Liang Dong, Yanqing Liu, and Robert J. Marks II.
"Reduction of Out-of-Bound Power and Peak-to-Average Ration in OFDM-Based Cognitive Radio Using Alternating Projections,"
2013 Texas Symposium on Wireless & Microwave Circuits & Systems, Waco, Texas, April 4-5, 2013 (4 pgs) DOI: 10.1109/WMCaS.2013.6563566

Reduction of Out-of-Band Power and Peak-to-Average Power Ratio in OFDM-Based Cognitive Radio Using Alternating Projections

Liang Dong, Yanqing Liu, and Robert J. Marks II
Department of Electrical and Computer Engineering
Baylor University, Waco Texas, 76798

***Abstract*—Non-contiguous OFDM technique can be used for the cognitive radio systems. The sidelobes of the OFDM-modulated tones cause the out-of-band power which can induce large interference to the incumbent communication systems. Another major drawback of OFDM-based systems is the high peak-to-average power ratio. In this paper, we propose a scheme to jointly reduce the out-of-band power and the peak-to-average power ratio of the OFDM transmission. A subset of the frequency tones do not carry any data but are weighted and the time-domain OFDM symbol is expanded at both edges with a few adjusting chips. The joint reduction scheme is developed based on the low-complexity method of alternating projections onto convex sets. In a few iterations, the transmitter removes the large peaks of the time-domain OFDM signal and reduces the out-of-band power. The receiver detects the data on the subcarriers with little performance degradation.**

I. INTRODUCTION

Non-contiguous OFDM (NC-OFDM) is a promising multi-carrier modulation technique for realizing cognitive radio systems [1], [2]. It reduces the dispersion effect of the multipath channel and simplifies the receiver equalizer. The system has high spectral efficiency with orthogonally overlapping subcarriers and easy implementation with existing Fourier transform modules. However, the out-of-band radiation in OFDM transmission interferes with wireless communications in adjacent channels and endangers the co-existence of the incumbent radio systems of the spectrum. There are several methods for reducing the out-of-band power (OBP) of the OFDM systems. OBP suppression can be achieved by the insertion of empty guard bands in the frequency domain. Alternatively, cancellation subcarriers can be used at the edges of the NC-OFDM bands. Cosovic and Mazzoni proposed a method of multiple choice sequences [3]. It transforms the original transmit sequence into a set of sequences and chooses the one with the lowest OBP. Yuan and Wyglinski proposed a method of combining cancellation subcarriers and modulated filter banks to suppress the sidelobes [4]. Pagadarai *et al.* proposed a method of constellation expansion to reduce the OBP [5]. It is not computationally intensive and no side information is needed. Li *et al.* proposed a method to reduce the OBP by iteratively adjusting the constellation points for the subcarriers that are close to the edges of the used bandwidth [6]. An orthogonal projections scheme was used by Zhang *et al.* to suppress the sidelobes of multicarrier systems [7].

Another drawback in OFDM systems is the high peak-to-average power ratio (PAPR). The high PAPR at the transmitter causes in-band signal distortion and out-of-band radiation due to the nonlinearity of the power amplifier. It also increases the complexity of the analog-to-digital and digital-to-analog converters. Senst *et al.* proposed a joint optimization to reduce the OBP and the PAPR [8]. It reserves a subset of the OFDM tones for the task. Ghassemi *et al.* proposed a method of joint reduction of the OBP and the PAPR using selected mapping [9]. It generates multiple representations of the transmit signal and select a sequence with low OBP and low PAPR.

In this paper, we propose a scheme of low-complexity joint reduction of the OBP and the PAPR of the NC-OFDM transmission using the method of alternating projections onto convex sets (POCS) [10]. In our method, we take advantage of the existing discrete Fourier transform (DFT) modules of the OFDM system. A subset of the subcarriers at the edges of each NC-OFDM frequency block are used as adjusting tones. The time-domain signal is expanded with a few adjusting chips on both edges. The projections in both time and frequency domains are simple to implement. The POCS iterations converge quickly and the method effectively reduces the OBP and removes the large signal peaks. The receiver needs very little modification and can accurately detect the data on the subcarriers.

II. OUT-OF-BAND POWER AND PEAK-TO-AVERAGE POWER RATIO

In an OFDM-based cognitive radio system, the data symbols of the secondary user are transmitted on the orthogonal subcarriers. The NC-OFDM transceiver activates those subcarriers that are not located in the bands occupied by the incumbent users. This is determined using dynamic spectrum sensing and channel estimation techniques. Suppose that an OFMD symbol has N subcarriers. Out of these N subcarriers, βN subcarriers can be used for the NC-OFDM transmission of the cognitive users. Suppose that the frequency indexes of these NC-OFDM subcarriers are in the set $\mathcal{B}$ such that the cardinality is $|\mathcal{B}| = \beta N$, $\beta \in (0, 1)$. The baseband OFDM signal in time-domain is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{kt}{T}} = \frac{1}{\sqrt{N}} \sum_{k\in\mathcal{B}} X_k e^{j2\pi \frac{kt}{T}} \qquad (1)$$

where T is the OFDM symbol duration and the subcarrier spacing is $1/T$. $\{X_k\}$ are the data on the subcarriers. For the NC-OFDM case, $X_k = 0$ for $k \notin \mathcal{B}$. The time-domain signal $x(t)$ has a support of $[0, T]$. That is, a rectangular window is applied to the time-domain signal. The spectrum is a convolution of the subcarrier data and the sinc function $S(f) = \mathrm{sinc}(f)$. We measure the out-of-band power (OBP) on M frequency samples adjacent to the NC-OFDM subcarriers.

These frequency samples can be taken more densely than how the subcarriers are spaced. Without any manipulation on the time-domain signal, the OBP can be given by

$$P_{\mathrm{OB}} = \sum_{m=1}^{M} \left| \sum_{k\in\mathcal{B}} X_k S(g_m - f_k) \right|^2 \tag{2}$$

where f_k is the normalized subcarrier frequency, g_m is the normalized frequency samples within the OBP range.

In order to reduce the OBP, for each sub-data-block in the frequency-domain in NC-OFDM, we choose the two most outside subcarriers to be the suppression tones. They do not carry any data but are weighted to suppress the sidelobes. In a low-complexity scheme as described later, we also expand the time-domain signal with a few additional chips to suppress the sidelobes. It should be noted that the calculation of the OBP in (2) is valid only when there is no manipulation on the time-domain signal such as peak clipping and chip adding. In our simulations, the OBP is directly calculated from the time-domain signal that is actually transmitted.

The continuous-time peak-to-average power ratio (PAPR) of an OFDM symbol is defined as

$$\mathrm{PAPR}[x(t)] = \frac{\max_{t\in[0,T)} |x(t)|^2}{\mathrm{E}[|x(t)|^2]} \tag{3}$$

where $\max |x(t)|^2$ is the maximum instantaneous power of the OFDM signal as $t \in [0, T)$. The continuous-time PAPR can be approximated by the discrete-time PAPR with oversampling on $x(t)$. The discrete-time OFDM signal with oversampling factor L is given by

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{kn}{LN}} = \frac{1}{\sqrt{N}} \sum_{k\in\mathcal{B}} X_k e^{j2\pi \frac{kn}{LN}} \tag{4}$$

where the discrete time-domain signal $x(n)$ has a support of $[0, LN-1]$. Usually, the oversampling factor can take an integer such that $L \geq 4$ for $\mathrm{PAPR}[x(n)]$ to approach the actural PAPR of the continuous-time signal. According to the Parseval's relation, we have

$$\mathrm{E}[|x(n)|^2] = \frac{1}{N} \sum_{k=0}^{N-1} |X_k|^2 = \frac{1}{N} \sum_{k\in\mathcal{B}} |X_k|^2. \tag{5}$$

With the data $\{X_k\}$ drawn with equal probability from an M-ary phase shift keying (MPSK) or an M-ary quadrature amplitude modulation (MQAM) constellation, the average power of an OFDM symbol $\mathrm{E}[|x(n)|^2]$ stays almost constant. This is in particular true for PSK-modulated data.

Suppose that the data symbols are chosen from an MPSK constellation with unit energy per data symbol, i.e. $|X_k| = 1$. Let vector $\mathbf{x}$ denote the sampled time-domain signal over a symbol period T. Because the high peaks in the time-domain OFDM signals occur rarely, a simple and effective approach is to set a PAPR threshold and clip the high peaks. However, in doing so, the data on the subcarriers are slightly distorted and the OBP is increased.

III. JOINT REDUCTION USING ALTERNATING PROJECTIONS ONTO CONVEX SETS

The OBP perceived in the frequency domain and PAPR perceived in the time domain can be jointly reduced by the method of alternating projections onto convex sets (POCS). Alternating projections method is an efficient algorithm that finds a point in the intersection of some convex sets of the sample space. For sets that do not intersect, POCS finds the point in one set that is closest to the other sets or finds a balanced point that is closest to each set. POCS uses a sequence of projections onto the sets and the convergence can be slow. Nevertheless, if the projections can be carried out by some analytical formula, the POCS method can be efficient.

The time-domain signal vector $\mathbf{x}$ is used as the vector point that is projected onto the convex sets. In fact, the time-domain signal vector over a symbol period $[0, T]$ is expanded on both ends with a few chip periods. For example, an OFDM time-domain symbol with N subcarriers may be expanded on each end with 2 chip periods, resulting in a signal vector over the period $[-2T/N, T + 2T/N]$. These additional chips are the adjusting chips that affect the OBP in the frequency domain. The expanded signal is actually transmitted and used in the POCS iterations. A few subcarriers of the OFDM frequency symbol are chosen as the adjusting tones. They do not carry any data but are automatically assigned with complex weights during the POCS iterations. These adjusting tones affect the OBP in the frequency domain as well as the PAPR in the time domain. We choose the adjusting tones that are the most outside subcarriers of each NC-OFDM sub-block since they have greater effects on the OBP. Suppose that the frequency indexes of these adjusting tones are in the set $\mathcal{A}$.

At the transmitter, the data are assigned to the subcarriers (in set $\mathcal{B}$) of the NC-OFDM symbol. The adjusting tones start with zero weights, i.e. $X_k = 0$ for $k \in \mathcal{A}$. The time-domain signal $\mathbf{x}$ is then calculated using the inverse discrete-time Fourier transform (IDFT) as in (4). For the first signal vector projection, the time-domain signal is clipped such that the maximum power is Γ. The threshold Γ can be determined by a preset PAPR value. As the signal is complex-valued, we keep the signal phase but change the signal amplitude to be within the threshold $\sqrt{\Gamma}$. We call this projection $\mathcal{P}_1$. Projection $\mathcal{P}_1$ projects the signal vector $\mathbf{x}$ onto the convex set

$$S_1 = \{\mathbf{x} \mid \|\mathbf{x}\|_\infty^2 \leq \Gamma\}. \tag{6}$$

The over-sampled time-domain signal is transferred to the frequency domain using DFT with frequency spacing $1/(2T)$. In doing so, we actually zero-pad in the time domain to make the signal over a period of $2T$ [11]. For the second signal vector projection, the subcarriers in set $\mathcal{B}$ are recovered with the original data X_k for $k \in \mathcal{B}$. The out-of-band is zeroed at frequencies with a $1/(2T)$ spacing. It is critical to evaluate the densely spaced spectrum with a factor of 2, because most OBP is located at frequencies that are $1/(2T)$, $3/(2T)$, $5/(2T)$, …away from the OFDM frequency block. We call this projection $\mathcal{P}_2$. Projection $\mathcal{P}_2$ projects the signal vector $\mathbf{x}$ onto the convex set

$$S_2 = \{\mathbf{x} \mid \{\mathbf{Qx}\}_k = X_k, k \in \mathcal{B}; \{\mathbf{Qx}\}_j = 0, j \in \mathcal{C}\} \tag{7}$$

where $\mathbf{Q}$ is the DFT matrix that takes into account both over-sampling and frequency-spacing, and $\mathcal{C}$ is the set of indexes of

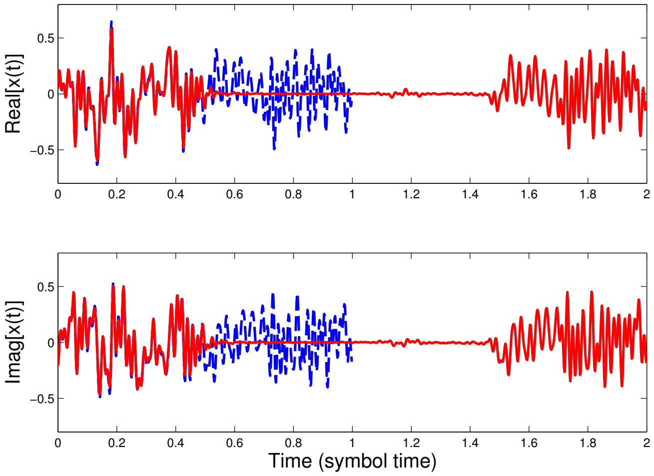

Fig. 1. Time-domain OFDM symbol.

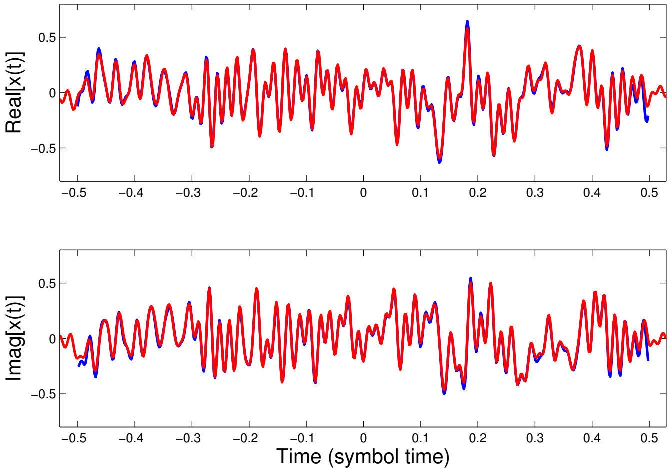

Fig. 2. Time-shifted and windowed OFDM symbol.

out-of-band frequencies that have spacing $1/(2T)$. Therefore, the POCS iterations are conducted between $\mathcal{P}_1$ and $\mathcal{P}_2$ to jointly reduce the OBP in the frequency domain and the PAPR in the time domain. Since the DFT and the IDFT can be carried out using the existing modules of the OFDM system, the POCS method provides a low-complexity scheme.

IV. DISCUSSION FOR IMPLEMENTATION

With practical concerns in a system implementation, there are three questions that arise naturally. (1) In Projection $\mathcal{P}_2$ and the following IDFT, we actually use the frequencies that are spaced $1/(2T)$ from the data subcarriers in the OFDM frequency block as more adjusting tones, and zero-force the $1/(2T)$ spaced frequencies in the out-of-band range. Does the transmitter need to extend its transmission time to $2T$ for each OFDM symbol? That would be a huge cost. (2) Does the receiver need major modification to detect the data? (3) The procedure is efficient in each POCS iteration. Does the method of POCS converge quickly for the particular problem of joint reduction of OBP and PAPR for a NC-OFDM system?

To answer Question (1), we realize that the out-of-band frequency components are generally much smaller than the data frequency components. Changing these out-of-band frequency components to zeros does not change much the time-domain signal. When the original frequency-domain signal is twice densely sampled and takes the IDFT, the time-domain signal is zero-padded such that one OFDM symbol is over period $[0, 2T]$. Forcing out-of-band frequency components to be zeros will change little in the time domain therefore there are still many time-chips that are close to zero. We omit these time-chips in the transmission by effectively windowing the time-domain signal with a window length slightly more than T. In Fig. 1, for example, the blue curve is the IDFT of the original OFDM symbol with 64 subcarriers. The red curve is the IDFT of twice densely sampled OFDM symbol with zero-forced out-of-band components. In Fig. 2, the signal is time-shifted and windowed by a window of a length slightly more than T. In this case, the window is 4 chips longer than a symbol period T such that the window is over $[-T/32-T/2, T/2+T/32]$. We call the additional 4 chips at the edges (2 at each edge) of the time-domain signal the adjusting chips. They are automatically assigned during the POCS iterations and have a great effect in reducing the OBP. The above time shifting has an effect that changes the sign of the data on each odd frequency subcarrier. This can be corrected accordingly in implementation.

To answer Question (2), we state that the receiver only needs to extend the time-domain symbol little beyond T to include the few adjusting chips. The receiver can still sample at the chip rate. With the above example, the receiver samples 68 chips for one OFDM symbol and processes with DFT. The orthogonality of the OFDM subcarriers is mostly preserved and the data on the subcarriers can be accurately detected.

Currently, we answer Question (3) with our simulation results. It appears that the method of POCS converges quickly. If the convex sets are far apart, the vector point will be projected back and forth onto the sets. In practice, we can stop at $\mathcal{P}_1$ or $\mathcal{P}_2$ depending on whether the OBP or the PAPR is more critical. In addition, we can use simultaneous weighted projections to find a compromised solution.

V. NUMERICAL RESULTS

We simulate a NC-OFDM system for the cognitive radios. There are $N = 64$ OFDM subcarriers with normalized frequencies $k = 0, 1, \ldots, 63$. Subcarriers $k \in [16, 23]$ and $k \in [40, 47]$ have to be silenced for the transmission of incumbent frequency users. The 6 tones at the edges of each NC-OFDM sub-frequency-block are selected to be the adjusting tones. Therefore, there are 42 data can be transmitted with one NC-OFDM symbol. The data are QPSK modulated. As the frequency-domain signal is twice densely sampled, the time-domain signal is shifted and windowed as described before. The signal period is extended such that there are 2 chips on each edge that are the adjusting chips. Therefore, the efficiency in the use of bandwidth of the proposed scheme is $\rho = 42/48 \times 64/68 = 0.8235$. The threshold PAPR is 5 dB to clip the peaks of the time-domain signal.

For a particular NC-OFDM symbol with randomly generated QPSK data, Fig. 3 shows the power spectrum density (PSD) over the range of normalized frequency $[-16, 80]$. The blue curve shows the PSD of the original NC-OFDM symbol. The green curve shows the PSD of the signal with

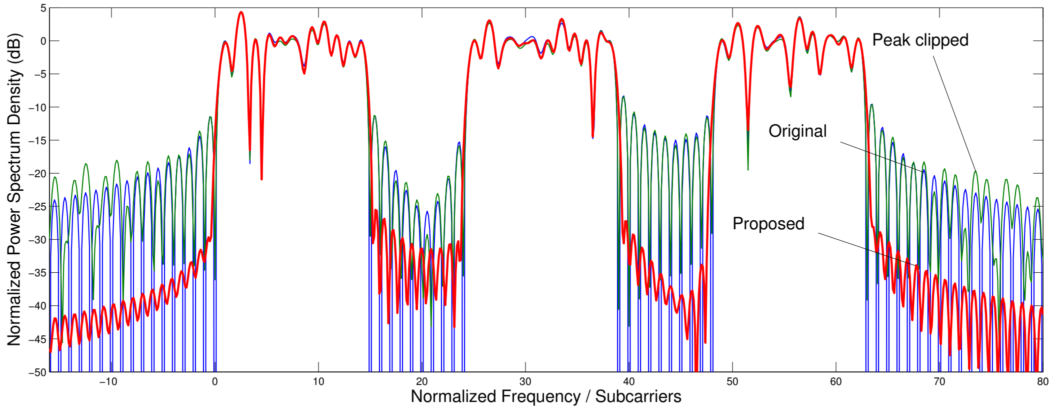

Fig. 3. Spectrum density of the NC-OFDM transmission.

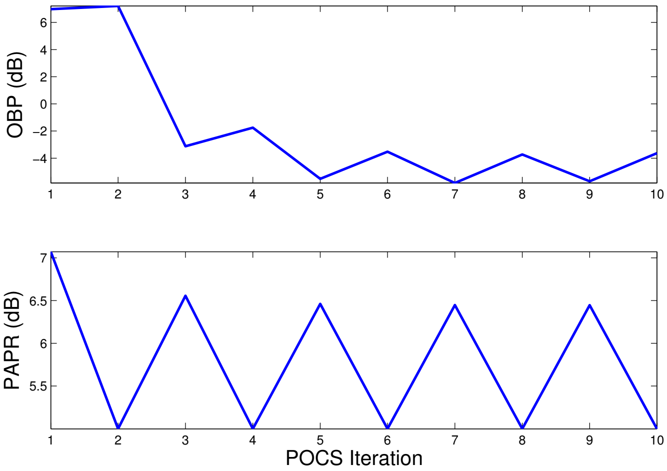

Fig. 4. OBP and PAPR at POCS iterations.

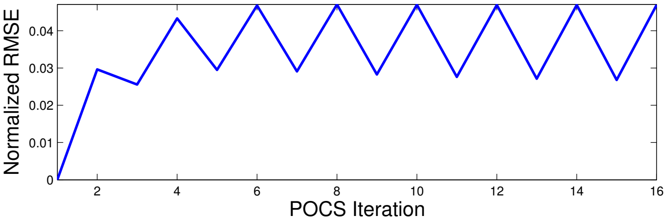

Fig. 5. Normalized RMSE of the received data.

only peak-clipping at PAPR 5 dB. The red curve shows the PSD of the proposed method of POCS after 5 iterations. The proposed scheme has the least out-of-band transmission with little degradation in data transmission. Fig. 4 shows the OBP and the PAPR at each POCS iteration. The method of POCS converges quickly such that the OBP is reduced more than 10 dB and the PAPR is brought down close to the threshold. In order to evaluate the reception performance, Fig. 5 shows the normalized root-mean-squared-error (RMSE) between the estimated data and the true QPSK data. At the receiver, the normalized RMSE is about 0.03 between the estimated data and the true QPSK data. The errors are small and they are due to the slight offset in the orthogonality in the OFDM symbol.

VI. CONCLUSION

A low-complexity scheme is proposed for joint reduction of out-of-band power and peak-to-average power ratio of OFDM-based cognitive radios. It takes advantage of the existing DFT modules of the OFDM system and is based on the method of alternating projections onto convex sets. The projections of the signal vector point are simple. The vector point is tracked with adjusting tones in the frequency domain and adjusting chips in the time domain. The transmission performance is improved with trade-off in bandwidth and offset in OFDM orthogonality.

REFERENCES

[1] H. Bogucka, A. M. Wyglinski, S. Pagadarai, and A. Kliks, “Spectrally agile multicarrier waveforms for opportunistic wireless access,” *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 108–115, Jun. 2011.

[2] D. Qu, J. Ding, T. Jiang, and X. Sun, “Detection of non-contiguous OFDM symbols for cognitive radio systems without out-of-band spectrum synchronization,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 693–701, Feb. 2011.

[3] I. Cosovic and T. Mazzoni, “Suppression of sidelobes in OFDM systems by multiple-choice sequences,” *Eur. Trans. Telecomms.*, vol. 17, pp. 623–630, 2006.

[4] Z. Yuan and A. M. Wyglinski, “On sidelobe suppression for multicarrier-based transmission in dynamic spectrum access networks,” *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1998–2006, May 2010.

[5] S. Pagadarai, R. Rajbanshi, A. M. Wyglinski, and G. J. Minden, “Sidelobe suppression for OFDM-based cognitive radios using constellation expansion,” in *IEEE Wireless Communications and Networking Conference (WCNC)*, Apr. 2008, pp. 888–893.

[6] D. Li, X. Dai, and H. Zhang, “Sidelobe suppression in NC-OFDM systems using constellation adjustment,” *IEEE Commun. Lett.*, vol. 13, no. 5, pp. 327–329, May 2009.

[7] J. A. Zhang, X. Huang, A. Cantoni, and Y. J. Guo, “Sidelobe suppression with orthogonal projection for multicarrier systems,” *IEEE Trans. Commun.*, vol. 60, no. 2, pp. 589–599, Feb. 2012.

[8] M. Senst, M. Jordan, M. Dorpinghaus, M. Farber, G. Ascheid, and H. Meyr, “Joint reduction of peak-to-average power ratio and out-of-band power in OFDM systems,” in *IEEE Global Telecommunications Conference (GLOBECOM)*, Nov. 2007, pp. 3812–3816.

[9] A. Ghassemi, L. Lampe, A. Attar, and T. A. Gulliver, “Joint sidelobe and peak power reduction in OFDM-based cognitive radio,” in *IEEE Vehicular Technology Conference (VTC)*, Sep. 2010, pp. 1–5.

[10] R. J. Marks, *Handbook of Fourier Analysis & Its Applications*. Oxford University Press, 2009.

[11] E. M. Lizarraga, A. A. Dowhuszko, and V. H. Sauchelli, “Improving out-of-band power emissions in OFDM systems using double-length symbols,” *IEEE Latin America Transactions*, vol. 10, no. 3, pp. 1710–1718, Apr. 2012.