residual energies of the nodes are identical, the objective function reduces to a “minimization of the maximum transmitter power” problem which can also be solved optimally in polynomial time.
- For $w_2 > 0$, it is unlikely that any optimal polynomial time algorithm exists, since the problem of minimizing the sum of transmitter powers ($w_1 = 0$) has been shown to be NP-complete [Cagalj et. al., *Mobicom 2002*].
- For the special case of equal residual energies and $w_1, w_2 \neq 0$, the problem reduces to a joint minimization of “maximum transmitter power and sum of transmitter powers”.

Inadequacy of the TTFF criterion

- Here we are concerned with the case when $w_1 \gg w_2 \neq 0$.
- For a proper choice of these parameter values, it is possible to obtain the "best possible" tree which maximizes the TTFF while ensuring that the solution is the most power efficient among the set of all trees with optimal TTFF.
- The concept of using a secondary optimization criterion, such as sum of transmitter powers, is not new [Ramanathan et. al, *Infocom 2000*] in the context of topology control of wireless ad-hoc networks.
- Using the most power efficient optimal TTFF tree also helps control the total interference power in the system.

MILP Model

- Let $\{F_{ij} : \forall(i \rightarrow j) \in \mathcal{E}\}$ be a set of flow variables (F_{ij} represents the flow from node i to node j).
- This problem can be interpreted as a single-origin multiple-destination uncapacitated flow problem – the source has D units of supply and the destination nodes have one unit of demand each.
- For other nodes, the net in-flow equals the net out-flow, since they serve only as relay nodes.
- This model can be viewed as a token allocation scheme where the source node generates as many tokens as there are destination nodes and distributes them along the “most efficient” tree such that each destination node gets to keep one token each.
- In Fig. 2, we have $F_{24} = 5, F_{41} = F_{43} = F_{45} = F_{46} = 1$ (rest are 0).

MILP Model

The single-origin multiple-destination flow problem can be solved using the usual *conservation of flow constraints*:

$$\sum_{j=1}^{N} F_{ij} \quad = \quad D; \qquad i = s, (i \rightarrow j) \in \mathcal{E} \tag{15}$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} \quad = \quad 1; \qquad \forall i \in \mathcal{D}, (i \rightarrow j) \in \mathcal{E} \tag{16}$$

$$\sum_{j=1}^{N} F_{ji} - \sum_{j=1}^{N} F_{ij} \quad = \quad 0; \qquad \forall i \notin \{\mathcal{D} \cup s\}, (i \rightarrow j) \in \mathcal{E} \tag{17}$$

MILP Model

Constraints linking the flow variables to the power variables, $\{Y_i\}$ are developed in two stages:

- First, we couple the flow variables and the indicator variables $\{X_{ij}\}$.
- Next, we link the $\{X_{ij}\}$ variables to the power variables.

Set of constraints which couple the flow variables and the X_{ij} variables are:

$$D \cdot X_{ij} - F_{ij} \geq 0; \ \forall (i \rightarrow j) \in \mathcal{E} \tag{18}$$

where D is the number of destination nodes.

- This constraint ensures that $X_{ij} = 1$ if $F_{ij} > 0$.
- The coefficient of X_{ij} here is due to the fact that the maximum flow out of any node on a single link is equal to the number of destination nodes.

MILP Model

- In Fig. 2, the status of the X_{ij} variables are $X_{24} = X_{41} = X_{43} = X_{45} = X_{46} = 1$, the rest being 0.

Next, we write down constraints linking the X_{ij} variables and the power variables.

- For an omni-directional antenna system, the cost of spanning in multiple nodes from node i is simply the cost incurred in reaching the farthest node. This condition is expressed as:

$$Y_i - \mathbf{P}_{ij} X_{ij} \geq 0;\ \forall i \in \mathcal{N}, \forall (i \rightarrow j) \in \mathcal{E} \tag{19}$$

- In order to relate the inverse TTFF parameter, σ, to the power variables, we note that $\sigma = \max_i Y_i / E_i$.

- As in Eq. (19), this condition can be written as:

$$\sigma - Y_i / E_i \geq 0;\ \forall i \in \mathcal{N} \tag{20}$$

MILP Model

- So far, we have implicitly assumed that the residual lifetimes of all transmitting nodes are greater than the (static) multicast duration.
- In other words, if L is the total number of bits to be transmitted during the session and R is the data rate in bps (assumed uniform throughout the network), we have assumed that:

$$E_i/Y_i \geq L/R \Longleftrightarrow Y_i/E_i \leq R/L \tag{21}$$

- Constraints of the form (21) can be explicitly added to the model to ensure that all nodes choose transmitter power levels such that their residual lifetimes are greater than or equal to the multicast session duration, L/R.

MILP Model

- The final set of constraints express the integrality of the X_{ij} variables and non-negativity of the F_{ij} and Y_i variables.
- The number of integer variables is equal to E while the number of continuous variables is equal to $E+N$. The number of constraints is equal to $2E+3N$.

$$X_{ij} \geq 0, \text{ integer}; \quad \forall i \in \mathcal{N} \qquad (22)$$

$$F_{ij} \geq 0; \quad \forall (i \rightarrow j) \in \mathcal{E} \qquad (23)$$

$$Y_i \geq 0; \quad \forall i \in \mathcal{N} \qquad (24)$$

Dealing with Prioritized Nodes

Our model so far assumes that all nodes enjoy equal priority in the network.

- We now consider the case where nodes may have unequal priorities, $e.g.$, depending on their location in the grid, or on their residual energies.
- For example, barycentric nodes may be assigned higher priorities to prevent their premature burn-out.

Let b_i be the priority associated with node i, $0 < b_i \leq 1$.

- The effective lifetime of node i, L_i^{eff}, is now defined as:

$$L_i^{\text{eff}} = E_i / b_i Y_i \tag{25}$$

- The actual lifetime of node i is still given by E_i/Y_i.
- The notion of effective lifetime is used only to guide the optimization process to choose a tree avoiding the nodes accorded the highest priorities

Dealing with Prioritized Nodes

- We now redefine the inverse TTFF parameter as follows:

$$\sigma = \text{max}_i \left(b_i Y_i / E_i \right) = 1/\tau \tag{26}$$

- This equation can be expressed as the following set of linear constraints:

$$\sigma - b_i Y_i / E_i \geq 0; \ \forall i \in \mathcal{N} \tag{27}$$

- Solving the optimization problem with Eq. (27) instead of Eq. (20) yields a node prioritized optimum solution.

Dealing with Prioritized Nodes

Consider the 3-node network below.

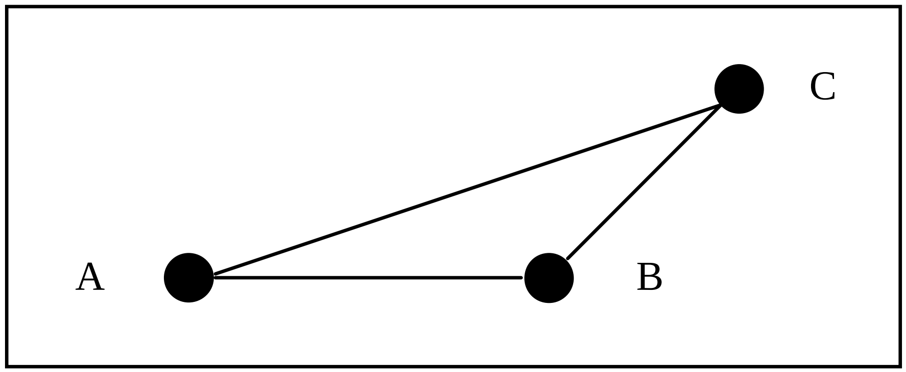

Figure 3: A 3-node network.

- Assume $\mathbf{P}_{AB} = 2$, $\mathbf{P}_{BC} = 1.5$, $\mathbf{P}_{AC} = 5$, $E_A = 10$ and $E_B = 5$. Let $b_A = b_B = 1$.

- The optimal TTFF broadcast tree, considering node A to be the source, is $\{A \rightarrow B, B \rightarrow C\}$, with a TTFF of $10/3$ (node B is the critical node).

Dealing with Prioritized Nodes

- If, however, $b_A = 0.5$ and $b_B = 1$ ($i.e.$, it is more important to preserve node B than A), the optimization process yields the broadcast tree $\{A \rightarrow C\}$, with node B reached implicitly.
- The effective lifetime of node A, as computed by the optimization process, is $E_A/b_A\mathbf{P}_{AC} = 10/(0.5 \times 5) = 4$ but its actual lifetime is $E_A/\mathbf{P}_{AC} = 10/5 = 2$.
- This example illustrates how node B can be preserved, at the expense of node A, by assigning suitable node weights.

Conclusion

- We have considered the problem of maximizing the time-to-first-failure in broadcast wireless networks.
- We showed that simply maximizing the TTFF criterion may not yield the best possible solution.
- This motivated us to consider a joint optimization problem involving the TTFF criterion and a secondary criterion such as the sum of transmitter powers.
- Finally, we presented a mixed integer linear programming model for solving the joint optimization problem and showed how the model can be modified to deal with prioritized nodes.

Future Work

- Currently, we are conducting extensive network simulations to quantify the effect of trading off total transmit power versus maximum transmit power on performance parameters such as throughput and end-to-end delay.
- Preliminary results confirm our intuition that multicast trees designed to minimize the maximum transmit power generally suffer from reduced throughput and higher latencies as the network load increases, compared to multicast trees which minimize the total transmit power.