Physics

R.J. Marks II Class Notes

(1968-1970)

Physics I. Assignments - Winter Quarter 1968-69

Date	Assignment & Problems
1/7	Chap I.
1/8	Chap II, 2,4,5,7
1/9	Chap II, 11,12,14,18
1/10	Chap II, 20,24 III, 2,5
1/14	Chap III, 6,8,9,10
1/15	Chap III, 12,13,15,16
1/16	Chap III, 23,24,29,31
1/17	Test II & III
1/21	~~Chap IV, 2,4,5,7.~~
1/22	~~Chap IV, 8,10,12,13~~
1/23	Chap IV, 22,24,29,34
1/24	Chap IV, 20, V, 1,4
1/28	Chap V, 5,6,7,8
1/29	~~Chap V, 10,12,15,17~~
1/30	Chap V, 19,20,22,29
1/31	Chap VI, 2,3,4,5
2/4	Chap VI, 9,10,11,12
2/5	Chap VI, 16,18,20,25
2/6	Test IV, V, VI
2/7	Chap VII, 2,3,5,6
2/11	Chap VII, 9,12,13,15
2/12	Chap VII, 19,20,21,22
2/13	Chap VIII, 1,2,5
2/14	Chap VIII, 3,6,7,9
2/18	Chap VIII, 11,12,13,14
2/19	Chap VIII, 15,24,25,26
2/20	Test VII and VIII
2/21	Chap IX, 1,5,6
2/25	Chap IX, 7,9,10,11
2/26	Chap IX, 14,16,17
2/27	Chap IX, 19,22,27
2/28	Chap X, 2,3,4,6
3/4	Chap X, 9,10,13,15
3/5	Chap X, 19,22,27
3/6	Chap X, 29,31,33
3/7	Test Chap. IX and X
3/11	Chap XI, 3,4,6,8
3/12	Chap XI, 9,10,13,16
3/13	Chap XII, 1,4,5
3/14	Chap XII, 13.

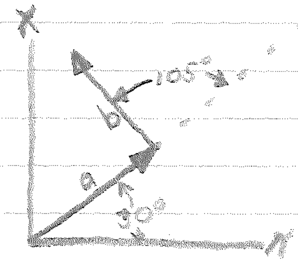
FINAL EXAM.

40 10 WKS

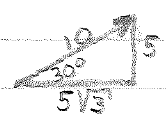
PHYS

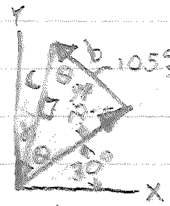
Pg. 28

4)



$\vec{a} \Rightarrow X = 5\sqrt{3} \quad Y = 5$



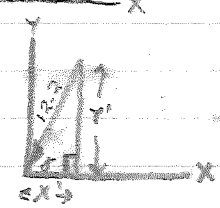


$\Theta = \frac{1}{2}(105^\circ) = 52\frac{1}{2}^\circ$

$|\vec{r}| = 2b\cos\Theta$

$= 20\cos 52\frac{1}{2}^\circ$

$|\vec{r}| \cong 20(.609) = 12.2$



$\gamma = 82.5^\circ$

$Y' = 12.2 \sin\gamma$

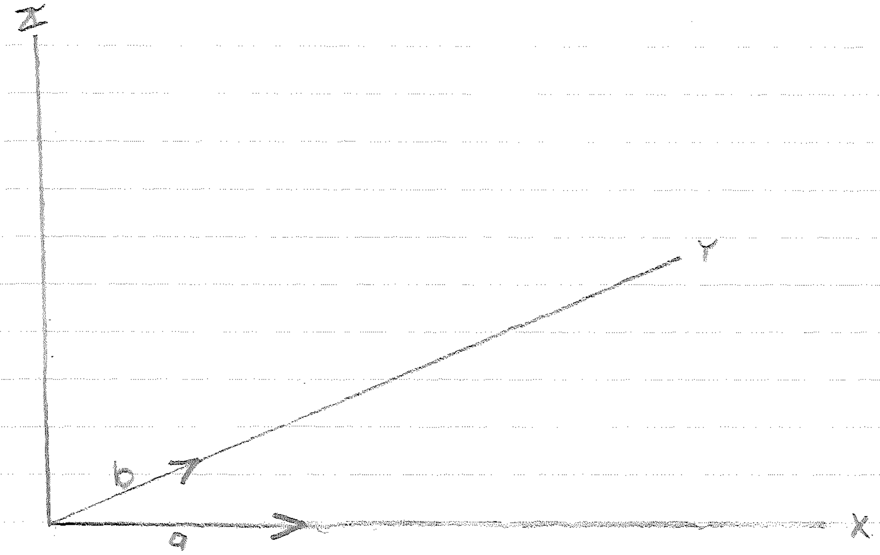
$Y' = 12.2(.991) = 12.1$

$X' = 12.2 \cos\gamma$

$X' = 12.2(.130) = 1.58$

Pg 28

20)



a) $+Z$

b) $-Z$

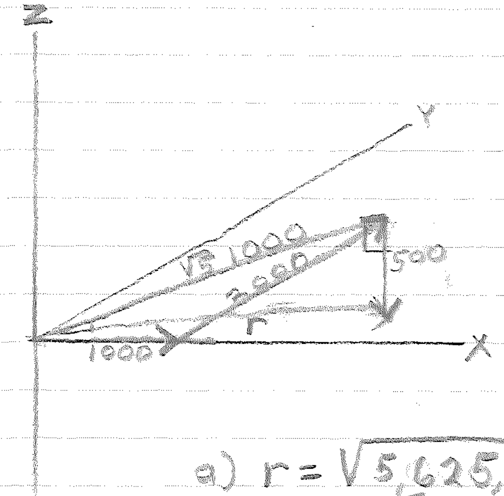
c) $+Y$

d) $a \cdot b = ab \cos \frac{\pi}{2} = 0$

BOB MARKS

Pg 29

12)



2/5

a) $r = \sqrt{5,625,000} = 10\sqrt{5,625} = 750$ ft

b) 0

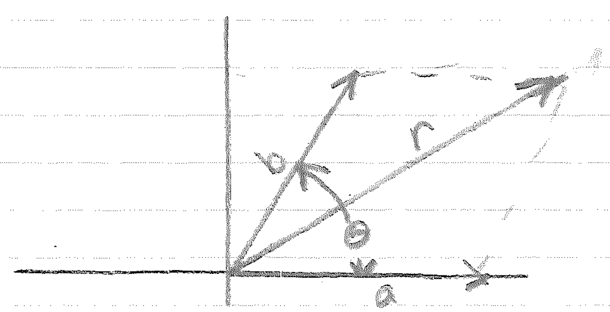
14) a) 2.5 m S

b) 1.25 m N

c) 6.25 m S

d) 10 m N

18)



$r_x = a_x + b_x$

$= a + b\cos\theta$

$r_y = a_y + b_y$

$= 0 + b\sin\theta$

$r = \sqrt{b^2\sin^2\theta + (a + b\cos\theta)^2}$

$r = \sqrt{b^2\sin^2\theta + a^2 + 2ab\cos\theta + b^2\cos^2\theta}$

$r = \sqrt{b^2(\sin^2\theta + \cos^2\theta) + a^2 + 2ab\cos\theta}$

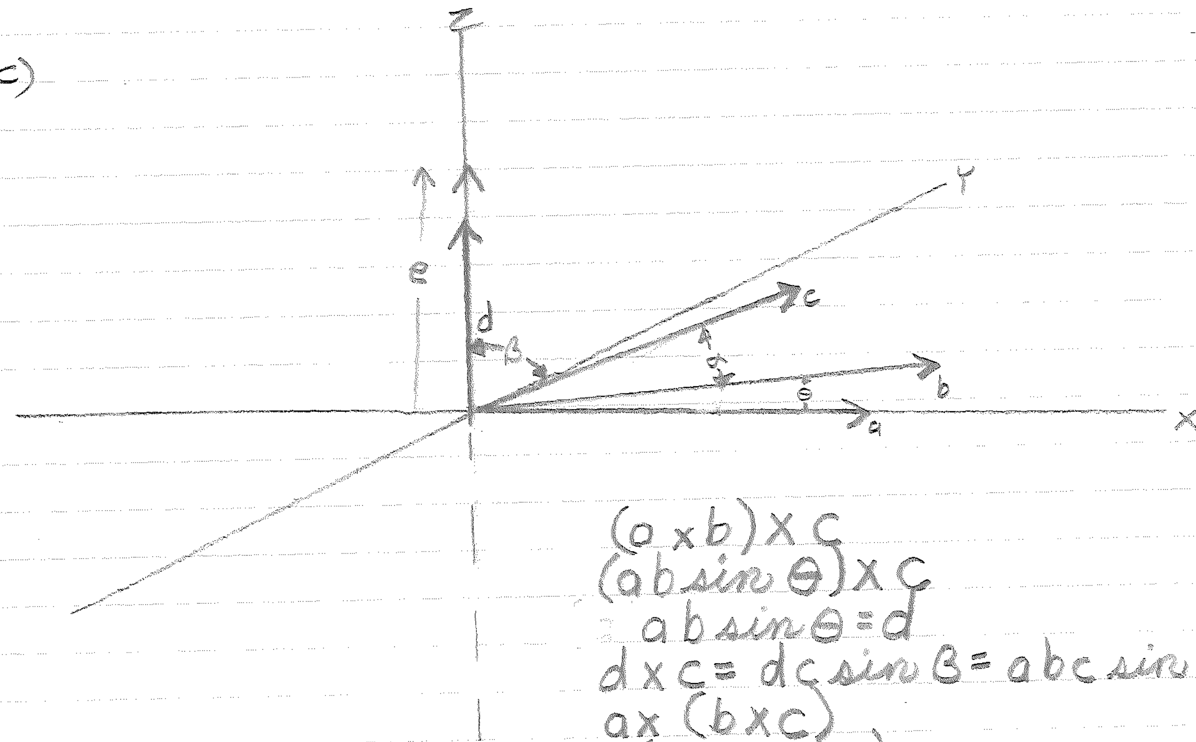
$r = \sqrt{b^2 + a^2 + 2ab\cos\theta}$

Pg 30

24) $a \cdot b = ab\cos\theta$ $b \cdot a = ba\cos\theta$

$ab\cos\theta = ab\cos\theta$

c)



$(a \times b) \times c$

$(ab\sin\theta) \times c$

$ab\sin\theta = d$

$d \times c = dc\sin\beta = abc\sin\theta\sin\beta$

$a \times (b \times c)$

$a \times (bc\sin\alpha)$

$e = bc\sin\alpha$

$a \times e = ae\sin\beta = abc\sin\alpha\sin\beta$

$\sin\beta = \sin\frac{\pi}{2} = 1$

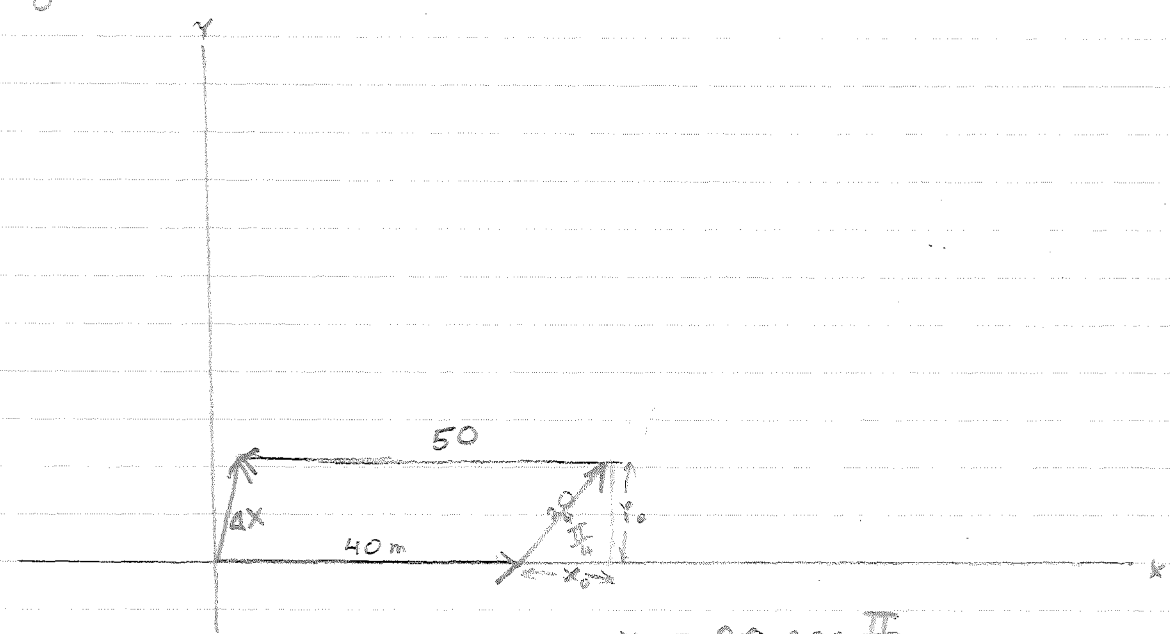
if $(a \times b) \times c = a \times (b \times c)$ then

$abc\sin\theta = abc\sin\alpha$

$\sin\theta = \sin\alpha$

Pg 52

2)



$V = 60\ mph$

$V_{AVE} = \frac{\Delta r}{\Delta t}$

$X_o = 20 \cos\frac{\pi}{4}$

$= 20 \frac{1}{\sqrt{2}} = \frac{20}{\sqrt{2}}$

$Y_o = X_o = \frac{20}{\sqrt{2}}$

$\vec{r}_a = t_1\, 60$

$= \frac{2}{3}\, 60 = 40$

$\vec{r}_b = t_2\, 60$

$= \frac{1}{3}\, 60 = 20$

$\vec{r}_c = t_3\, 60 = 50$

$r_{ax} = 40$

$r_{bx} = \frac{20}{\sqrt{2}}$

$r_{cx} = -50$

$\Delta r_x = \frac{20}{\sqrt{2}} - 10 \cong 4.14$

$r_{ay} = r_{cy} = 0$

$r_{by} = 20/\sqrt{2} \cong 14.1$

$\Delta r_y = 14.1$

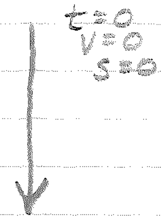
$\Delta\vec{r} = \sqrt{200 + 17.1} = \sqrt{217} = 14.3$

$\Delta t = \frac{40}{60} + \frac{20}{60} + \frac{50}{60} = \frac{11}{6}\ hrs.$

$V_{AVE} = \frac{14.3}{\frac{11}{6}} = \frac{6(14.3)}{11} = 7.8\ MPH$

Pg 52

5)



$a = 32$

$V = 32t + c_1$

$c_1 = 0$

$V = 32t$

$s = 16t^2 + c_2$

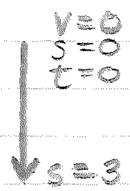
$c_2 = 0$

$s = 16t^2$

$s = 4$

$4 = 16t^2 \quad t = \frac{1}{2}$

$\underline{V = 16}$



$a = 32$

$V = 32t + C_1 \quad c_1 = 0$

$V = 32t$

$S = 16t^2 + C_2 \quad C_2 = 0$

$S = 16t^2$

$s = 3$

$3 = 16t^2$

$\frac{3}{16} = t^2$

$t = \frac{\sqrt{3}}{4}$

$V = 32 \frac{\sqrt{3}}{4} = 8\sqrt{3}$

$a_{AVE} = \frac{\Delta V}{\Delta t}$

$\Delta V = 16 + 8\sqrt{3} \quad \Delta t = .01$

$\cong 30$

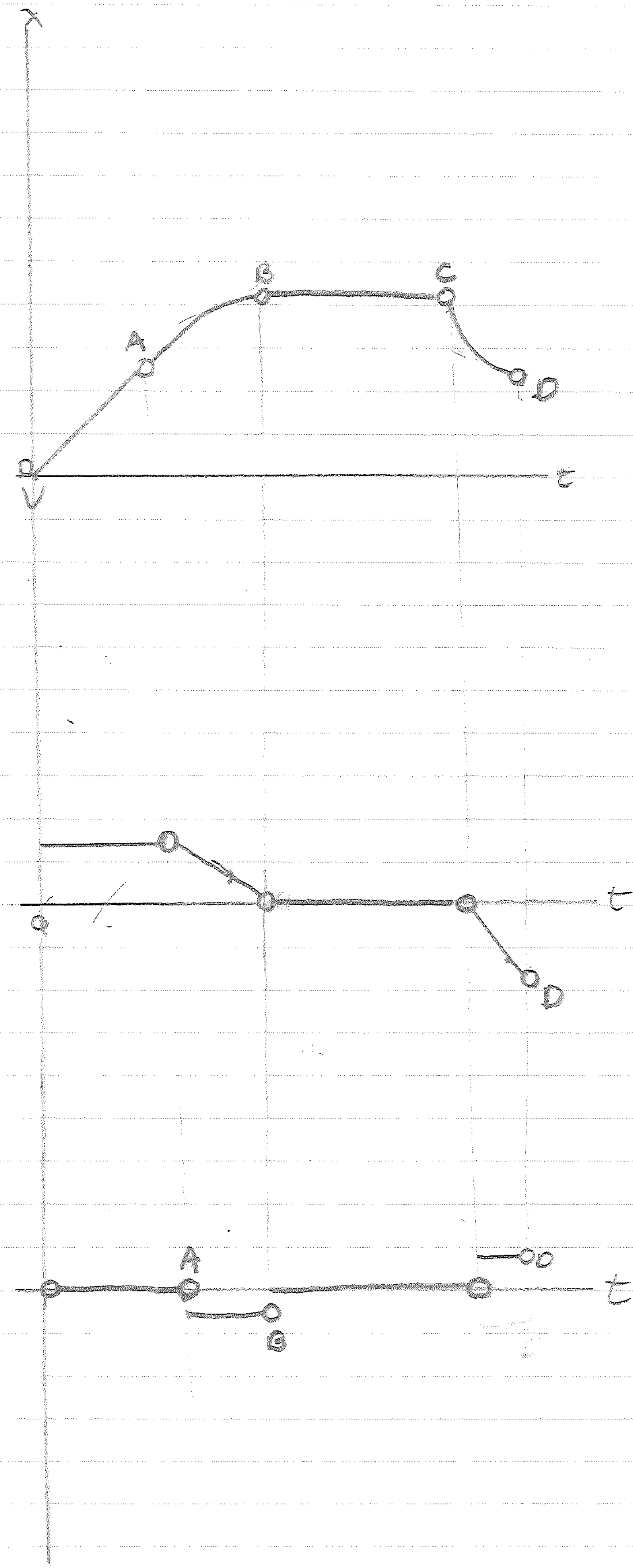
$a_{AVE} = \frac{30}{.01} = 3{,}000 \text{ ft/sec}^2$

Pg 53

6) o-a +
a-b +
b-c 0
c-d −

o-a 0
a-b −
b-c 0
c-d +

no



Pg 53

8) $a_{AVE} = \frac{\Delta V}{\Delta t}$

$V_0 = 0 \qquad V_f = 200$

$a = k$

$V = kt + c$

at $V=0,\ t=0$

$\therefore c = 0$

$V = kt$

$s = \frac{k}{2}t^2 + C_2$

at $s=0,\ t=0 \therefore C_2 = 0$

$200 = kt \qquad 2 = \frac{k}{2}t^2$

$t = \frac{200}{k} \qquad t^2 = \frac{4}{k}$

$t = \frac{2}{\sqrt{k}}$

$\frac{200}{k} = \frac{2}{\sqrt{k}}$

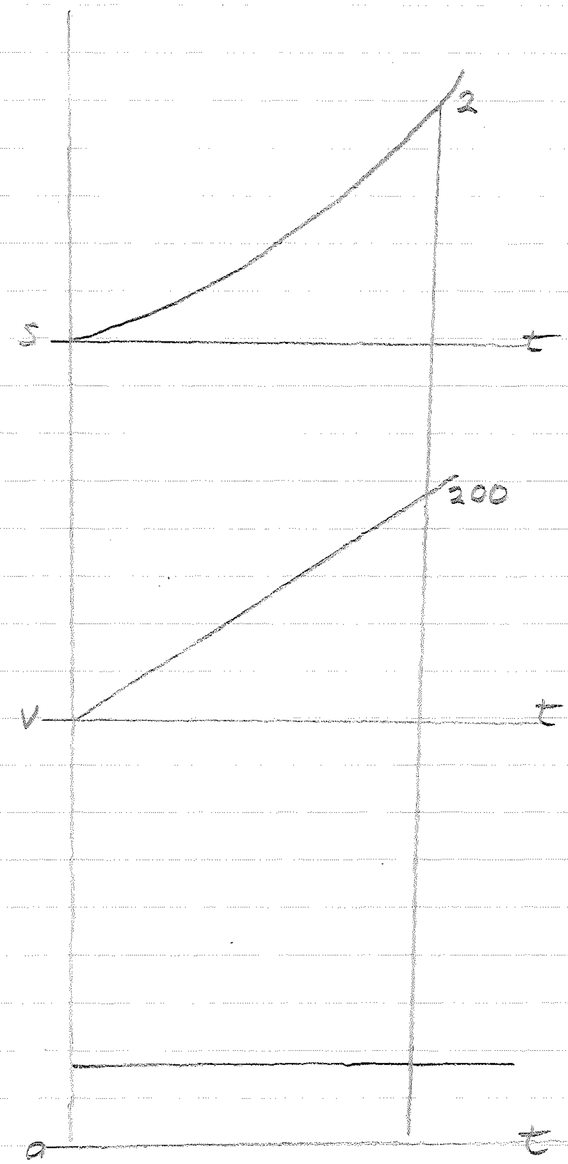
$200\sqrt{k} = 2k$

$100\sqrt{k} = k$

$10,000k = k^2$

$k^2 - 10,000k = 0$

$k = 10,000 \frac{ft}{sec^2}$



Pg 53

9) $\Delta v = 4.0 \times 10^6 - 1.0 \times 10^4$

$= 400 \times 10^4 - 1.0 \times 10^4$

$= 399 \times 10^4 = 3.99 \times 10^6$

$a = k$

$v = kt + c$

at $v = 10^4$, $t = 0$ ∴ $c = 10^4$

$v = kt + 10^4$

$s = \frac{k}{2}t^2 + 10^4 t + c_2$

at $s = 0$, $t = 0$ ∴ $c_2 = 0$

$s = 10^{-2} = \frac{k}{2}t^2 + 10^4 t$ $\qquad v = 4 \times 10^6 = kt + 10^4$

$\frac{k}{2}t^2 + 10^4 t - 10^{-2} = 0$ $\qquad t = \frac{4 \times 10^6 - 10^4}{k}$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-10^4 \pm \sqrt{10^8 + 2k(10^{-2})}}{k} = \frac{4 \times 10^6 - 10^4}{k}$

$-10^4 \pm \sqrt{10^8 + 2k(10^{-2})} = 4 \times 10^6 - 10^4$

$\pm \sqrt{10^8 + 2k(10)^{-2}} = 4 \times 10^6$

$10^8 + 2k(10)^{-2} = 16 \times 10^{12}$

$2k10^{-2} = 16 \times 10^{12} - 10^8$

$= 160000 \times 10^8 - 10^8$

$= 159999 \times 10^8$

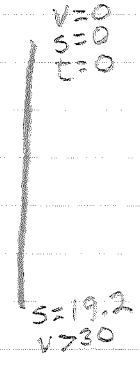
$2k = 159999 \times 10^{10}$

$k = 79999.5 \times 10^{10}$

$\approx 8.00 \times 10^{14}$

Pg 53

10) $30\,\frac{M}{H} \times \frac{1.47\,\frac{ft}{sec}}{1\,\frac{M}{H}} = 44.1\,\frac{ft}{sec}$



$a = k$

$v = kt$

$s = \frac{k}{2}t^2$

at $s = 19.2$

$38.4 = kt^2$

$t = \frac{6.20}{(\sqrt{k})}$

$v = k^{\frac{1}{2}}(6.20)$

if $k = 32$

$v = (5.66)(6.20) > 30$

∴ mom was speeding.

Pg 53

12)

$V=0$
$s=0$
$t=0$

$a = 32$

$V = 32t + C$

$C = 0$

$V = 32t$

$1.86 \times 10^5 \frac{M}{S} \times \frac{5280 ft}{M} = 9.81 \times 10^8 \frac{ft}{sec}$

$\frac{1}{4}\left(9.81 \times 10^8 \frac{ft}{sec}\right) = 2.45 \times 10^8 \frac{ft}{sec}$

$2.45 \times 10^8 = 32t$

$245 \times 10^6 = 32t$

$t = 7.66 \times 10^6 \text{ sec}$

$s = 16t^2$

$= 16(7.66 \times 10^6)^2$

$= 939 \times 10^{12}$

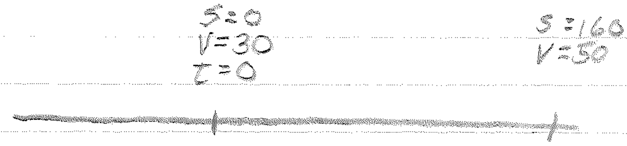
$= 9.39 \times 10^{14}$

5

Pg 53-54

13)

a)



$a = k$

$V = kt + C$

$V = kt + 30$

$s = \frac{k}{2}t^2 + 30t + C_2$

$s = \frac{k}{2}t^2 + 30t$

$160 = \frac{k}{2}t^2 + 30t$ $\qquad$ $50 = kt + 30$

$\frac{k}{2}t^2 + 30t - 160 = 0$ $\qquad$ $kt = 20$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad$ $t = \frac{20}{k}$

$t = \frac{-30 \pm \sqrt{900 + 320k}}{k}$

$-30 \pm \sqrt{900 + 320k} = 20$

$\pm\sqrt{900 + 320k} = 50$

$900 + 320k = 2500$

$32k = 160$

$k = 5.0 = a$ ✓

b) $s = \frac{k}{2}t^2 + 30t$

$k = 5 \quad s = 160$

$160 = \frac{5}{2}t^2 + 30t$

$5t^2 + 60t - 320 = 0$

$t^2 + 12t - 64 = 0$

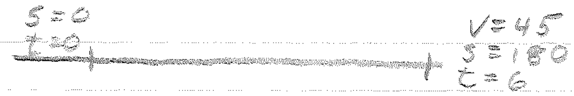
$t = \frac{-12 \pm \sqrt{144 + 256}}{2}$

$= \frac{-12 \pm \sqrt{400}}{2}$

$= \frac{20 - 12}{2} = 4$ sec ✓

Pg 54

15)



$a = K$

$V = Kt + C_1$

at $V = 45$, $t = 6$ $\therefore C_1 = 45 - 6K$

$V = Kt + 45 - 6K$

$s = \frac{K}{2}t^2 + (45 - 6K)t + C_2$

at $s = 180$, $t = 6$

$180 = \frac{K}{2}(36) + (45 - 6K)6 + C_2$

$180 = 18K + 270 - 36K + C_2$

$C_2 = 18K - 90$

$s = \frac{K}{2}t^2 + (45 - 6K)t + (18K - 90)$

$s = 0$ $t = 0$

$18K = 90$

$K = 5$

b) $a = 5\ ft/sec^2$

$V = Kt + 45 - 6K$

$K = 5$ $t = 0$

a) $V = 45 - 30 = 15\ ft/sec$

c) $a = 5$

$V = 5t$

$s = \frac{5}{2}t^2$

$V = 15$ $\therefore t = 3$

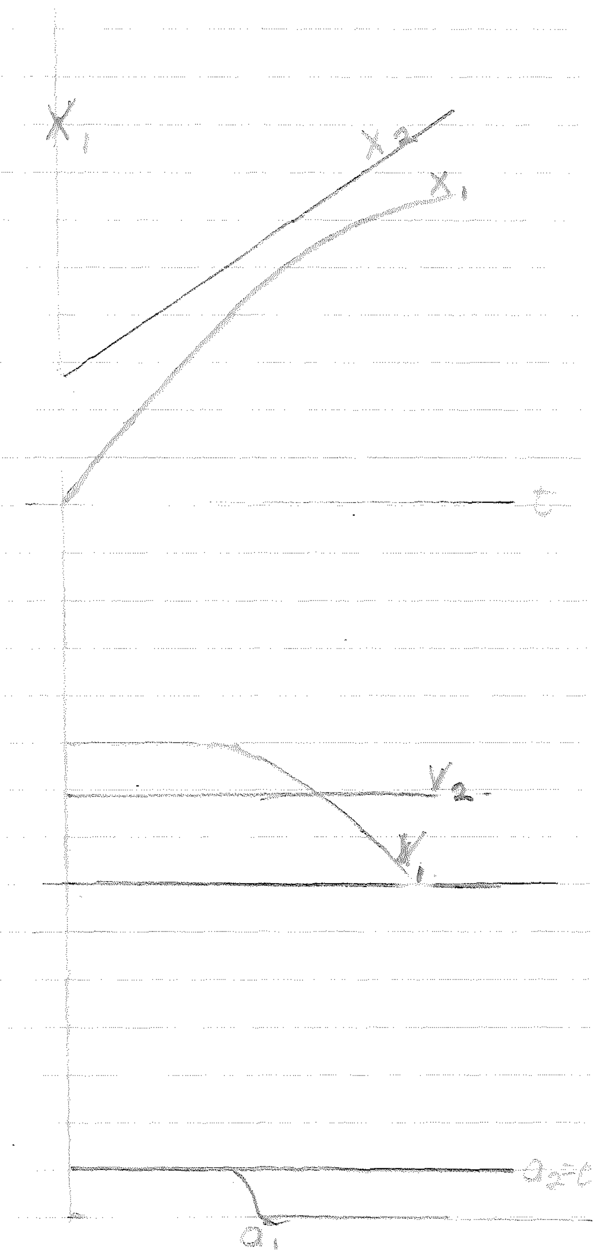
$s = \frac{5}{2}(9) = \frac{45}{2} = 22.5\ ft$

Pg 54

(6)

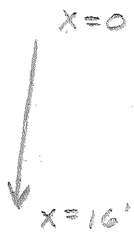
$a = k$

$V = kt \quad v_1$



Pg 54-5

24)



a) $X = \frac{a}{2}t^2 + V_{ox}t + x_o$

$X = \frac{a_x}{2}t^2 = 16$

$32t^2 = 32$

$t = 1$ SEC

$V_x = a_x t + V_{ox}$

$= a_x t$

$= 32$ ft/sec

$32 \frac{ft}{sec} \times 5$ sec $= 160$ ft

b) $V_{AVE} = \frac{\Delta x}{\Delta t} = \frac{176 \text{ FT}}{6 \text{ SEC}} = 29.4 \frac{FT}{SEC}$

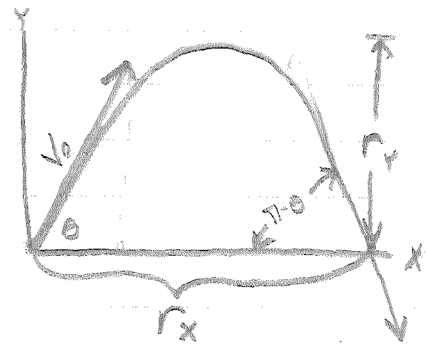
c) $X = \frac{a}{2}t^2 + V_{ox}t + X_o$

$176 = 16(5)^2 + V_{ox}5 + 0$

$176 = 400 + V_{ox}5$

Pg 75

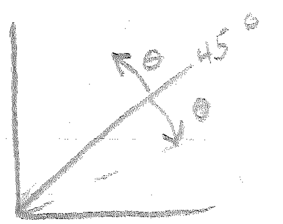
7)



$r_y = r_x$

$$r_y = \frac{a}{2}t^2 \qquad r_x = V_0 \sin\theta\, t$$
$$r_y = 16t^2 \qquad r_x = V_0 \tfrac{1}{4}\sqrt{r_y}\, \sin\theta$$
$$t = \tfrac{1}{4}\sqrt{r_y} \qquad \sqrt{r_x} = \tfrac{1}{4} V_0 \sin\theta$$
$$r_y = r_x = \tfrac{1}{16} V_0^2 \sin^2\theta$$

8)



$$R = \frac{V_0^2 \sin 2\theta}{g}$$

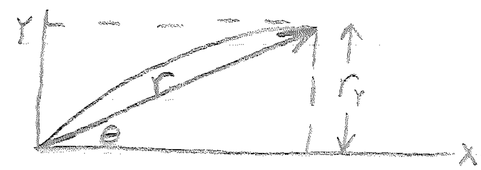
$$R_1 = \frac{V_0^2 \sin 2(\frac{\pi}{4}+\theta)}{g} \qquad R_2 = \frac{V_0^2 \sin 2(\frac{\pi}{4}-\theta)}{g}$$

$$\because R_2 = R_1 \Rightarrow \frac{2V_0^2 \sin(\frac{\pi}{4}+\theta)\cos(\frac{\pi}{4}+\theta)}{g} = \frac{2V_0^2 \sin(\frac{\pi}{4}-\theta)\cos(\frac{\pi}{4}-\theta)}{g}$$

$$(\sin\tfrac{\pi}{4}\cos\theta + \sin\theta\cos\tfrac{\pi}{4})(\cos\tfrac{\pi}{4}\cos\theta - \sin\tfrac{\pi}{4}\sin\theta) =$$
$$(\sin\tfrac{\pi}{4}\cos\theta - \cos\tfrac{\pi}{4}\sin\theta)(\cos\tfrac{\pi}{4}\cos\theta + \sin\tfrac{\pi}{4}\sin\theta)$$
$$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$$

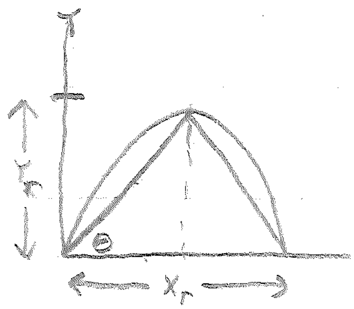
Pg 74

5)



$$a = g$$
$$V_y = gt - V_{oy}$$
$$V_{oy} = V_0 \sin\theta$$
$$r_y = \frac{g}{2}t^2 + V_0 \cos\theta$$
$$\frac{dr_y}{d\theta} = gt - V_0 \sin\theta = 0$$
$$gt = V_0 \sin\theta$$
$$t = \frac{V_0 \sin\theta}{g}$$
$$Y_{MAX} = \frac{g}{2}\left(\frac{V_0 \sin\theta}{g}\right)^2 + V_0 \cos\theta$$
$$= \frac{(V_0 \sin\theta)^2}{2g}$$

7)

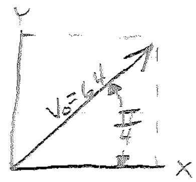


$$Y_r = X_r$$
$$V_r = 2V_0 \csc\theta$$
$$X_r = 2V_0 t \csc\theta$$
$$a_y = g$$
$$V_y = gt$$
$$Y_r = \frac{g}{2}t^2$$

Bob Markow

Pg 75

12)

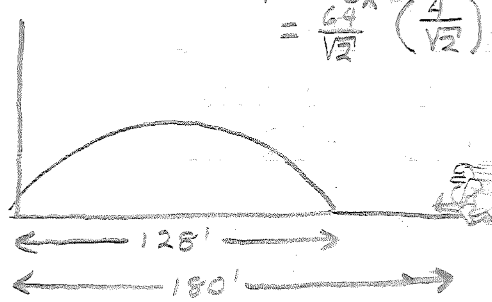


$$V_{oy} = V_{ox} = 64 \cos\frac{\pi}{4} = \frac{64}{\sqrt{2}}$$

$V=0$ $s=0$
$t=0$
$a=32$
$v=\frac{64}{\sqrt{2}}$

$$V = at_1$$
$$\frac{64}{\sqrt{2}} = 32t$$
$$t_1 = \frac{2}{\sqrt{2}}$$
$$t = 2t_1 = \frac{4}{\sqrt{2}}$$

$$r_x = V_{ox}t$$
$$= \frac{64}{\sqrt{2}}\left(\frac{4}{\sqrt{2}}\right) = 128\text{ ft}$$ ✓



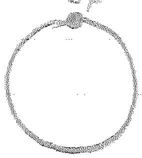
$$r = 180' - 128' = 52'$$
$$t = \frac{4}{\sqrt{2}}$$
$$V_{foot} = \frac{52\sqrt{2}}{4} \cong 18.3\text{ ft/sec}$$ ✓

5

Pg 77

22)

$S_p = 2.18 \times 10^{6}$



$\longleftarrow 5.28 \times 10^{-11} = r$

$$a = \frac{V^2}{r} = \frac{(2.18 \times 10^{6})^2}{5.28 \times 10^{-11}} = .902 \times 10^{23} \frac{m}{sec^2}$$
$$= 9.02 \times 10^{22} \frac{m}{sec^2}$$

24) $g = 9.8 \frac{m}{sec^2}$

$r_{\text{of earth}} = 6.37 \times 10^{6} m$

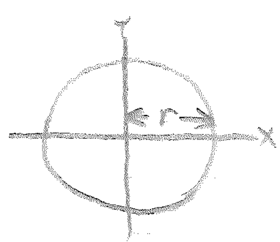
$$a = 9.8 \frac{m}{sec^2} = \frac{V^2}{r}$$
$$9.8 \frac{m}{sec^2} = \frac{V^2}{6.37 \times 10^{6} m}$$
$$V^2 = 62.4 \times 10^{6} \frac{m^2}{sec^2}$$
$$V = 7.90 \times 10^{3} \frac{m}{sec}$$

$$a = 3 \times 10^{-2} \frac{m}{sec^2} = \frac{V^2}{6.37 \times 10^{6} m}$$
$$19.11 \times 10^{4} \frac{m^2}{sec^2} = V^2$$
$$V = 4.37 \times 10^{2} \frac{m}{sec}$$

$$4.37 \times 10^{2} f = 7.90 \times 10^{3}$$

$f = 1.81 \times 10 = 18.1$ *times as fast*

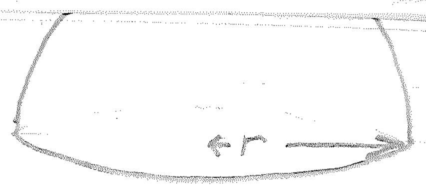
29)



$$x^2 j + y^2 j = r^2$$
$$y^2 = r^2 - x^2$$
$$y = (r^2 - x^2)^{\frac{1}{2}}$$
$$\frac{dr}{dx} = V$$
$$U = r^2 - x^2 \qquad y = U^{\frac{1}{2}}$$
$$V = -2x \left[\tfrac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}\right]$$
$$= -\frac{x}{\sqrt{r^2 - x^2}} = -\frac{x}{y}$$
$$a = \frac{dV}{dt} = \frac{y^2 - \frac{x^2}{y}}{r^2 - x^2} = \frac{-y^2 - xV}{-y^2} = 1 + \frac{xV}{y^2}$$

Pg 76

20)



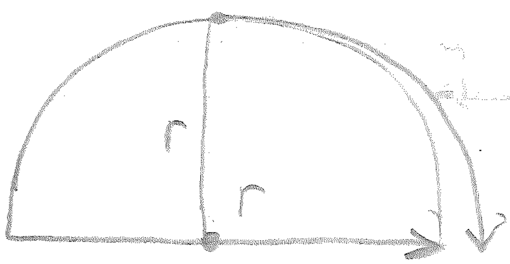
$$r = \frac{g}{2}t^2 + V_{ox}t$$

$$r = 16t^2 + V_{ox}t$$

$$gt^2 + 2V_{ox}t - 2r = 0$$

$$t = \frac{-2V_{ox} \pm \sqrt{V_{ox}^2 t^2 + 8rg}}{2g}$$

$$= \frac{-2V_{ox} \pm \sqrt{V_{ox}^2 t^2 + 8rg}}{2g}$$

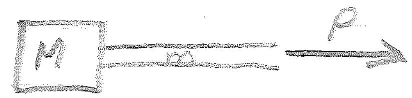


$$V_{ox}t = r$$

$$V_{ox} = \frac{r\,2g}{-2V_{ox} \pm \sqrt{V_{ox}^2 t^2 + 8rg}}$$

Pg 105 V

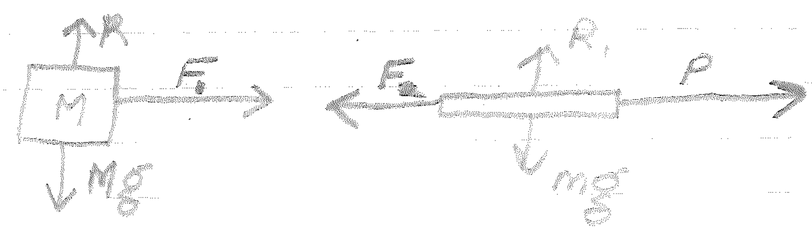
4)



a) $F = ma$

$P = (M + m)\,a$

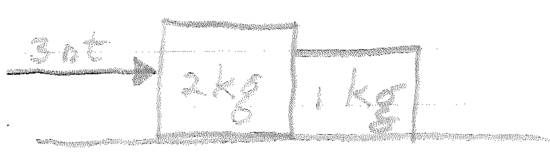
$a = \frac{P}{M+m}$

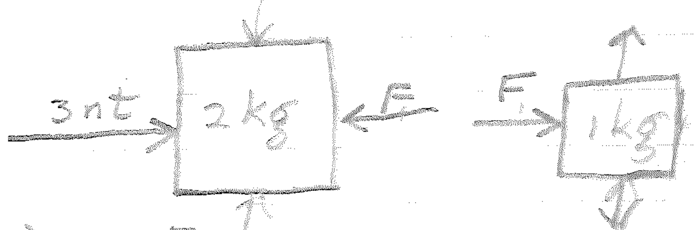


$F_1 = F_2$

$F_1 = M\left(\frac{P}{M+m}\right) = \frac{PM}{M+m}$

10)





a) $F = ma$

$F = ma$

$3\,nt = 3a$

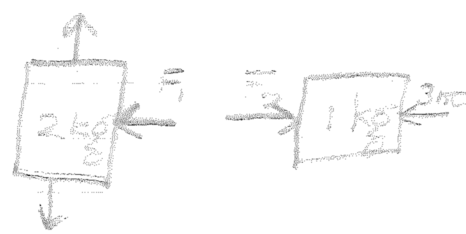
$a = \frac{1\,m}{sec^2}$

$F_1 = F_2 = ma$

$= 1\,kg\,\frac{1\,m}{sec^2}$

$= 1\,nt$

b)



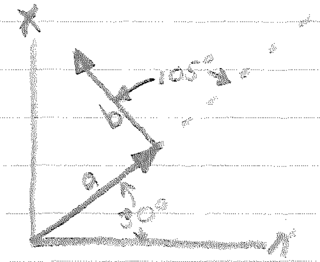
$F_1 = F_2 = ma$

$= 2\,nt$

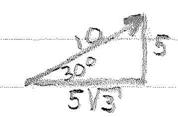
PHYS

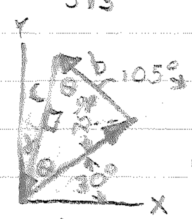
Pg. 28

4)



$\vec{a} \Rightarrow X = 5\sqrt{3} \quad Y = 5$



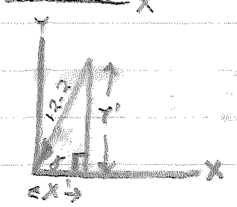


$\Theta = \frac{1}{2}(105^\circ) = 52\frac{1}{2}^\circ$

$|\vec{r}| = 2b\cos\Theta$

$= 20\cos 52\frac{1}{2}^\circ$

$|\vec{r}| \cong 20(.609) = 12.2$



$\gamma = 82.5^\circ$

$Y' = 12.2 \sin\gamma$

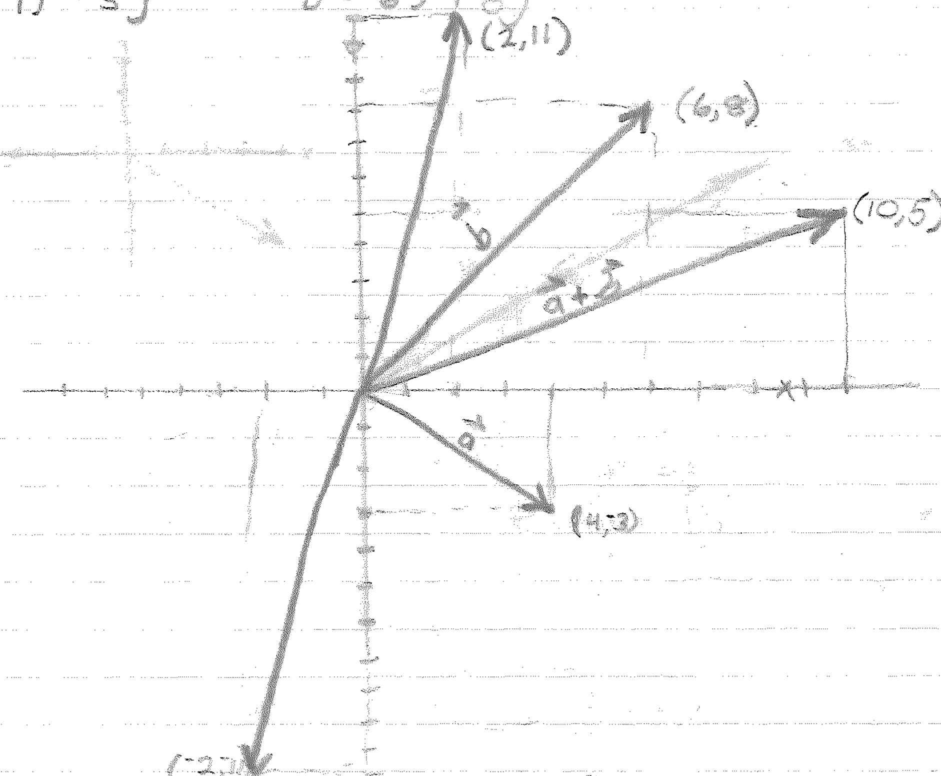
$Y' = 12.2(.991) = 12.1$

$X' = 12.2\cos\gamma$

$X' = 12.2(.130) = 1.58$

Pg 28

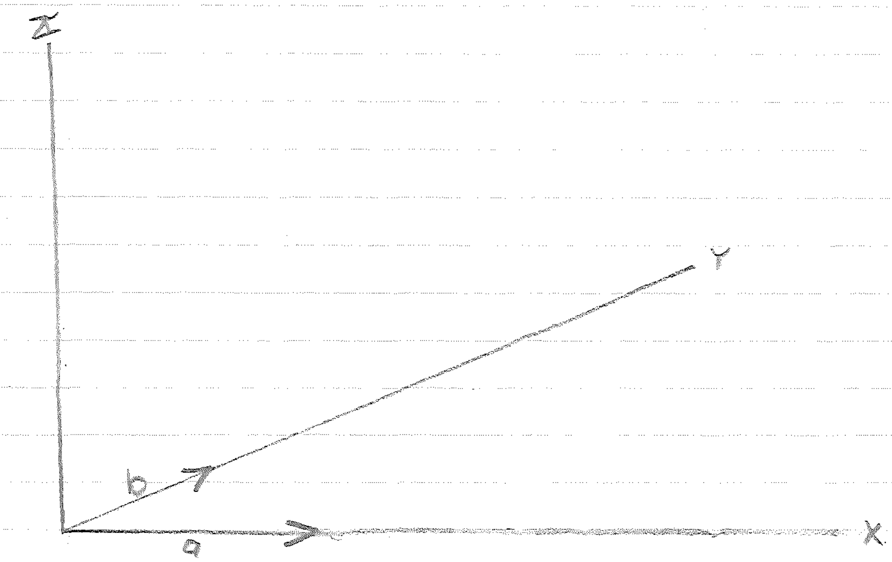
5) $a = 4i - 3j$ $b = 6i + 8j$



a) $\vec{a} = 5$ at $-36.0°$
b) $\vec{b} = 10$ at $53.1°$
c) $\vec{a} + \vec{b} = 11.1$ at $26.6°$
d) $\vec{b} - \vec{a} = 11.2$ at $79.7°$
e) $\vec{a} - \vec{b} = 11.2$ at 259.7

Pg 28

20)

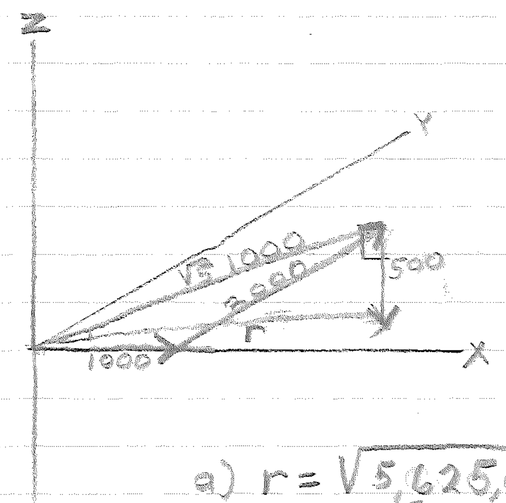


a) +Z
b) -Z
c) +Y
d) $a \cdot b = ab\cos\frac{\pi}{2} = 0$

BOB MARKS

Pg 29

12)



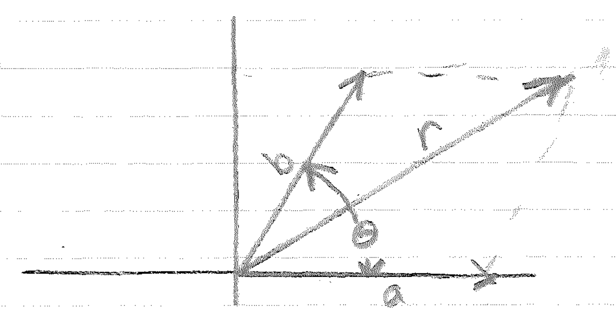
2/5

a) $r = \sqrt{5{,}625{,}000} = 10\sqrt{5{,}625} = 750$ ft

b) Θ

14) a) 2.5 m S
b) 1.25 m N
c) 6.25 m S
d) 10 m N

18)



$r_x = a_x + b_x$
$= a + b\cos\Theta$

$r_y = a_y + b_y$
$= 0 + b\sin\Theta$

$r = \sqrt{b^2\sin^2\Theta + (a + b\cos\Theta)^2}$

$r = \sqrt{b^2\sin^2\Theta + a^2 + 2ab\cos\Theta + b^2\cos^2\Theta}$

$r = \sqrt{b^2(\sin^2\Theta + \cos^2\Theta) + a^2 + 2ab\cos\Theta}$

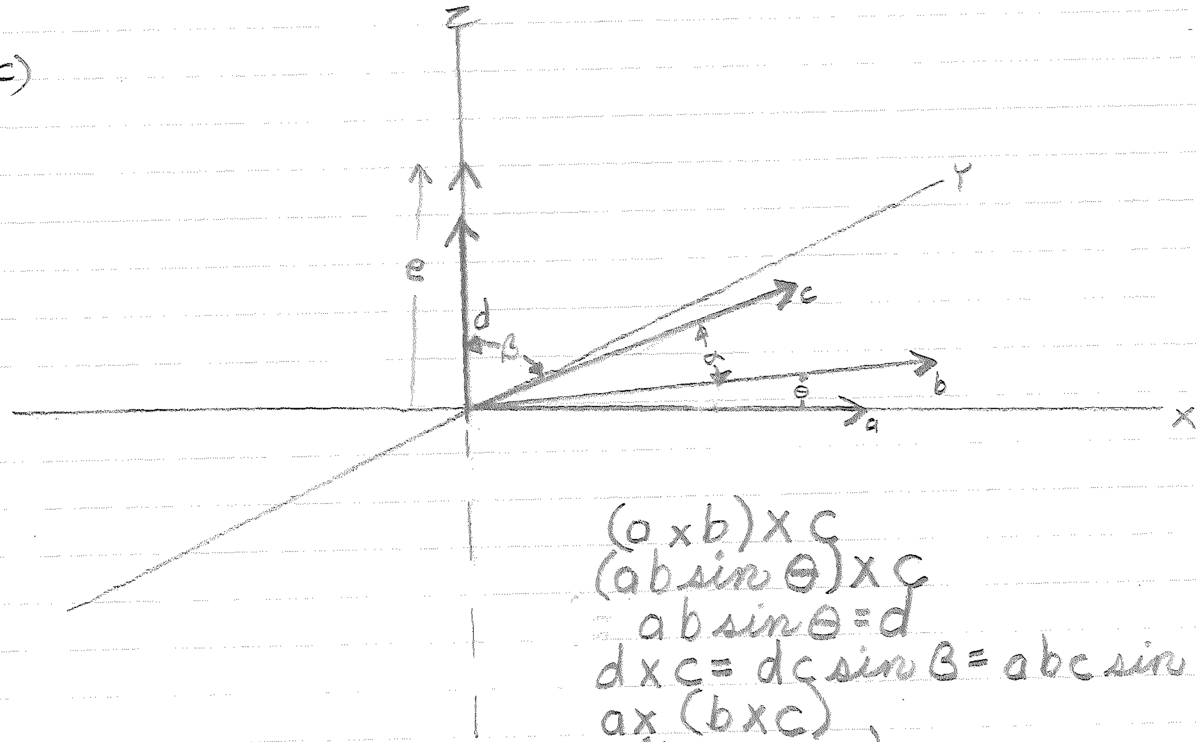
$r = \sqrt{b^2 + a^2 + 2ab\cos\Theta}$

Pg 30

24) $a \cdot b = ab\cos\theta$ $b \cdot a = ba\cos\theta$

$ab\cos\theta = ab\cos\theta$

c)



$(a \times b) \times c$

$(ab\sin\theta) \times c$

$ab\sin\theta = d$

$d \times c = dc\sin\beta = abc\sin\theta\sin\beta$

$a \times (b \times c)$

$a \times (bc\sin\alpha)$

$e = bc\sin\alpha$

$a \times e = ae\sin\beta = abc\sin\alpha\sin\beta$

$\sin\beta = \sin\frac{\pi}{2} = 1$

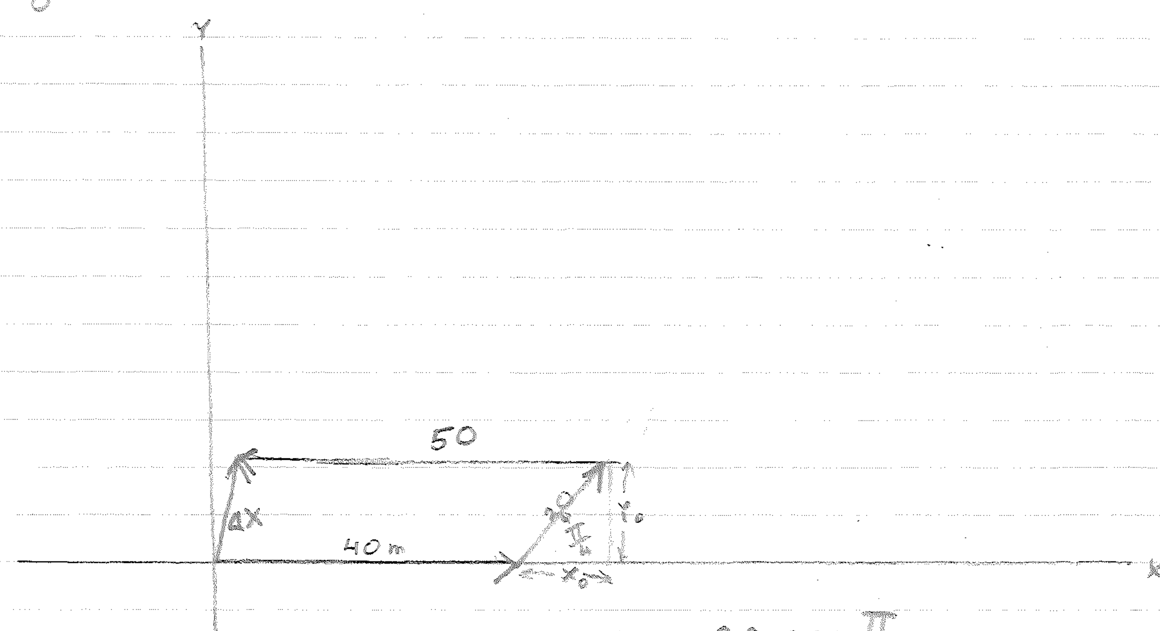
if $(a \times b) \times c = a \times (b \times c)$ then

$abc\sin\theta = abc\sin\alpha$

$\sin\theta = \sin\alpha$

Pg 52

2)



$V = 60$ mph

$V_{AVE} = \frac{\Delta r}{\Delta t}$

$X_0 = 20 \cos \frac{\pi}{4}$
$= 20 \frac{1}{\sqrt{2}} = \frac{20}{\sqrt{2}}$

$Y_0 = X_0 = \frac{20}{\sqrt{2}}$

$\vec{r}_a = t_1\, 60$
$= \frac{2}{3}\, 60 = 40$

$\vec{r}_b = t_2\, 60$
$= \frac{1}{3}\, 60 = 20$

$\vec{r}_c = t_3\, 60 = 50$

$r_{a_x} = 40$
$r_{b_x} = \frac{20}{\sqrt{2}}$
$r_{c_x} = -50$

$\Delta r_x = \frac{20}{\sqrt{2}} - 10 \cong 4.14$

$r_{a_Y} = r_{c_x} = 0$
$r_{b_Y} = 20/\sqrt{2} \cong 14.1$
$\Delta r_Y = 14.1$

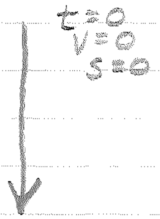
$\Delta \vec{r} = \sqrt{200 + 17.1} = \sqrt{217} = 14.3$

$\Delta t = \frac{40}{60} + \frac{20}{60} + \frac{50}{60} = \frac{11}{6}$ hr.

$V_{AVE} = \frac{14.3}{\frac{11}{6}} = \frac{6(14.3)}{11} = 7.8$ MPH

Pg 52

5)



$a = 32$

$V = 32t + C_1$

$C_1 = 0$

$V = 32t$

$s = 16t^2 + C_2$

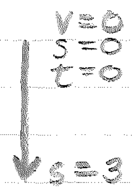
$C_2 = 0$

$s = 16t^2$

$s = 4$

$4 = 16t^2 \qquad t = \frac{1}{2}$

$\underline{V = 16}$



$a = 32$

$V = 32t + C_1 \qquad C_1 = 0$

$V = 32t$

$s = 16t^2 + C_2 \qquad C_2 = 0$

$s = 16t^2$

$s = 3$

$3 = 16t^2$

$\frac{3}{16} = t^2$

$t = \frac{\sqrt{3}}{4}$

$V = 32\frac{\sqrt{3}}{4} = 8\sqrt{3}$

$a_{AVE} = \frac{\Delta V}{\Delta t}$

$\Delta V = 16 + 8\sqrt{3} \qquad \Delta t = .01$

$\simeq 30$

$a_{AVE} = \frac{30}{.01} = 3{,}000$ ft/[illegible]^2

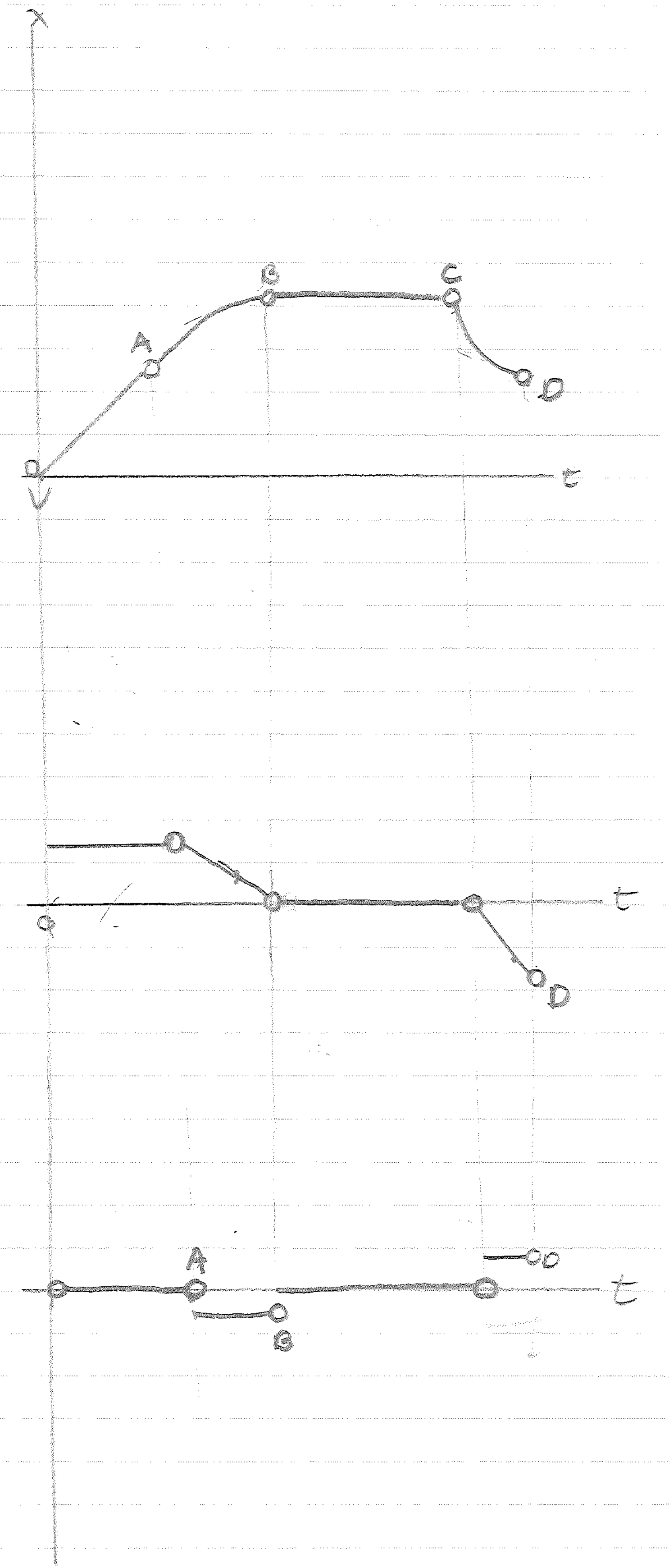
Pg 53

6)

o-a	+
a-b	+
b-c	0
c-d	−

o-a	0
a-b	−
b-c	0
c-d	+

no



Pg 53

8) $a_{AVE} = \frac{\Delta V}{\Delta t}$

$V_0 = 0$ $\quad$ $V_f = 200$

$a = k$

$V = kt + c$

at $V=0,\ t=0$

$\therefore c = 0$

$V = kt$

$s = \frac{k}{2}t^2 + C_2$

at $s=0,\ t=0 \therefore C_2 = 0$

$200 = kt$ $\qquad$ $2 = \frac{k}{2}t^2$

$t = \frac{200}{k}$ $\qquad$ $t^2 = \frac{4}{k}$

$t = \frac{2}{\sqrt{k}}$

$\frac{200}{k} = \frac{2}{\sqrt{k}}$

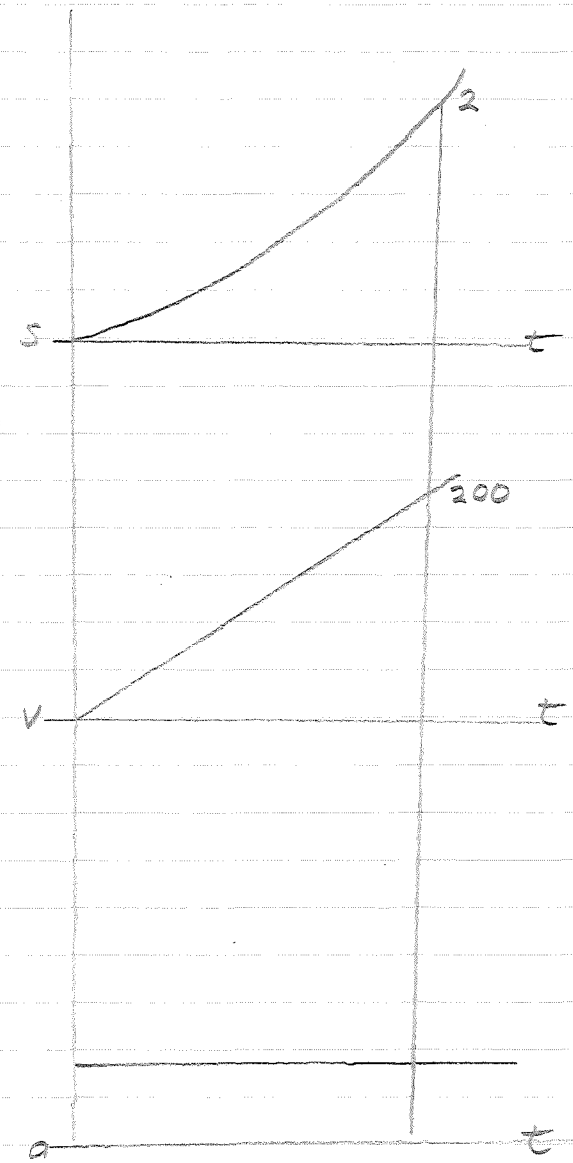
$200\sqrt{k} = 2k$

$100\sqrt{k} = k$

$10{,}000k = k^2$

$k^2 - 10{,}000k = 0$

$k = 10{,}000 \frac{ft}{sec^2}$



Pg 53

9) $\Delta V = 4.0\times10^{6} - 1.0\times10^{4}$

$= 400\times10^{4} - 1.0\times10^{4}$

$= 399\times10^{4} = 3.99\times10^{6}$

$a = k$

$V = kt + c$

at $V = 10^{4}$, $t = 0$ $\therefore$ $c = 10^{4}$

$V = kt + 10^{4}$

$s = \frac{k}{2}t^{2} + 10^{4}t + c_2$

at $s = 0$; $t = 0$ $\therefore$ $c_2 = 0$

$s = 10^{-2} = \frac{k}{2}t^{2} + 10^{4}t$ $\qquad V = 4\times10^{6} = kt + 10^{4}$

$\frac{k}{2}t^{2} + 10^{4}t - 10^{-2} = 0$ $\qquad t = \frac{4\times10^{6} - 10^{4}}{k}$

$t = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$

$t = \frac{-10^{4} \pm \sqrt{10^{8} + 2k(10^{-2})}}{k} = \frac{4\times10^{6} - 10^{4}}{k}$

$-10^{4} \pm \sqrt{10^{8} + 2k(10^{-2})} = 4\times10^{6} - 10^{4}$

$\pm\sqrt{10^{8} + 2k(10)^{-2}} = 4\times10^{6}$

$10^{8} + 2k(10)^{-2} = 16\times10^{12}$

$2k10^{-2} = 16\times10^{12} - 10^{8}$

$= 160000\times10^{8} - 10^{8}$

$= 159999\times10^{8}$

$2k = 159999\times10^{10}$

$k = 79999.5\times10^{10}$

$\approx 8.00\times10^{14}$

Pg 53

10) $30 \frac{M}{H} \times \frac{1.47 \frac{ft}{sec}}{1 \frac{M}{H}} = 44.1 \frac{ft}{sec}$

$v = 0$
$s = 0$
$t = 0$

$s = 19.2$
$v > 30$

$a = k$

$v = kt$

$s = \frac{k}{2} t^2$

at $s = 19.2$

$38.4 = kt^2$

$t = \frac{6.20}{(\sqrt{k})}$

$v = k^{\frac{1}{2}}(6.20)$

if $k = 32$

$v = (5.66)(6.20) > 30$

∴ mane was speeding.

Pg 53

12)

$a = 32$

$V = 32t + C$

$C = 0$

$V = 32t$

$V=0$
$S=0$
$t=0$

$1.86 \times 10^5 \frac{M}{S} \times \frac{5280 ft}{M} = 9.81 \times 10^8 \frac{ft}{sec}$

$\frac{1}{4}\left(9.81 \times 10^8 \frac{ft}{sec}\right) = 2.45 \times 10^8 \frac{ft}{sec}$

$2.45 \times 10^8 = 32t$

$2.45 \times 10^6 = 32t$

$t = 7.66 \times 10^6$ sec

$S = 16t^2$

$= 16(7.66 \times 10^6)^2$

$= 939 \times 10^{12}$

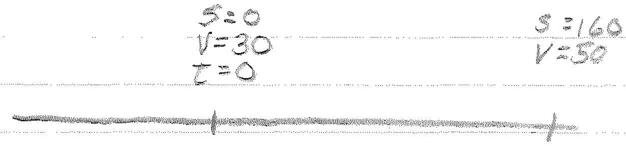
$= 9.39 \times 10^{14}$

5

Pg 53-54

13)

a)



$a = k$

$v = kt + c$

$v = kt + 30$

$s = \frac{k}{2}t^2 + 30t + c_2$

$s = \frac{k}{2}t^2 + 30t$

$160 = \frac{k}{2}t^2 + 30t$ $\qquad$ $50 = kt + 30$

$\frac{k}{2}t^2 + 30t - 160 = 0$ $\qquad$ $kt = 20$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad$ $t = \frac{20}{k}$

$t = \frac{-30 \pm \sqrt{900 + 320k}}{k}$

$-30 \pm \sqrt{900+320k} = 20$

$\pm\sqrt{900+320k} = 50$

$900 + 320k = 2500$

$32k = 160$

$k = 5.0 = a$ ✓

b) $s = \frac{k}{2}t^2 + 30t$

$k = 5 \quad s = 160$

$160 = \frac{5}{2}t^2 + 30t$

$5t^2 + 60t - 320 = 0$

$t^2 + 12t - 64 = 0$

$t = \frac{-12 \pm \sqrt{144 + 256}}{2}$

$= \frac{-12 \pm \sqrt{400}}{2}$

$= \frac{20 - 12}{2} = 4$ sec ✓

c) $v=0$, $t=0$, $s=0$ ——— $v=30$

$a=5$

$v=5t$

$v=30$

$t=6$ sec ✓

d) $s=\frac{5}{2}t^2+C_2$ $C_2=0$

$s=\frac{5}{2}t^2$

$s=\frac{5}{2}(6)^2=5(18)=90$ ft ✓

Pg 54

15) $s=0$, $t=0$ ——— $V=45$, $s=180$, $t=6$

$a=k$

$V=kt+C_1$

at $V=45$, $t=6$ $\therefore$ $C_1=45-6K$

$V=kt+45-6K$

$s=\frac{K}{2}t^2+(45-6K)t+C_2$

at $s=180$, $t=6$

$180=\frac{K}{2}(36)+(45-6K)6+C_2$

$180=18K+270-36K+C_2$

$C_2=18K-90$

$s=\frac{K}{2}t^2+(45-6K)t+(18K-90)$

$s=0$ $t=0$

$18K=90$

$K=5$

b) $a=5\ ft/sec^2$

$V=kt+45-6K$

$K=5$ $t=0$

a) $V=45-30=15\ FT/SEC$

$V=0$, $t=0$, $s=0$, $a=5$ ↓ $V=15$

c) $a=5$

$V=5t$

$s=\frac{5}{2}t^2$

$V=15$ $\therefore$ $t=3$

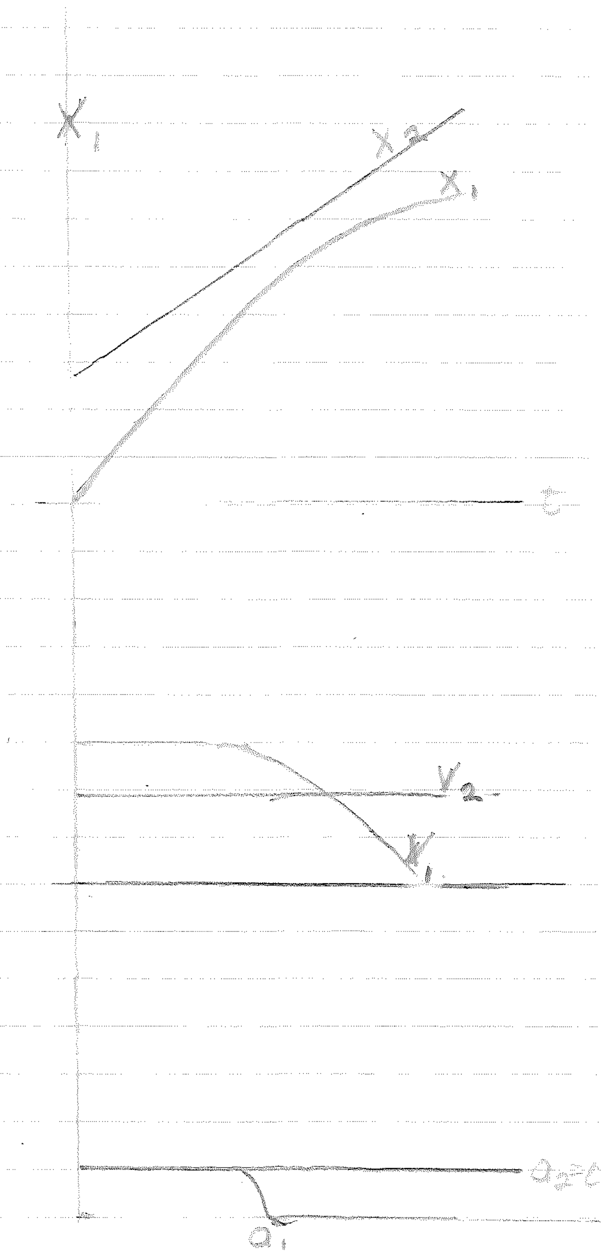
$s=\frac{5}{2}(9)=\frac{45}{2}=22.5\ ft$

Pg 54

1(a)

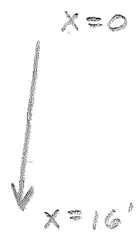
$a = k$

$V = kt$ v_1



Pg 54-5

24)



a) $X = \frac{a}{2}t^2 + V_{ox}t + x_o$

$X = \frac{a_x}{2}t^2 = 16$

$32t^2 = 32$

$t = 1 \text{ SEC}$

$V_x = a_x t + V_{ox}$

$= a_x t$

$= 32 \text{ ft/sec}$

$32 \frac{ft}{sec} \times 5 \text{ sec} = 160 \text{ ft}$

b) $V_{AVE} = \frac{\Delta x}{\Delta t} = \frac{176 \text{ FT}}{6 \text{ SEC}} = 29.4 \frac{FT}{SEC}$

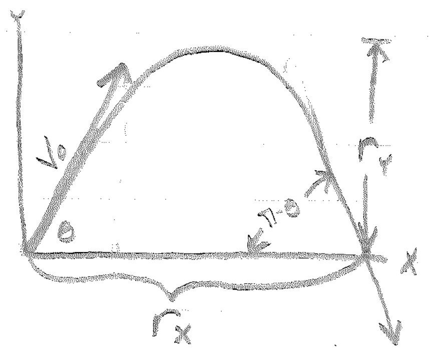
c) $X = \frac{a}{2}t^2 + V_{ox}t + X_o$

$176 = 16(5)^2 + V_{ox}5 + 0$

$176 = 400 + V_{ox}5$

Pg 75

7)



$r_y = r_x$

$r_y = \frac{a}{2}t^2$

$r_y = 16t^2$

$t = \frac{1}{4}\sqrt{r_y}$

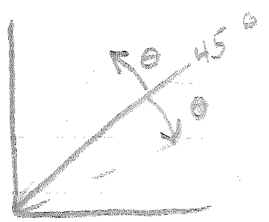
$r_x = V_0 \sin\theta \, t$

$r_x = V_0 \frac{1}{4}\sqrt{r_y} \sin\theta$

$\sqrt{r_x} = \frac{1}{4} V_0 \sin\theta$

$r_y = r_x = \frac{1}{16} V_0^2 \sin^2\theta$

8)



$R = \frac{V_0^2 \sin 2\theta}{g}$

$R_1 = \frac{V_0^2 \sin 2(\frac{\pi}{4}+\theta)}{g}$ $\quad R_2 = \frac{V_0^2 \sin 2(\frac{\pi}{4}-\theta)}{g}$

if $R_2 = R_1 \Rightarrow \frac{2V_0^2 \sin(\frac{\pi}{4}+\theta)\cos(\frac{\pi}{4}+\theta)}{g} = \frac{2V_0^2 \sin(\frac{\pi}{4}-\theta)\cos(\frac{\pi}{4}-\theta)}{g}$

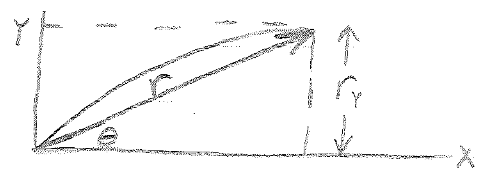
$(\sin\frac{\pi}{4}\cos\theta + \sin\theta\cos\frac{\pi}{4})(\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta) =$

$(\sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta)(\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta)$

$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

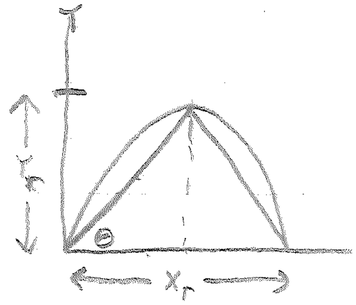
Pg 74

5)



$$a = g$$
$$V_Y = gt - V_{oY}$$
$$V_{oY} = V_o \sin\theta$$
$$r_Y = \frac{g}{2}t^2 + V_o \cos\theta$$
$$\frac{dr_Y}{d\theta} = gt - V_o \sin\theta = 0$$
$$gt = V_o \sin\theta$$
$$t = \frac{V_o \sin\theta}{g}$$
$$Y_{MAX} = \frac{g}{2}\left(\frac{V_o \sin\theta}{g}\right)^2 + V_o \cos\theta$$
$$= \frac{(V_o \sin\theta)^2}{2g}$$

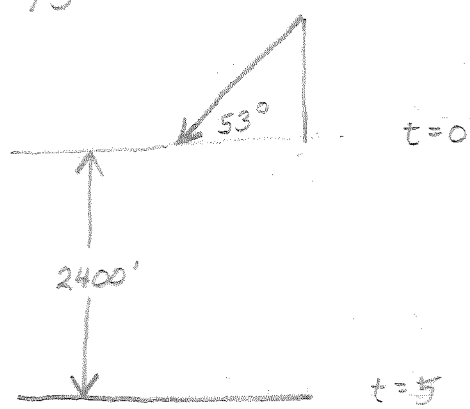
7)



$$Y_r = X_r$$
$$V_r = 2V_o \csc\theta$$
$$X_r = 2V_o t \csc\theta$$
$$a_Y = g$$
$$V_Y = gt$$
$$Y_r = \frac{g}{2}t^2$$

Pg 75

10)

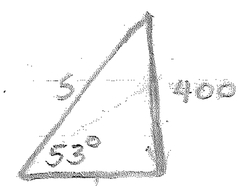


$X_y = \frac{a}{2}t^2 + V_{ox}t$

$2400 = 16(25) + V_{ox}5$

$2000 = 5V_{ox}$

$V_{ox} = 400$



a) $S = 400 \csc 53°$

$S = 502$ ft/sec

b) $d_v = S \cos 53°$

$d_v = 302$ ft/sec

$d = 302 \frac{ft}{sec}$ 3 sec $= 1695$ ft

c) horizonal

$V_x = 565 \frac{ft}{sec}$

verticle

$V_y = \frac{a}{2}t^2 + V_{ox}t$

$= 16(9) + (752)(3)$

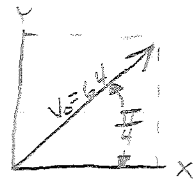
$= 144 + 2256$

$= 3400 \frac{ft}{sec}$

Bob Markey

Pg 75

12)



$V_{oy} = V_{ox} = 64 \cos \frac{\pi}{4} = \frac{64}{\sqrt{2}}$

$V = 0$ $s = 0$
$t = 0$
$a = 32$
$V = \frac{64}{\sqrt{2}}$

$V = a t_1$

$\frac{64}{\sqrt{2}} = 32 t$

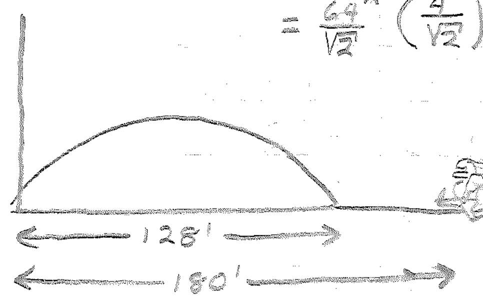
$t_1 = \frac{2}{\sqrt{2}}$

$t = 2 t_1 = \frac{4}{\sqrt{2}}$

$r_x = V_{ox} t$

$= \frac{64}{\sqrt{2}} \left(\frac{4}{\sqrt{2}}\right) = 128 \text{ ft}$ ✓

5

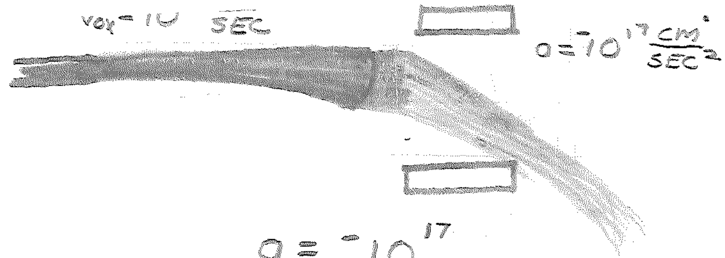


$r = 180' - 128' = 52'$

$t = \frac{4}{\sqrt{2}}$

$V_{foot} = \frac{52\sqrt{2}}{4} \cong 18.3 \text{ ft/sec}$ ✓

15)



$a_y = -10^{17}$

$S_y = \frac{a}{2} t^2$

$Vt = S_1$

$10^9 t = 2$

$t = 2 \times 10^{-9}$

a) $S_y = -.5 \times 10^{17} (2 \times 10^{-9})^2$

$= -.5 \times 10^{17} \times 4 \times 10^{-18}$

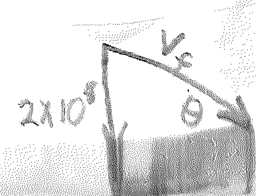
$= -2 \times 10^{-1} = -.2$ cm

b) $V_y = a_y t$

$= (10^{17})(2 \times 10^{-9})$

$= 2 \times 10^8 \frac{cm}{sec}$

$V_x = 10^9$ cm/sec



$= \sqrt{10 \ldots}$

$= \sqrt{1.04 \times 10^{18}}$ cm/

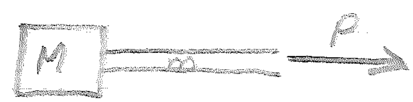
$\ldots \times 10^8$ sec

$\tan\theta = \frac{2 \times 10^8}{10^9} = .20$

$\theta = 11.5°$ S of E

Pg 105 V

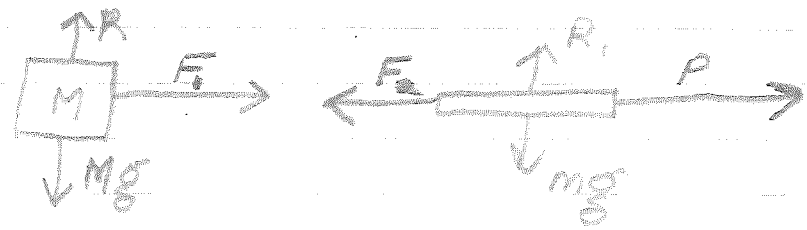
4)



a) $F = ma$

$P = (M + m)\,a$

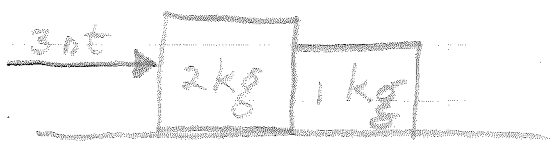
$a = \frac{P}{M+m}$

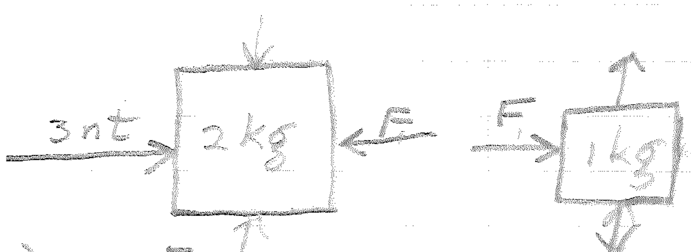


$F_1 = F_2$

$F_1 = M\left(\frac{P}{M+m}\right) = \frac{PM}{M+m}$

10)





a) $F = ma$

$F = ma$

$3\,nt = 3a$

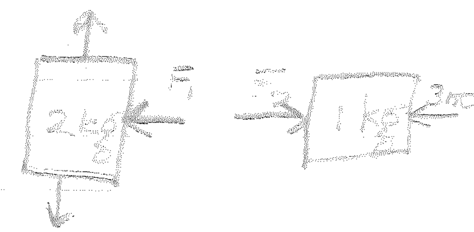
$a = 1\,\frac{m}{sec^2}$

$F_1 = F_2 = ma$

$= 1\,kg\,\frac{m}{sec^2}$

$= 1\,nt$

b)

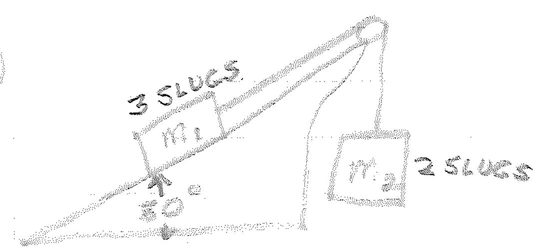


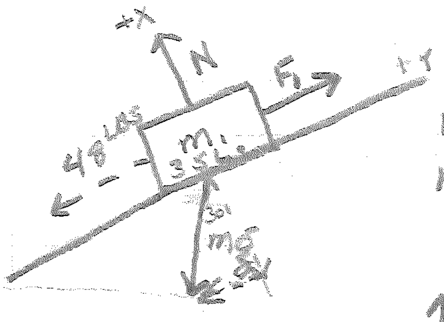
$F_1 = F_2 = ma$

$= 2\,nt$

106 V

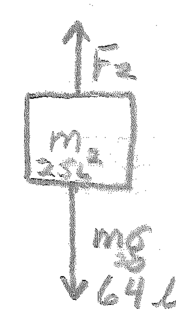
15)





$mg = 3 \text{ SLUGS } 32 \frac{ft}{sec^2} = 96$ lbs

$mg_x = mg \sin 30° = 48$ lbs



$x^2 = a^2 + b^2 - 2ab\cos x$

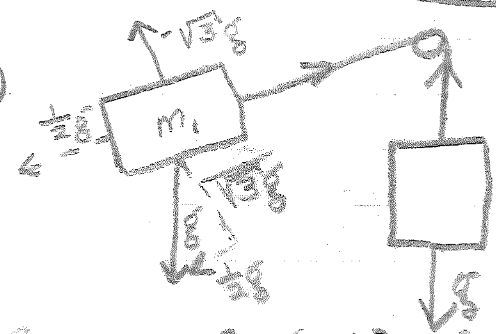
$= 2300 + 4100 - 3040$

$= 6400 - 3040 = 3360$

b) $x = 58$ lbs

~~$\Sigma F_1' = \Sigma F_2'$~~

a)



$a^2 = (16)^2 + (32)^2 - 2(16)(32)\frac{\sqrt{3}}{2}$

b) $64 - 48 = 16$ lbs

$\frac{16 \text{ lbs}}{5 \text{ slug}} = 3.2 \frac{ft}{sec^2}$

Pg 105 V

5) $F = ma$

$V=0$
$t=0$
$s=0$
$x=0$

$V = 74.5$ ft/sec
$x = 200$ ft
$t =$

$a = ?$

$V = at$ $\quad x = \frac{a}{2}t^2$

$74.5 = at$ $\quad 200 = \frac{a}{2}t^2$

$t = \frac{74.5}{a}$ $\quad t^2 = \frac{400}{a}$ $\quad t = \frac{20}{\sqrt{a}}$

$\frac{74.5}{a} = \frac{20}{\sqrt{a}}$

$74.5\sqrt{a} = 20a$

$5550a = 400a^2$

$\frac{555}{40} = a = 13.9$ ft/sec^2

$F = m\,a$ $\quad$ $\frac{3000\text{ lb}}{13.9\frac{ft}{sec^2}}$ $\quad$ $\frac{1\text{ slug}}{32.2\text{ lb}} = 93.3$

$F = 93.3$ slugs $\quad t = \frac{74.5}{13.9} = 5.4$ sec

$F = 1{,}300$ lbs

b) $1{,}300$ lbs $= 93.3$ slugs a

$a = 13.9$ ft/sec^2

$25\frac{M}{H} \times \frac{1.47\frac{ft}{sec}}{M/H} = 36.8\frac{ft}{sec}$

$V = at$

$36.8 = 13.9t$

$t = 2.7$ sec

$x = \frac{a}{2}t^2$

$= (7)(2.7)^2 = 51$ ft.

Pg 105 IV

6) $V=0$, $x=0$ ⟶ $6\times10^{6}\frac{m}{sec}=V$, $x=.01\,m$

$$V=at \qquad x=\frac{a}{2}t^2$$

$$6\times10^{6}\frac{m}{sec}=at \qquad .01\,m=\frac{a}{2}t^2$$

$$t=\frac{6\times10^{6}}{a} \qquad t^2=\frac{.02}{a}$$

$$\frac{6\times10^{6}}{a}=\frac{.141}{\sqrt{a}}$$

$$6\times10^{6}\sqrt{a}=.141\,a$$

$$36\times10^{12}a=.02a^2$$

$$a=1800\times10^{12}=1.8\times10^{15}$$

$$F=ma$$

$$=(9.1\times10^{-31}\,kg)(1.8\times10^{15}\frac{m}{sec^2})$$

$$=16.4\times10^{-16}\,nt=1.64\times10^{-15}\,nt$$

$$V=at$$

$$6\times10^{6}\frac{m}{sec}=(1.8\times10^{15})t$$

$$t=3.33\times10^{-9}\,sec$$

$$a=9.8\frac{m}{sec^2}$$

$$F=ma$$

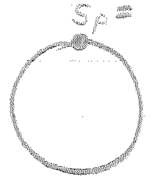
$$=9.1\times10^{-31}\,kg\,(9.8\frac{m}{sec^2})$$

$$=89.2\times10^{-31}=8.92\times10^{-30}\,nt$$

Yes. Since the magnitude of the force due to gravity is about 10^{-16} times that of the electrical force, it is negligible.

Pg 77

22) $S_p = 2.18 \times 10^6$



$\longleftarrow 5.28 \times 10^{-11} = r$

$$a = \frac{v^2}{r} = \frac{(2.18 \times 10^6)^2}{5.28 \times 10^{-11}} = .902 \times 10^{23} \frac{m}{sec^2}$$
$$= 9.02 \times 10^{22} \frac{m}{sec^2}$$

24) $g = 9.8 \frac{m}{sec^2}$

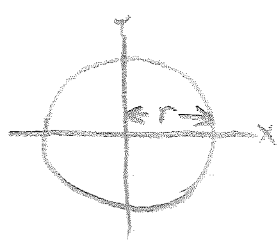
$r_{\text{of earth}} = 6.37 \times 10^6 m$

$$a = 9.8 \frac{m}{sec^2} = \frac{v^2}{r}$$
$$9.8 \frac{m}{sec^2} = \frac{v^2}{6.37 \times 10^6 m}$$
$$v^2 = 62.4 \times 10^6 \frac{m^2}{sec^2}$$
$$v = 7.90 \times 10^3 \frac{m}{sec}$$

$$a = 3 \times 10^{-2} \frac{m}{sec^2} = \frac{v^2}{6.37 \times 10^6 m}$$
$$19.11 \times 10^4 \frac{m^2}{sec^2} = v^2$$
$$v = 4.37 \times 10^2 \frac{m}{sec}$$

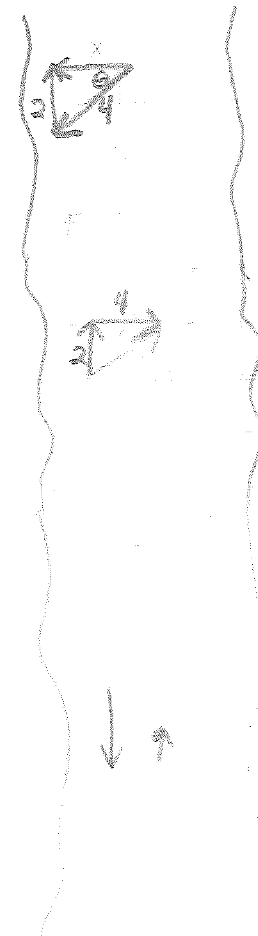
$$4.37 \times 10^2 f = 7.90 \times 10^3$$
$f = 1.81 \times 10 = 18.1$ times as fast

29)



$$x^2 j + y^2 j = r^2$$
$$y^2 = r^2 - x^2$$
$$y = (r^2 - x^2)^{\frac{1}{2}}$$
$$\frac{dy}{dx} = v$$
$$u = r^2 - x^2 \qquad y = u^{\frac{1}{2}}$$
$$v = -2x\left[\tfrac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}\right]$$
$$= -\frac{x}{\sqrt{r^2 - x^2}} = -\frac{x}{y}$$
$$a = \frac{dv}{dt} = \frac{y^2 - \frac{x^2}{y}}{r^2 - x^2} = \frac{-y^2 - xv}{-y^2} = 1 + \frac{xv}{y^2}$$

34) a)



$\sin\theta = \frac{1}{2}$

$\theta = 30°$ S of W

a) If time is at a min:

$4\frac{m}{H}\, t = 4M$

$t = 1$ HR

b) If distance traveled is at a min:

$2\sqrt{3}\, t = 4M$

$t = \frac{2}{\sqrt{3}}$ HR $= 1.16$ HR

c)

c) $V_1 = 6$ M/HR

$t_1 = \frac{2m}{6\,m/HR} = \frac{1}{3}$ HR

$V_2 = 2$ m/HR

$t_2 = 1$ HR

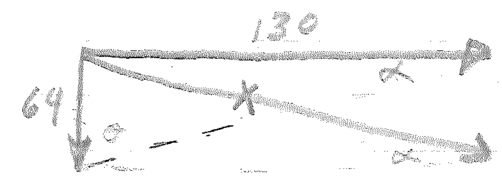
$t_1 + t_2 = \frac{4}{3}$ HR

d) $t = \frac{4}{3}$ HR

Pg 105 IX

7) $F = ma$

$= 2 \text{ slugs} \cdot 32 \frac{ft}{sec^2} = 64 \text{ lbs}$



$\tan\alpha = \frac{64}{130} = .492$

$\alpha = 26.2°$

$|x| = (\csc\alpha) 64 = \frac{64}{\sin\alpha} = 145 \text{ lbs}$

$F = ma$

$145 = 2a$

$a = 72.5 \frac{ft}{sec^2}$ } at 26.2° S of E

$V = 72.5 \frac{ft}{sec^2} t$ }

Pg 105

5)

$F = ma$

$4.5\times10^{-15}\,nt = 9.1\times10^{-31}\,kg\,(a)$

$a = .495\times10^{16}\ \frac{m}{sec^2}$

$x = \frac{a}{2}t^2$

$Vt = x$

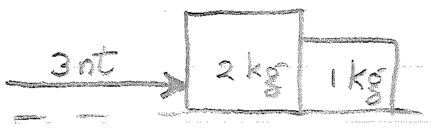
$1.2\times10^{7}\ \frac{m}{s}\,t = 3\times10^{-2}\,m$

$t = 2.5\times10^{-9}$ sec

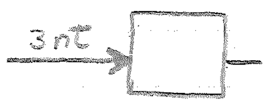
$x = .247\times10^{15}(2.5\times10^{-9})^2$

$= 15.4\times10^{-3}\,m$

Pg 105 V



a)



$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

$$\frac{2\,kg}{1\,kg} = \frac{3-F}{F}$$

$$3-F = 2F$$

$$F = 1\,nt$$

b)

$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

$$\frac{2}{1} = \frac{F}{3-F}$$

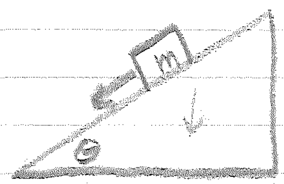
$$6-2F = F$$

$$6 = 3F$$

$$F = 2\,nt$$

Pg 106

19)



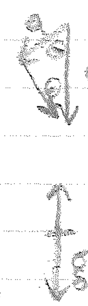
a) $a = g \sin\theta$

b) $a = g \sin\theta$

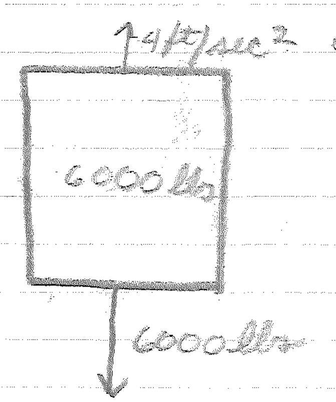
c) $A = (g - a) \sin\theta$

d) $A = (g + a) \sin\theta$

e) 0



20)



a) $F = ma$

$= \left(\frac{6000}{32.2}\right) 4 = 746$ lbs

6746 lbs

b) 5254 lbs

Pg 126 VI

2) $\mu_k = .25$

$F_f = \mu_k N$

$ma = \mu_k mg$

$a = \mu_k g$

$= (.25)(32.2) = 8.05 \frac{ft}{sec^2}$

$30 \frac{mi}{hr} \times 1.47 = 44.1 \frac{ft}{sec}$

$V = at$

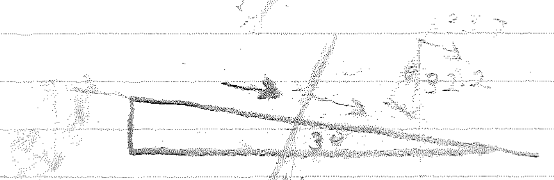
$44.1 \frac{ft}{sec} = 8.05 \frac{ft}{sec^2} t$

$t = 5.48 \ sec$

3) $X = \frac{a}{2} t^2$

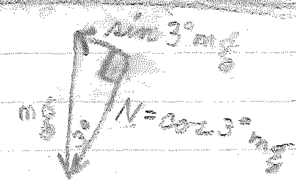
$1400 = a(61)^2$ $\qquad$ $1400 = a(42)^2$

$a = .38 \frac{ft}{sec^2}$ $\qquad$ $a = .79 \frac{ft}{sec^2}$



$f_k = \mu_k N$

$\Sigma F = f_f + N - WT = 0$

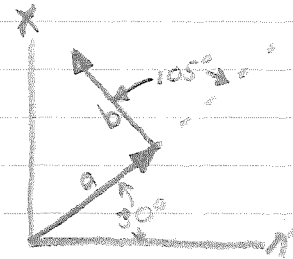


$f_k = \mu_k N$

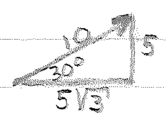
PHYS

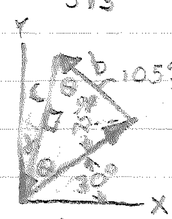
Pg. 28

4)



$\vec{a} \Rightarrow X = 5\sqrt{3} \quad Y = 5$



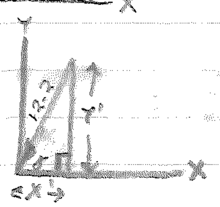


$\Theta = \frac{1}{2}(105°) = 52\frac{1}{2}°$

$|\vec{r}| = 2b\cos\Theta$

$= 20\cos 52\frac{1}{2}°$

$|\vec{r}| \cong 20(.609) = 12.2$



$\gamma = 82.5°$

$Y' = 12.2 \sin\gamma$

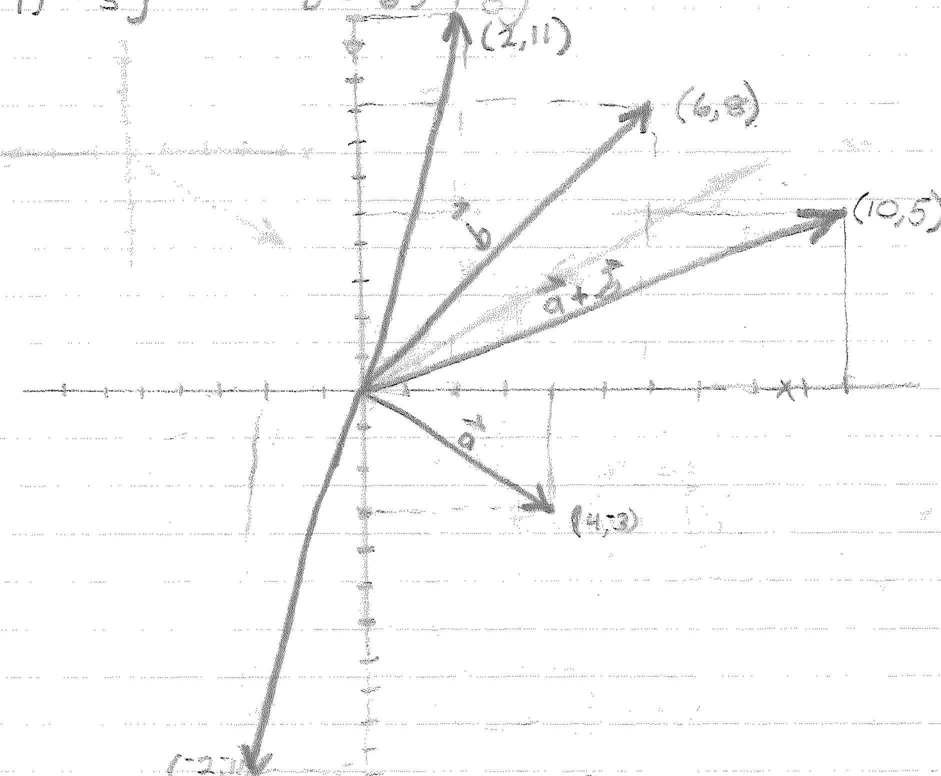
$Y' = 12.2(.991) = 12.1$

$X' = 12.2 \cos\gamma$

$X' = 12.2(.130) = 1.58$

Pg 28

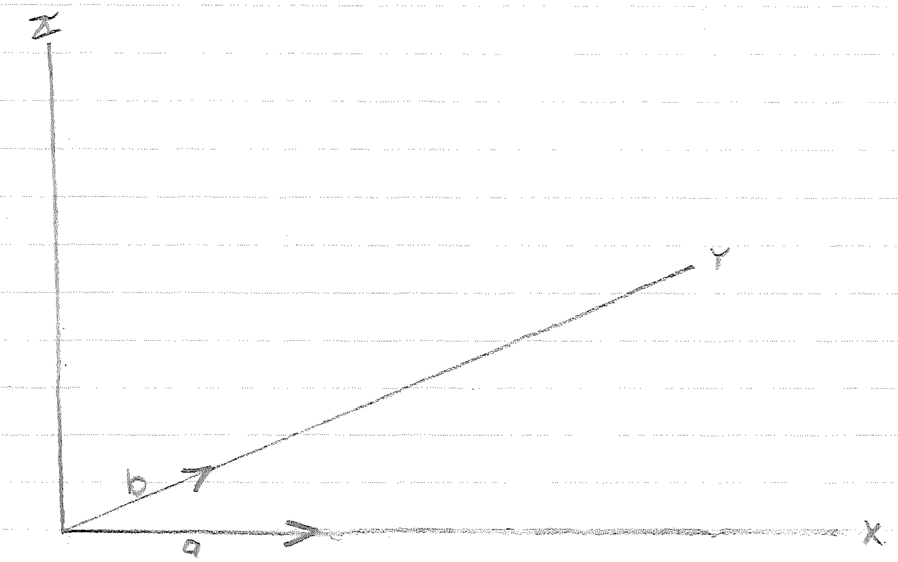
5) $a = 4i - 3j$ $b = 6i + 8j$



a) $\vec{a} = 5$ at $-36.0°$
b) $\vec{b} = 10$ at $53.1°$
c) $\vec{a} + \vec{b} = 11.1$ at $26.6°$
d) $\vec{b} - \vec{a} = 11.2$ at $79.7°$
e) $\vec{a} - \vec{b} = 11.2$ at 259.7

Pg 28

20)

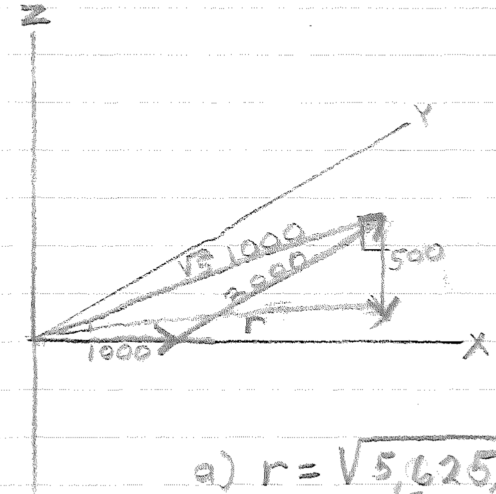


a) $+Z$
b) $-Z$
c) $+Y$
d) $a \cdot b = ab\cos\frac{\pi}{2} = 0$

Bob Marks

Pg 29

12)



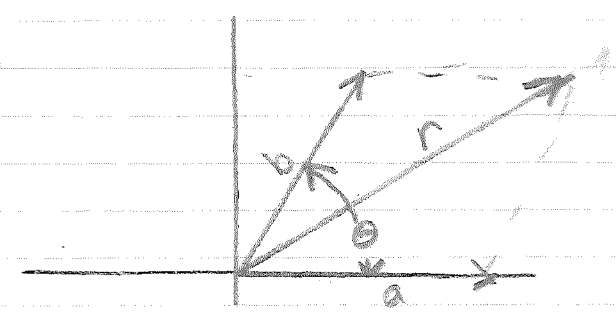
2/5

a) $r = \sqrt{5{,}625{,}000} = 10\sqrt{5{,}625} = 750$ ft

b) θ

14) a) 2.5 m S
b) 1.25 m N
c) 6.25 m S
d) 10 m N

18)



$r_x = a_x + b_x$
$= a + b\cos\theta$

$r_y = a_y + b_y$
$= 0 + b\sin\theta$

$r = \sqrt{b^2\sin^2\theta + (a + b\cos\theta)^2}$

$r = \sqrt{b^2\sin^2\theta + a^2 + 2ab\cos\theta + b^2\cos^2\theta}$

$r = \sqrt{b^2(\sin^2\theta + \cos^2\theta) + a^2 + 2ab\cos\theta}$

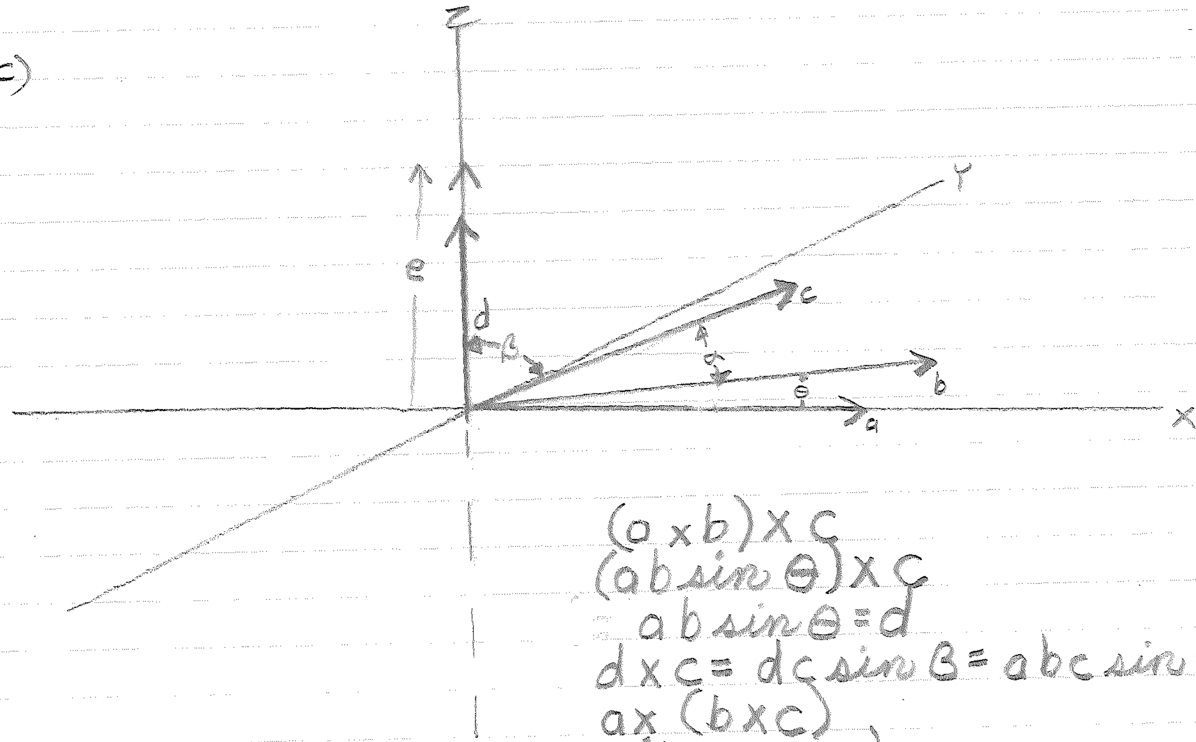
$r = \sqrt{b^2 + a^2 + 2ab\cos\theta}$

Pg 30

24) $a \cdot b = ab\cos\theta$ $b \cdot a = ba\cos\theta$

$ab\cos\theta = ab\cos\theta$

c)



$(a \times b) \times c$

$(ab\sin\theta) \times c$

$ab\sin\theta = d$

$d \times c = dc\sin\beta = abc\sin\theta\sin\beta$

$a \times (b \times c)$

$a \times (bc\sin\alpha)$

$e = bc\sin\alpha$

$a \times e = ae\sin\beta = abc\sin\alpha\sin\beta$

$\sin\beta = \sin\frac{\pi}{2} = 1$

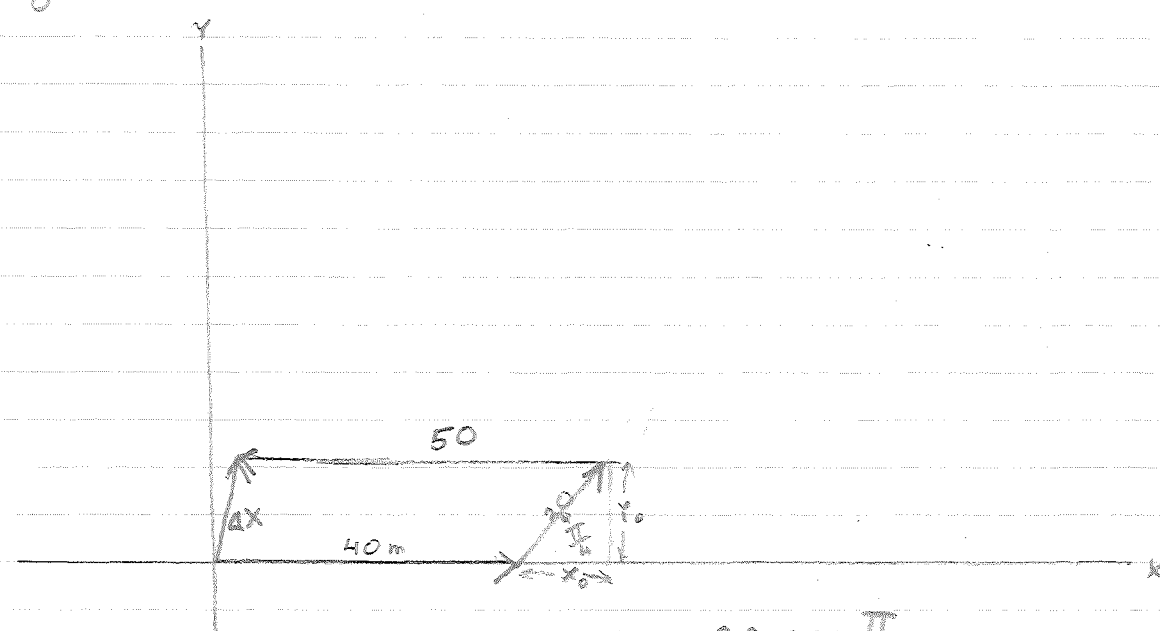
if $(a \times b) \times c = a \times (b \times c)$ then

$abc\sin\theta = abc\sin\alpha$

$\sin\theta = \sin\alpha$

Pg 52

2)



$V = 60$ mph

$V_{AVE} = \frac{\Delta r}{\Delta t}$

$X_0 = 20 \cos \frac{\pi}{4}$

$= 20 \frac{1}{\sqrt{2}} = \frac{20}{\sqrt{2}}$

$Y_0 = X_0 = \frac{20}{\sqrt{2}}$

$\vec{r}_a = t_1\, 60$

$= \frac{2}{3}\, 60 = 40$

$\vec{r}_b = t_2\, 60$

$= \frac{1}{3}\, 60 = 20$

$\vec{r}_c = t_3\, 60 = 50$

$r_{a_x} = 40$

$r_{b_x} = \frac{20}{\sqrt{2}}$

$r_{c_x} = -50$

$\Delta r_x = \frac{20}{\sqrt{2}} - 10 \cong 4.14$

$r_{a_Y} = r_{c_x} = 0$

$r_{b_Y} = 20/\sqrt{2} \cong 14.1$

$\Delta r_Y = 14.1$

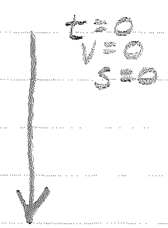
$\Delta \vec{r} = \sqrt{200 + 17.1} = \sqrt{217} = 14.3$

$\Delta t = \frac{40}{60} + \frac{20}{60} + \frac{50}{60} = \frac{11}{6}$ hr.

$V_{AVE} = \frac{14.3}{\frac{11}{6}} = \frac{6\,(14.3)}{11} = 7.8$ MPH

Pg 52

5)



$a = 32$

$V = 32t + C_1$

$C_1 = 0$

$V = 32t$

$s = 16t^2 + C_2$

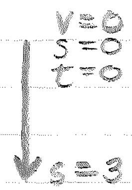
$C_2 = 0$

$s = 16t^2$

$s = 4$

$4 = 16t^2 \quad t = \frac{1}{2}$

$\underline{V = 16}$



$a = 32$

$V = 32t + C_1 \quad C_1 = 0$

$V = 32t$

$s = 16t^2 + C_2 \quad C_2 = 0$

$s = 16t^2$

$s = 3$

$3 = 16t^2$

$\frac{3}{16} = t^2$

$t = \frac{\sqrt{3}}{4}$

$V = 32 \frac{\sqrt{3}}{4} = 8\sqrt{3}$

$a_{AVE} = \frac{\Delta V}{\Delta t}$

$\Delta V = 16 + 8\sqrt{3} \quad \Delta t = .01$

≈ 30

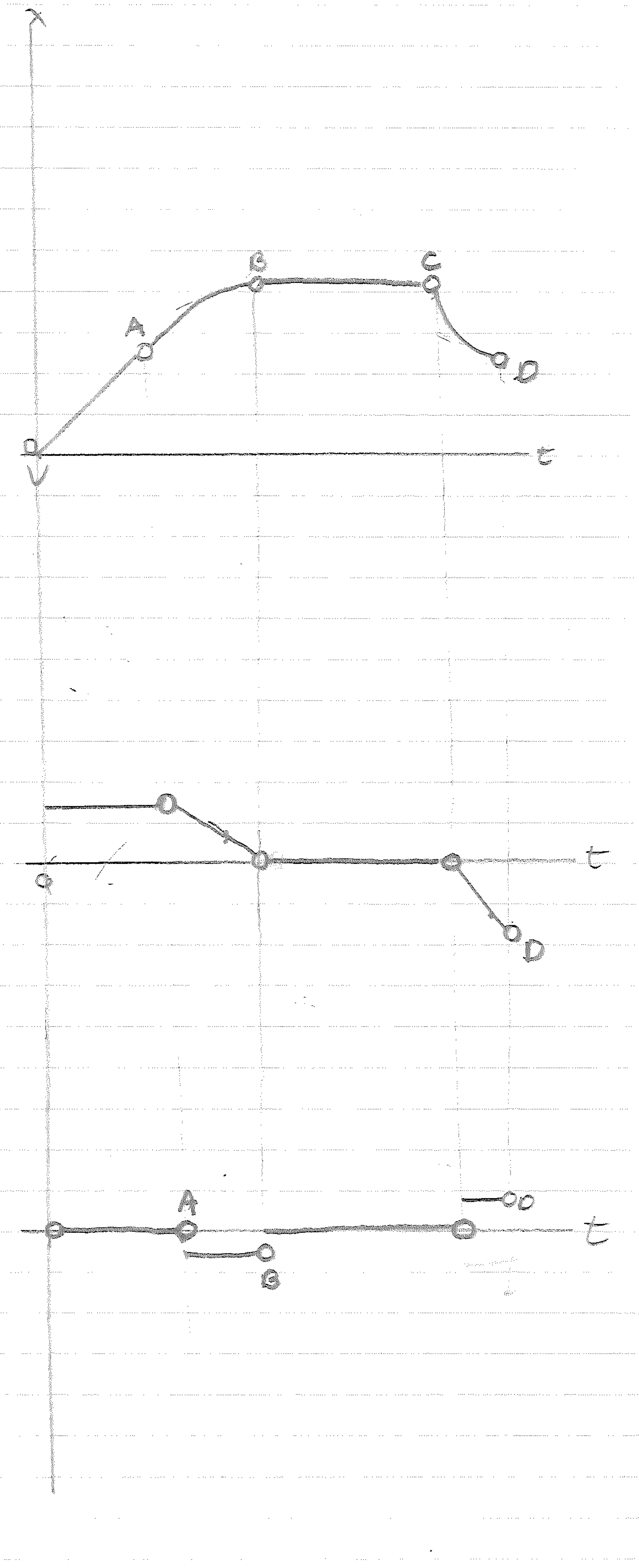
$a_{AVE} = \frac{30}{.01} = 3{,}000\ ft/sec^2$

Pg 53

6) o-a +
a-b +
b-c 0
c-d −

o-a 0
a-b −
b-c 0
c-d +

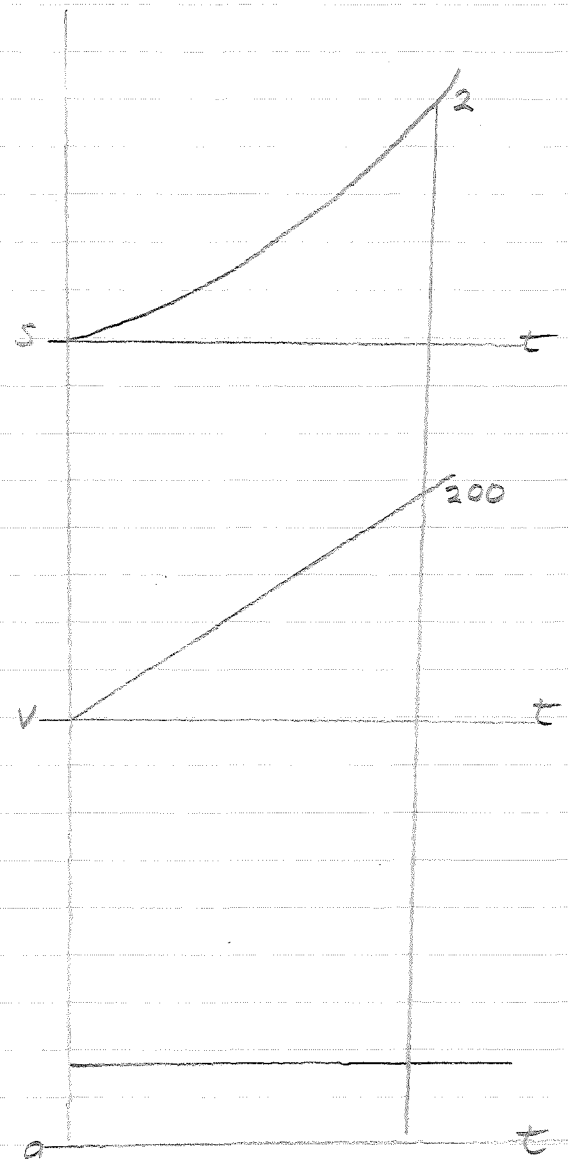
no



Pg 53

8) $a_{AVE} = \frac{\Delta v}{\Delta t}$

$V_0 = 0$ $V_f = 200$



$a = k$

$V = kt + c$

at $V=0$, $t=0$

$\therefore c = 0$

$V = kt$

$s = \frac{k}{2}t^2 + C_2$

at $s=0$, $t=0$ $\therefore C_2 = 0$

$200 = kt$ $\quad 2 = \frac{k}{2}t^2$

$t = \frac{200}{k}$ $\quad t^2 = \frac{4}{k}$

$t = \frac{2}{\sqrt{k}}$

$\frac{200}{k} = \frac{2}{\sqrt{k}}$

$200\sqrt{k} = 2k$

$100\sqrt{k} = k$

$10,000k = k^2$

$k^2 - 10,000k = 0$

$k = 10,000 \frac{ft}{sec^2}$

Pg 53

9) $\Delta V = 4.0 \times 10^6 - 1.0 \times 10^4$

$= 400 \times 10^4 - 1.0 \times 10^4$

$= 399 \times 10^4 = 3.99 \times 10^6$

$a = k$

$V = kt + c$

at $V = 10^4$, $t = 0$ $\therefore$ $c = 10^4$

$V = kt + 10^4$

$s = \frac{k}{2}t^2 + 10^4 t + c_2$

at $s = 0$; $t = 0$ $\therefore$ $c_2 = 0$

$s = 10^{-2} = \frac{k}{2}t^2 + 10^4 t$ $\qquad$ $V = 4 \times 10^6 = kt + 10^4$

$\frac{k}{2}t^2 + 10^4 t - 10^{-2} = 0$ $\qquad$ $t = \frac{4 \times 10^6 - 10^4}{k}$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-10^4 \pm \sqrt{10^8 + 2k(10^{-2})}}{k} = \frac{4 \times 10^6 - 10^4}{k}$

$-10^4 \pm \sqrt{10^8 + 2k(10^{-2})} = 4 \times 10^6 - 10^4$

$\pm \sqrt{10^8 + 2k(10)^{-2}} = 4 \times 10^6$

$10^8 + 2k(10)^{-2} = 16 \times 10^{12}$

$2k 10^{-2} = 16 \times 10^{12} - 10^8$

$= 160000 \times 10^8 - 10^8$

$= 159999 \times 10^8$

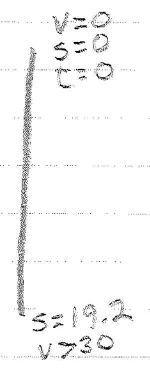
$2k = 159999 \times 10^{10}$

$k = 79999.5 \times 10^{10}$

$\approx 8.00 \times 10^{14}$

Pg 53
§

10) $30 \frac{M}{H} \times \frac{1.47 \frac{ft}{sec}}{1 \frac{M}{H}} = 44.1 \frac{ft}{sec}$



$a = k$

$V = kt$

$S = \frac{k}{2} t^2$

at $S = 19.2$

$38.4 = kt^2$

$t = \frac{6.20}{(\sqrt{k})}$

$V = k^{\frac{1}{2}}(6.20)$

if $k = 32$

$V = (5.66)(6.20) > 30$

∴ man was speeding.

Pg 53

12)

$a = 32$

$V = 32t + C$

$C = 0$

$V = 32t$

$v=0$, $s=0$, $t=0$

$1.86 \times 10^5 \frac{M}{S} \times \frac{5280 ft}{M} = 9.81 \times 10^8 \frac{ft}{sec}$

$\frac{1}{4}(9.81 \times 10^8 \frac{ft}{sec}) = 2.45 \times 10^8 \frac{ft}{sec}$

$2.45 \times 10^8 = 32t$

$2.45 \times 10^6 = 32t$

$t = 7.66 \times 10^6$ sec

$s = 16t^2$

$= 16(7.66 \times 10^6)^2$

$= 939 \times 10^{12}$

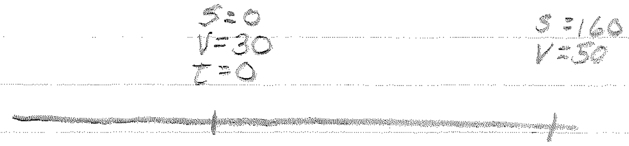
$= 9.39 \times 10^{14}$

5

Pg 53-54

13)

a)



$a = k$

$v = kt + c$

$v = kt + 30$

$s = \frac{k}{2}t^2 + 30t + c_2$

$s = \frac{k}{2}t^2 + 30t$

$160 = \frac{k}{2}t^2 + 30t$ $\qquad$ $50 = kt + 30$

$\frac{k}{2}t^2 + 30t - 160 = 0$ $\qquad$ $kt = 20$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad$ $t = \frac{20}{k}$

$t = \frac{-30 \pm \sqrt{900 + 320k}}{k}$

$-30 \pm \sqrt{900+320k} = 20$

$\pm\sqrt{900+320k} = 50$

$900 + 320k = 2500$

$32k = 160$

$k = 5.0 = a$ ✓

b) $s = \frac{k}{2}t^2 + 30t$

$k = 5 \quad s = 160$

$160 = \frac{5}{2}t^2 + 30t$

$5t^2 + 60t - 320 = 0$

$t^2 + 12t - 64 = 0$

$t = \frac{-12 \pm \sqrt{144 + 256}}{2}$

$= \frac{-12 \pm \sqrt{400}}{2}$

$= \frac{20 - 12}{2} = 4$ sec ✓

c) $v=0$, $t=0$, $s=0$ ———————— $v=30$

$a=5$

$v=5t$

$v=30$

$t=6$ sec ✓

d) $s=\frac{5}{2}t^2+C_2$ $C_2=0$

$s=\frac{5}{2}t^2$

$s=\frac{5}{2}(6)^2=5(18)=90$ ft ✓

Pg 54

15) $s=0$, $t=0$ ———— $V=45$, $s=180$, $t=6$

$a = K$

$V = Kt + C_1$

at $V = 45$, $t = 6$ $\therefore C_1 = 45 - 6K$

$V = Kt + 45 - 6K$

$s = \frac{K}{2}t^2 + (45 - 6K)t + C_2$

at $s = 180$, $t = 6$

$180 = \frac{K}{2}(36) + (45 - 6K)6 + C_2$

$180 = 18K + 270 - 36K + C_2$

$C_2 = 18K - 90$

$s = \frac{K}{2}t^2 + (45 - 6K)t + (18K - 90)$

$s = 0$ $t = 0$

$18K = 90$

$K = 5$

b) $a = 5\ ^{ft}/_{sec^2}$

$V = Kt + 45 - 6K$

$K = 5$ $t = 0$

a) $V = 45 - 30 = 15\ ^{FT}/_{SEC}$

$V=0$, $t=0$, $s=0$, $a=5$ ↓ $V=15$

c) $a = 5$

$V = 5t$

$s = \frac{5}{2}t^2$

$V = 15$ $\therefore t = 3$

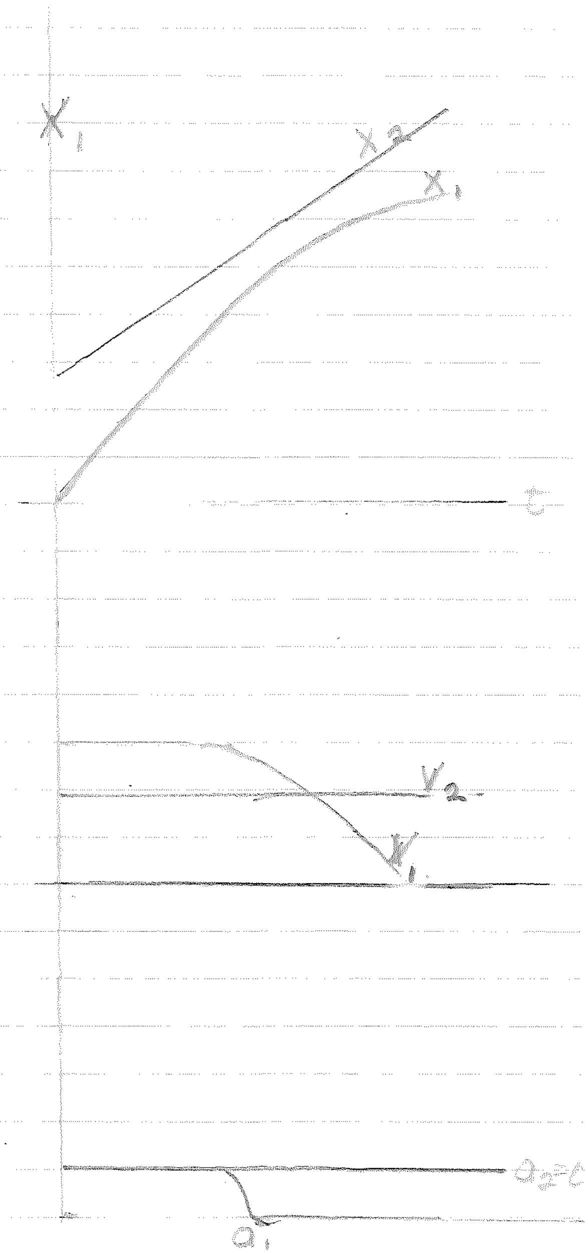
$s = \frac{5}{2}(9) = \frac{45}{2} = 22.5$ ft

Pg 54

(a)

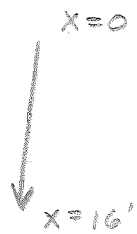
$a = k$

$V = kt$ v_1



Pg 54-5

24)



a) $X = \frac{a}{2}t^2 + V_{0x}t + x_0$

$X = \frac{a_x}{2}t^2 = 16$

$32t^2 = 32$

$t = 1$ SEC

$V_x = a_x t + V_{0x}$

$= a_x t$

$= 32$ ft/sec

$32 \frac{ft}{sec} \times 5$ sec $= 160$ ft

b) $V_{AVE} = \frac{\Delta x}{\Delta t} = \frac{176 \text{ FT}}{6 \text{ SEC}} = 29.4 \frac{FT}{SEC}$

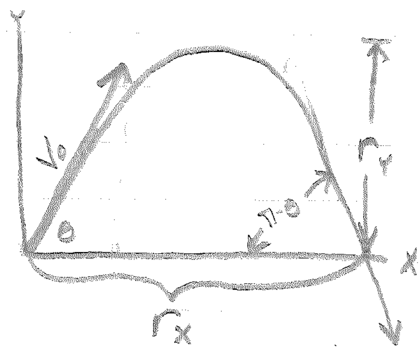
c) $X = \frac{a}{2}t^2 + V_{0x}t + X_0$

$176 = 16(5)^2 + V_{0x}5 + 0$

$176 = 400 + V_{0x}5$

Pg 75

7)



$r_y = r_x$

$r_y = \frac{a}{2}t^2$

$r_y = 16t^2$

$t = \frac{1}{4}\sqrt{r_y}$

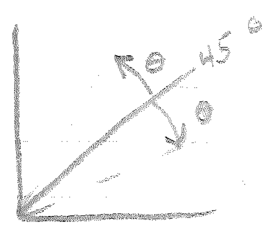
$r_x = V_0 \sin\theta\, t$

$r_x = V_0 \frac{1}{4}\sqrt{r_y} \sin\theta$

$\sqrt{r_x} = \frac{1}{4} V_0 \sin\theta$

$r_y = r_x = \frac{1}{16} V_0^2 \sin^2\theta$

8)



$R = \frac{V_0^2 \sin 2\theta}{g}$

$R_1 = \frac{V_0^2 \sin 2(\frac{\pi}{4}+\theta)}{g}$ $\quad R_2 = \frac{V_0^2 \sin 2(\frac{\pi}{4}-\theta)}{g}$

if $R_2 = R_1 \Rightarrow \frac{2V_0^2 \sin(\frac{\pi}{4}+\theta)\cos(\frac{\pi}{4}+\theta)}{g} = \frac{2V_0^2 \sin(\frac{\pi}{4}-\theta)\cos(\frac{\pi}{4}-\theta)}{g}$

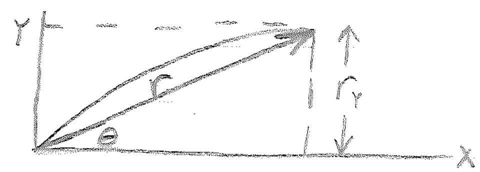
$(\sin\frac{\pi}{4}\cos\theta + \sin\theta\cos\frac{\pi}{4})(\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta) =$

$(\sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta)(\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta)$

$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

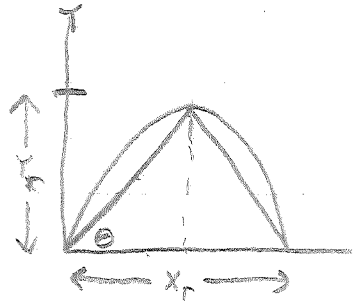
Pg 74

5)



$$a = g$$
$$V_y = gt - V_{oy}$$
$$V_{oy} = V_o \sin\theta$$
$$r_y = \frac{g}{2}t^2 + V_o \cos\theta$$
$$\frac{dr_y}{d\theta} = gt - V_o \sin\theta = 0$$
$$gt = V_o \sin\theta$$
$$t = \frac{V_o \sin\theta}{g}$$
$$Y_{max} = \frac{g}{2}\left(\frac{V_o \sin\theta}{g}\right)^2 + V_o \cos\theta$$
$$= \frac{(V_o \sin\theta)^2}{2g}$$

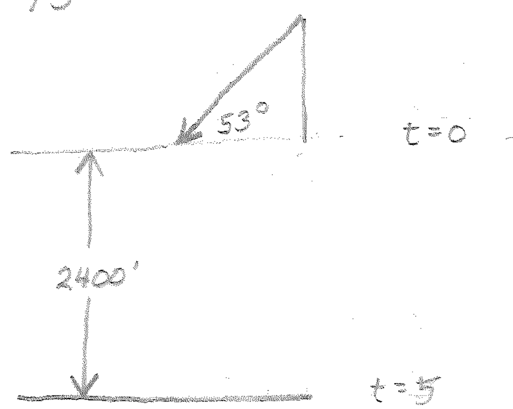
7)



$$Y_r = X_r$$
$$V_r = 2V_o \csc\theta$$
$$X_r = 2V_o t \csc\theta$$
$$a_y = g$$
$$V_y = gt$$
$$Y_r = \frac{g}{2}t^2$$

Pg 75

10)

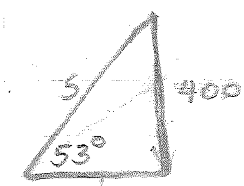


$x_y = \frac{a}{2}t^2 + V_{ox}t$

$2400 = 16(25) + V_{ox}5$

$2000 = 5V_{ox}$

$V_{ox} = 400$



a) $s = 400 \csc 53°$

$s = 502\ ft/sec$

b) $d_v = s \cos 53°$

$d_v = 302\ ft/sec$

$d = 302\ \frac{ft}{sec}\ 3\ sec = 1695\ ft$

c) horizonal

$V_x = 565\ \frac{ft}{sec}$

verticle

$V_y = \frac{a}{2}t^2 + V_{ox}t$

$= 16(9) + (752)(3)$

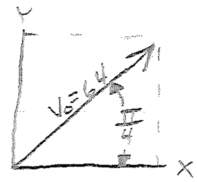
$= 144 + 2256$

$= 3400\ \frac{ft}{sec}$

Bob Markow

Pg 75

12)



$V_{0y} = V_{0x} = 64 \cos \frac{\pi}{4} = \frac{64}{\sqrt{2}}$

$V = 0$, $t = 0$, $s = 0$

$a = 32$

$V = \frac{64}{\sqrt{2}}$

$V = a t_1$

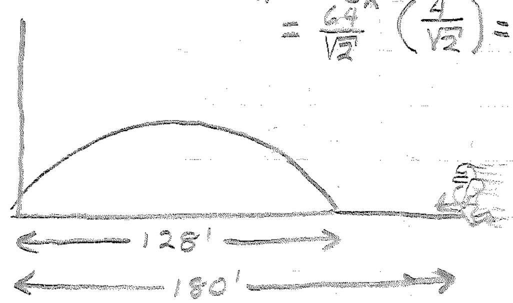
$\frac{64}{\sqrt{2}} = 32 t$

$t_1 = \frac{2}{\sqrt{2}}$

$t = 2 t_1 = \frac{4}{\sqrt{2}}$

$r_x = V_{0x} t$

$= \frac{64}{\sqrt{2}} \left(\frac{4}{\sqrt{2}}\right) = 128 \text{ ft}$ ✓



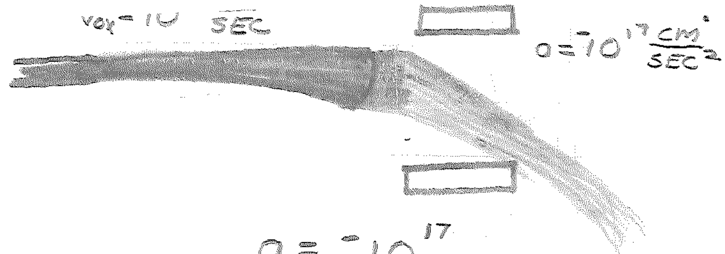
$r = 180' - 128' = 52'$

$t = \frac{4}{\sqrt{2}}$

$V_{foot} = \frac{52\sqrt{2}}{4} \cong 18.3 \text{ ft/sec}$ ✓

5

15)



$a_y = {}^{-}10^{17}$

$S_y = \frac{a}{2}t^2$

$Vt = S_1$

$10^9 t = 2$

$t = 2 \times 10^{-9}$

a) $S_y = {}^{-}.5 \times 10^{17}(2 \times 10^{-9})^2$

$= {}^{-}.5 \times 10^{17} \times 4 \times 10^{-18}$

$= {}^{-}2 \times 10^{-1} = {}^{-}.2\ cm$

b) $V_y = a_y t$

$= (10^{17})(2 \times 10^{-9})$

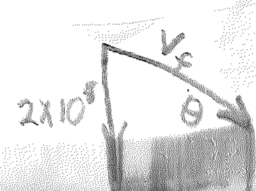
$= 2 \times 10^8\ \frac{cm}{sec}$

$V_x = 10^9\ cm/sec$

$= \sqrt{10^{\ldots}}$ [illegible]

$= \sqrt{1.04 \times 10^{18}}$

$1.02 \times 10^9\ cm/sec$

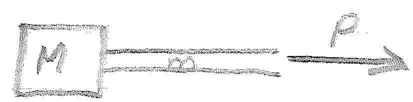


$\tan\theta = \frac{2 \times 10^8}{10^9} = .20$

$\theta = 11.5°$ S of E

Pg 105 V

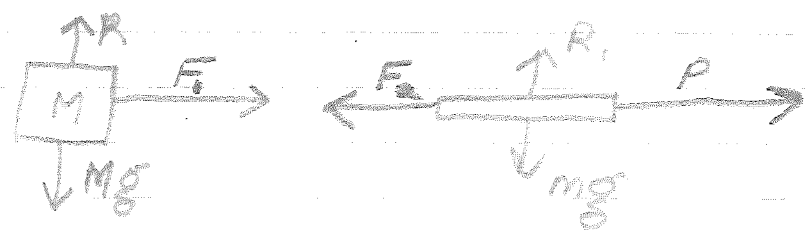
4)



a) $F = ma$

$P = (M + m)a$

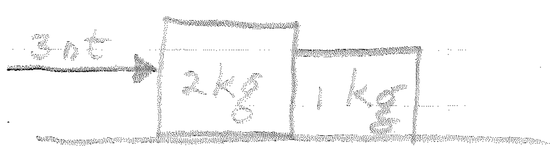
$a = \frac{P}{M+m}$

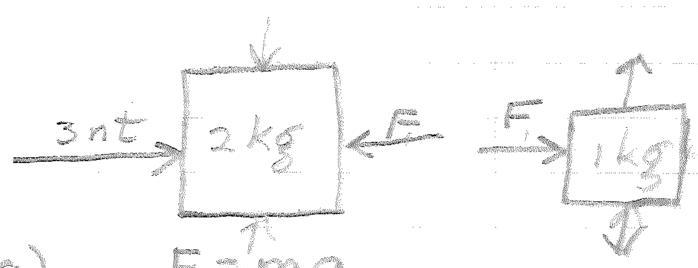


$F_1 = F_2$

$F_1 = M\left(\frac{P}{M+m}\right) = \frac{PM}{M+m}$

10)





a) $F = ma$

$F = ma$

$3nt = 3a$

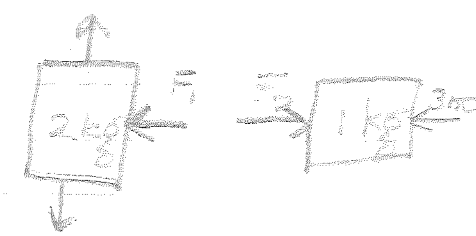
$a = 1 \frac{m}{sec^2}$

$F_1 = F_2 = ma$

$= 1 kg \frac{m}{sec^2}$

$= 1 nt$

b)

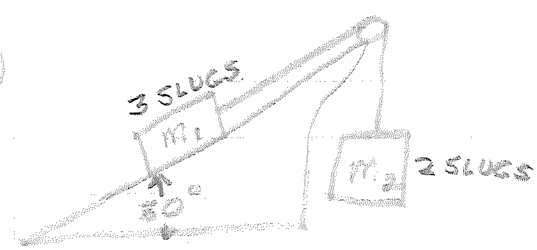


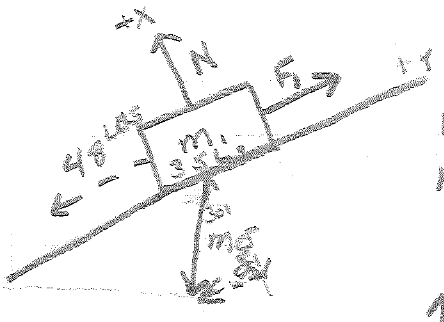
$F_1 = F_2 = ma$

$= 2 nt$

106 V

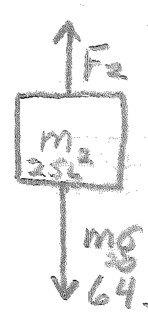
15)





$mg = 3\text{SLUGS}\ 32\frac{ft}{sec^2} = 96$ lbs

$mg_x = mg \sin 30° = 48$ lbs



$x^2 = a^2 + b^2 - 2ab\cos x$

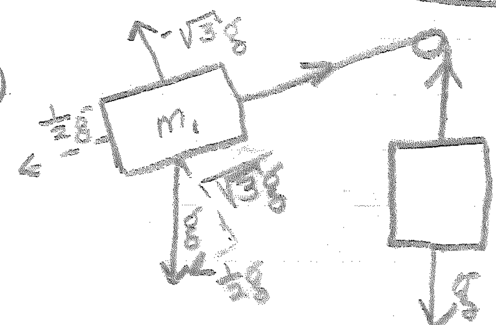
$= 2300 + 4100 - 3040$

$= 6400 - 3040 = 3360$

~~$\Sigma F_1' = \Sigma F_2'$~~

b) $x = 58$ lbs

a)



$a^2 = (16)^2 + (32)^2 - 2(16)(32)\frac{\sqrt{3}}{2}$

b) $64 - 48 = 16$ lbs

$\frac{16 \text{ lbs}}{5 \text{ slug}} = 3.2 \frac{ft}{sec^2}$

Pg 105 V

5) $F = ma$

$v = 0$, $t = 0$, $s = 0$, $x = 0$

↓

$v = 74.5$ ft/sec, $x = 200$ ft, $t =$

$a = ?$

$v = at$ $\quad x = \frac{a}{2}t^2$

$74.5 = at$ $\quad 200 = \frac{a}{2}t^2$

$t = \frac{74.5}{a}$ $\quad t^2 = \frac{400}{a}$ $\quad t = \frac{20}{\sqrt{a}}$

$\frac{74.5}{a} = \frac{20}{\sqrt{a}}$

$74.5\sqrt{a} = 20a$

$5550a = 400a^2$

$\frac{555}{40} = a = 13.9$ ft/sec^2

$F = m\,a$ $\quad \frac{3000 \text{ lb}}{32.2 \text{ lb}/\text{slug}} = 93.3$

$F = 93.3 \text{ slugs} \cdot 13.9 \frac{ft}{sec^2}$ $\quad t = \frac{74.5}{13.9} = 5.4$ sec

$F = 1{,}300$ lbs

6) $1{,}300 \text{ lbs} = 93.3 \text{ slugs } a$

$a = 13.9$ ft/sec^2

$25 \frac{M}{H} \times \frac{1.47 \frac{ft}{sec}}{M/H} = 36.8 \frac{ft}{sec}$

$v = at$

$36.8 = 13.9t$

$t = 2.7$ sec

$x = \frac{a}{2}t^2$

$= (7)(2.7)^2 = 51$ ft.

Pg 105 IV

6) $V=0$, $x=0$ ⟶ $6\times10^6 \frac{m}{sec} = V$, $x = .01\,m$

$$V = at \qquad x = \frac{a}{2}t^2$$

$$6\times10^6 \frac{m}{sec} = at \qquad .01\,m = \frac{a}{2}t^2$$

$$t = \frac{6\times10^6}{a} \qquad t^2 = \frac{.02}{a}$$

$$\frac{6\times10^6}{a} = \frac{.141}{\sqrt{a}}$$

$$6\times10^6\sqrt{a} = .141\,a$$

$$36\times10^{12}\,a = .02\,a^2$$

$$a = 1800\times10^{12} = 1.8\times10^{15}$$

$$F = ma$$

$$= (9.1\times10^{-31}\,kg)(1.8\times10^{15}\frac{m}{sec^2})$$

$$= 16.4\times10^{-16}\,nt = 1.64\times10^{-15}\,nt$$

$$V = at$$

$$6\times10^6\frac{m}{sec} = (1.8\times10^{15})\,t$$

$$t = 3.33\times10^{-9}\,sec$$

$$a = 9.8\frac{m}{sec^2}$$

$$F = ma$$

$$= 9.1\times10^{-31}\,kg\,(9.8\frac{m}{sec^2})$$

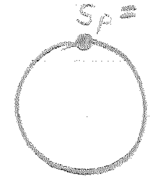
$$= 89.2\times10^{-31} = 8.92\times10^{-30}\,nt$$

Yes. Since the magnitude of the force due to gravity is about 10^{-16} times that of the electrical force, it is negligible.

Pg 77

22)

$S_p = 2.18 \times 10^6$



$\longleftarrow 5.28 \times 10^{-11} = r$

$$a = \frac{v^2}{r} = \frac{(2.18 \times 10^6)^2}{5.28 \times 10^{-11}} = .902 \times 10^{23} \frac{m}{sec^2}$$
$$= 9.02 \times 10^{22} \frac{m}{sec^2}$$

24) $g = 9.8 \frac{m}{sec^2}$

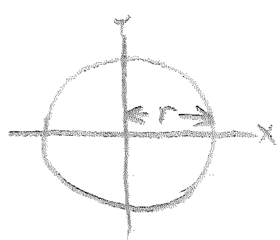
$r_{\text{of earth}} = 6.37 \times 10^6 m$

$$a = 9.8 \frac{m}{sec^2} = \frac{v^2}{r}$$
$$9.8 \frac{m}{sec^2} = \frac{v^2}{6.37 \times 10^6 m}$$
$$v^2 = 62.4 \times 10^6 \frac{m^2}{sec^2}$$
$$v = 7.90 \times 10^3 \frac{m}{sec}$$

$$a = 3 \times 10^{-2} \frac{m}{sec^2} = \frac{v^2}{6.37 \times 10^6 m}$$
$$19.11 \times 10^4 \frac{m^2}{sec^2} = v^2$$
$$v = 4.37 \times 10^2 \frac{m}{sec}$$

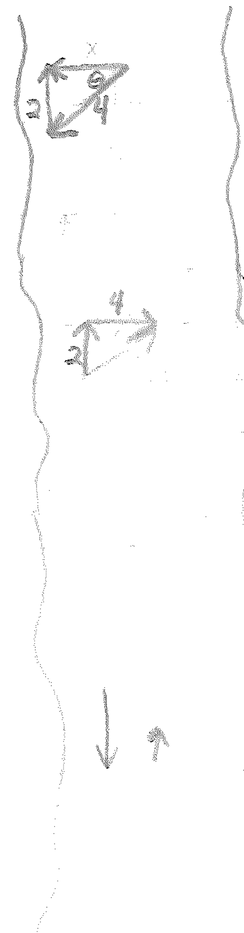
$$4.37 \times 10^2 f = 7.90 \times 10^3$$
$f = 1.81 \times 10 = 18.1$ *times as fast*

29)



$$x^2 \hat{j} + y^2 \hat{j} = r^2$$
$$y^2 = r^2 - x^2$$
$$y = (r^2 - x^2)^{\frac{1}{2}}$$
$$\frac{dy}{dx} = v$$
$$u = r^2 - x^2 \qquad y = u^{\frac{1}{2}}$$
$$v = -2x \left[\frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}\right]$$
$$= -\frac{x}{\sqrt{r^2 - x^2}} = -\frac{x}{y}$$
$$a = \frac{dv}{dt}$$
$$= \frac{y^2 - \frac{x^2}{y}}{r^2 - x^2} = \frac{-y^2 - xv}{-y^2} = 1 + \frac{xv}{y^2}$$

34) a)



$\sin\theta = \frac{1}{2}$

$\theta = 30°$ S of W

a) Of time is at a min:

$4 \frac{M}{H} t = 4M$

$t = 1$ HR

b) Of distance traveled is at a min:

$2\sqrt{3}\, t = 4M$

$t = \frac{2}{\sqrt{3}}$ HR $= 1.16$ HR

c)

c) $V_1 = 6$ M/HR

$t_1 = \frac{2M}{6 M/HR} = \frac{1}{3}$ HR

$V_2 = 2$ M/HR

$t_2 = 1$ HR

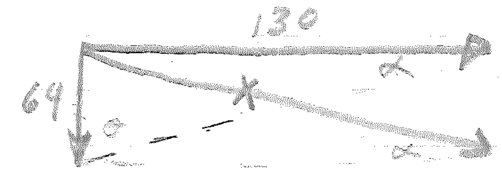
$t_1 + t_2 = \frac{4}{3}$ HR

d) $t = \frac{4}{3}$ HR

Pg 105 IV

7) $F = ma$

$= 2\,\text{slugs} \cdot 32\,\frac{ft}{sec^2} = 64\,\text{lbs}$



$\tan\alpha = \frac{64}{130} = .492$

$\alpha = 26.2°$

$|X| = (\csc\alpha)\,64 = \frac{64}{\sin\alpha} = 145\,\text{lbs}$

$F = ma$

$145 = 2a$

$a = 72.5\,\frac{ft}{sec^2}$ } at 26.2° S of E

$V = 72.5\,\frac{ft}{sec^2}\,t$

Pg 105

5)

$F = ma$

$4.5\times10^{-15}\,nt = 9.1\times10^{-31}\,kg\,(a)$

$a = .495\times10^{16}\,\frac{m}{sec^2}$

$x = \frac{a}{2}t^2$

$Vt = x$

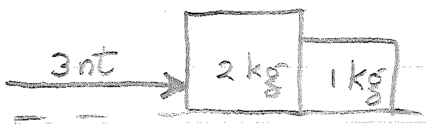
$1.2\times10^7 \frac{m}{s}\,t = 3\times10^{-2}\,m$

$t = 2.5\times10^{-9}$ sec

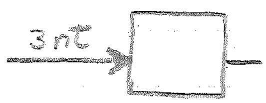
$x = .247\times10^{15}(2.5\times10^{-9})^2$

$= 15.4\times10^{-3}\,m$

Pg 105 V



a)



$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

$$\frac{2\,kg}{1\,kg} = \frac{3-F}{F}$$

$$3 - F = 2F$$

$$F = 1\,nt$$

b) $$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

$$\frac{2}{1} = \frac{F}{3-F}$$

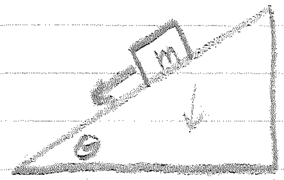
$$6 - 2F = F$$

$$6 = 3F$$

$$F = 2\,nt$$

Pg 106

19)



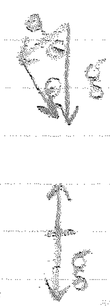
a) $a = g \sin\theta$

b) $a = g \sin\theta$

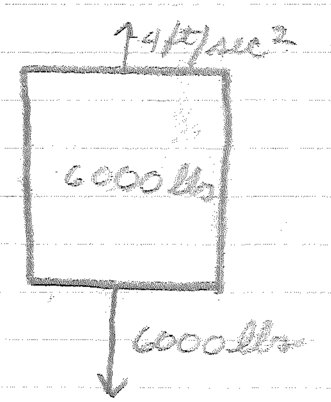
c) $A = (g - a) \sin\theta$

d) $A = (g + a) \sin\theta$

e) 0



20)



a) $F = ma$

$= \left(\frac{6000}{32.2}\right) 4 = 746$ lbs

6746 lbs

b) 5254 lbs

Pg 126 VI

2) $\mu_k = .25$

$F_f = \mu_k N$

$ma = \mu_k mg$

$a = \mu_k g$

$= (.25)(32.2) = 8.05 \frac{ft}{sec^2}$

$30 \frac{mi}{hr} \times 1.47 = 44.1 \frac{ft}{sec}$

$V = at$

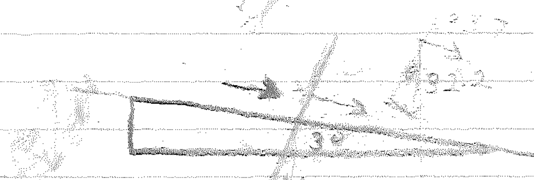
$44.1 \frac{ft}{sec} = 8.05 \frac{ft}{sec^2} t$

$t = 5.48 sec$

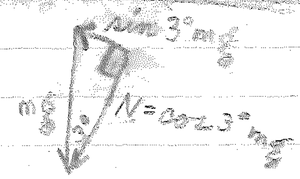
3) $X = \frac{a}{2} t^2$

$1400 = a(61)^2 \qquad 1400 = a(42)^2$

$a = .38 \frac{ft}{sec^2} \qquad a = .79 \frac{ft}{sec^2}$



$f_k = \mu_k N \qquad \Sigma F = f_f + N - WT = 0$



$f_k = \mu_k N$

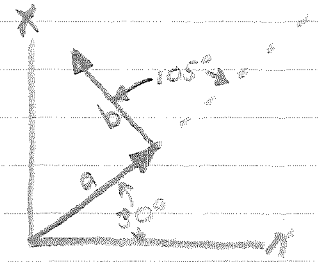
(1.16) ... m

μ_k ... .62

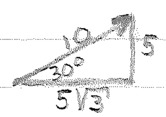
PHYS

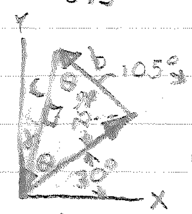
Pg. 28

4)



$\vec{a} \Rightarrow X = 5\sqrt{3} \quad Y = 5$



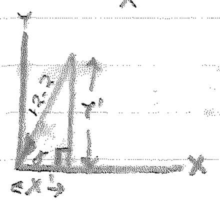


$\theta = \frac{1}{2}(105°) = 52\frac{1}{2}°$

$|\vec{r}| = 2b\cos\theta$

$= 20\cos 52\frac{1}{2}°$

$|\vec{r}| \cong 20(.609) = 12.2$



$\gamma = 82.5°$

$Y' = 12.2\sin\gamma$

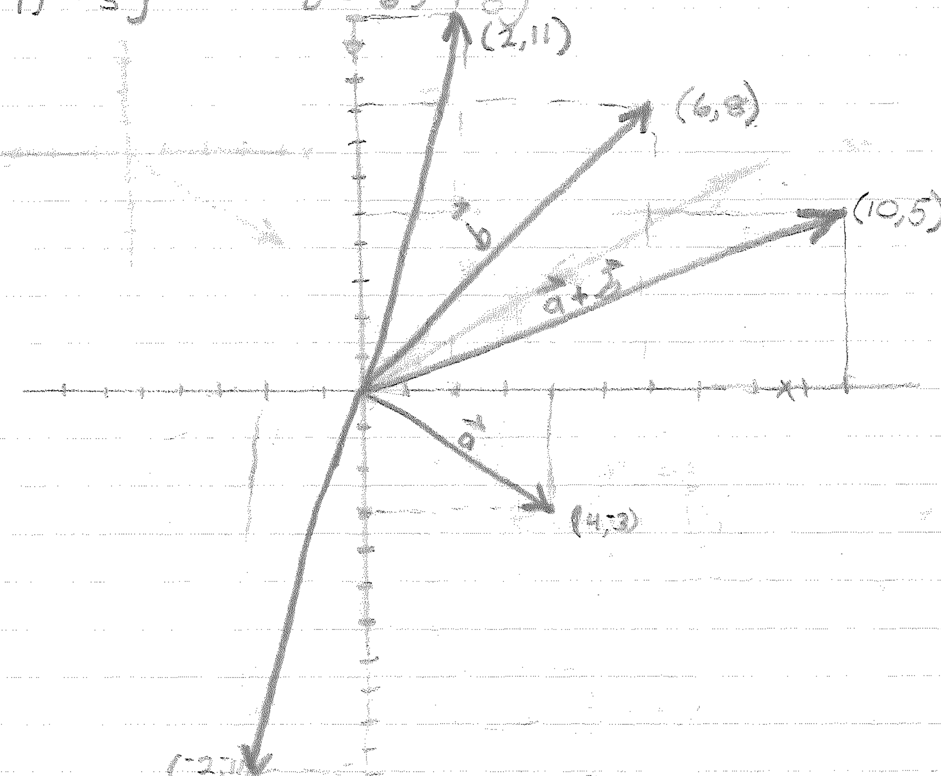
$Y' = 12.2(.991) = 12.1$

$X' = 12.2\cos\gamma$

$X' = 12.2(.130) = 1.58$

Pg 28

5) $a = 4i - 3j$ $b = 6i + 8j$



a) $\vec{a} = 5$ at $-36.0°$

b) $\vec{b} = 10$ at $53.1°$

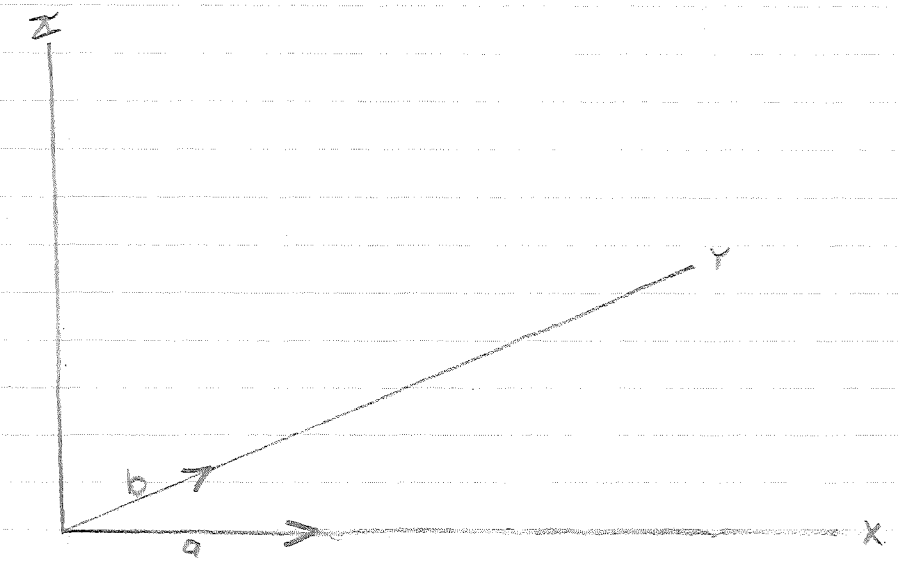
c) $\vec{a} + \vec{b} = 11.1$ at $26.6°$

d) $\vec{b} - \vec{a} = 11.2$ at $79.7°$

e) $\vec{a} - \vec{b} = 11.2$ at 259.7

Pg 28

20)

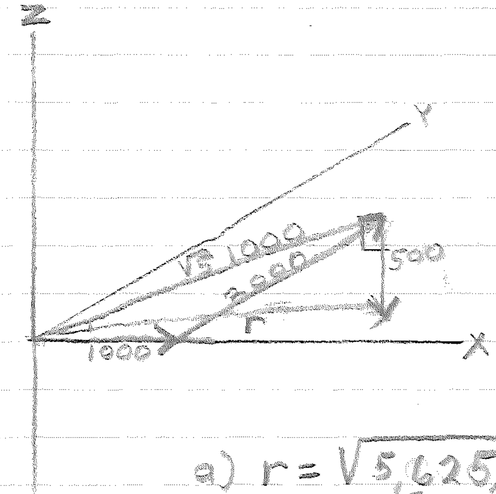


a) $+Z$
b) $-Z$
c) $+Y$
d) $a \cdot b = ab \cos \frac{\pi}{2} = 0$

Bob Marks

Pg 29

12)



2/5

a) $r = \sqrt{5,625,000} = 10\sqrt{5,625} = 750$ ft

b) Θ

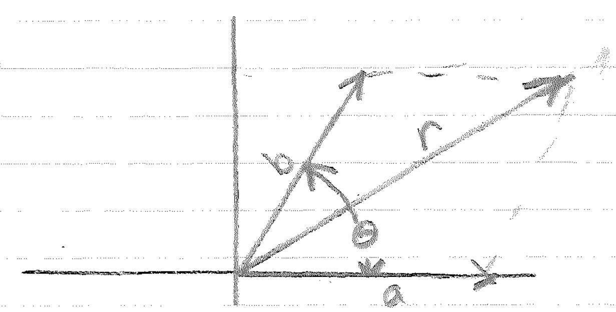
14) a) 2.5 m S

b) 1.25 m N

c) 6.25 m S

d) 10 m N

18)



$r_x = a_x + b_x$

$= a + b\cos\Theta$

$r_y = a_y + b_y$

$= 0 + b\sin\Theta$

$r = \sqrt{b^2\sin^2\Theta + (a + b\cos\Theta)^2}$

$r = \sqrt{b^2\sin^2\Theta + a^2 + 2ab\cos\Theta + b^2\cos^2\Theta}$

$r = \sqrt{b^2(\sin^2\Theta + \cos^2\Theta) + a^2 + 2ab\cos\Theta}$

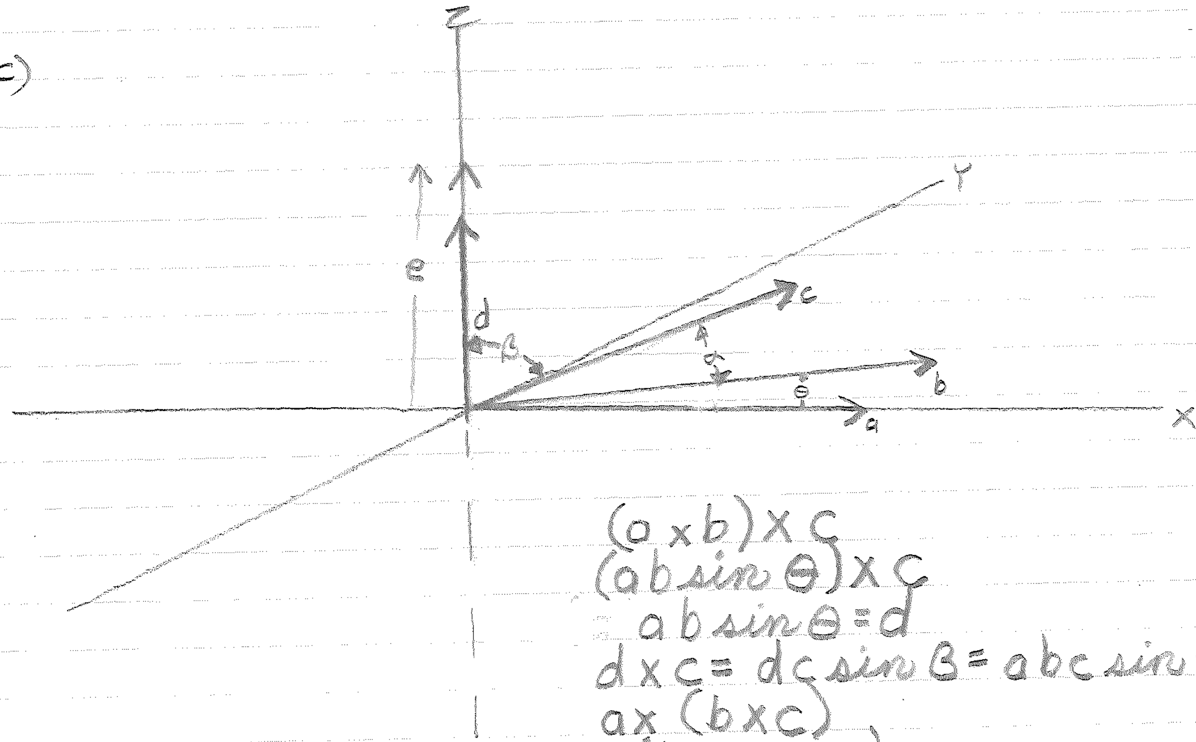
$r = \sqrt{b^2 + a^2 + 2ab\cos\Theta}$

Pg 30

24) $a \cdot b = ab\cos\theta$ $\quad$ $b \cdot a = ba\cos\theta$

$ab\cos\theta = ab\cos\theta$

c)



$(a \times b) \times c$

$(ab\sin\theta) \times c$

$ab\sin\theta = d$

$d \times c = dc\sin\beta = abc\sin\theta\sin\beta$

$a \times (b \times c)$

$a \times (bc\sin\alpha)$

$e = bc\sin\alpha$

$a \times e = ae\sin\beta = abc\sin\alpha\sin\beta$

$\sin\beta = \sin\frac{\pi}{2} = 1$

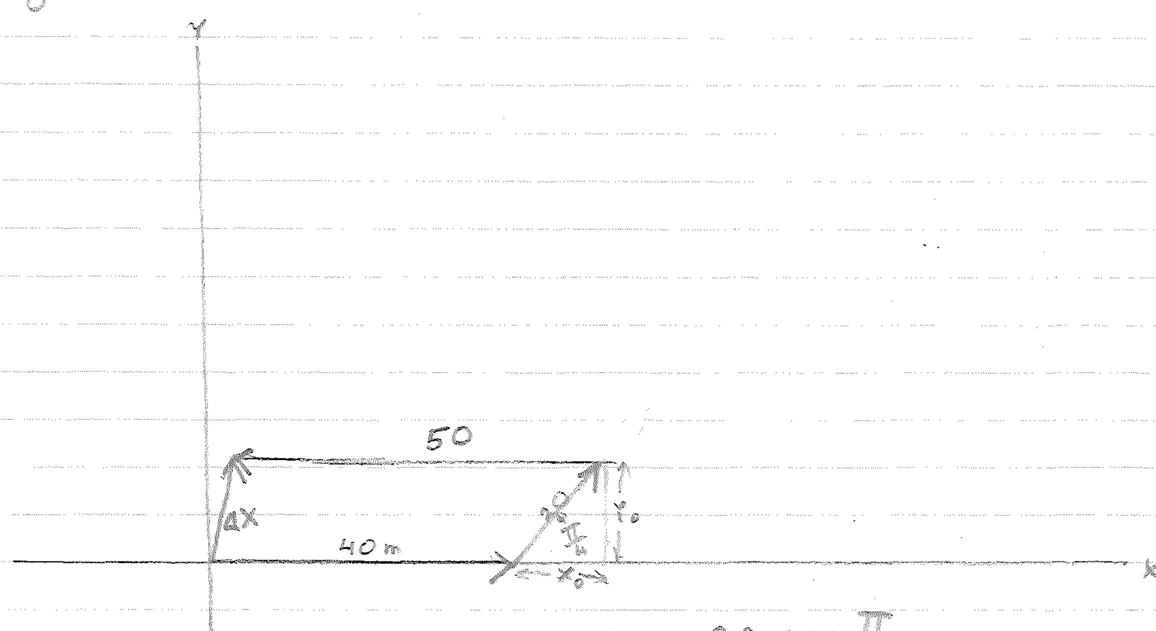
if $(a \times b) \times c = a \times (b \times c)$ then

$abc\sin\theta = abc\sin\alpha$

$\sin\theta = \sin\alpha$

Pg 52

2)



$x_0 = 20 \cos \frac{\pi}{4}$
$= 20 \frac{1}{\sqrt{2}} = \frac{20}{\sqrt{2}}$
$Y_0 = X_0 = \frac{20}{\sqrt{2}}$

$V = 60$ mph
$V_{AVE} = \frac{\Delta r}{\Delta t}$

$\vec{r}_a = t_1 \, 60$
$= \frac{2}{3} \, 60 = 40$
$\vec{r}_b = t_2 \, 60$
$= \frac{1}{3} \, 60 = 20$
$\vec{r}_c = t_3 60 = 50$

$r_{a_x} = 40$
$r_{b_x} = \frac{20}{\sqrt{2}}$
$r_{c_x} = -50$

$\Delta r_x = \frac{20}{\sqrt{2}} - 10 \cong 4.14$

$r_{a_Y} = r_{c_Y} = 0$
$r_{b_Y} = 20/\sqrt{2} \cong 14.1$
$\Delta r_Y = 14.1$

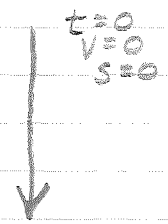
$\Delta \vec{r} = \sqrt{200 + 17.1} = \sqrt{217} = 14.3$

$\Delta t = \frac{40}{60} + \frac{20}{60} + \frac{50}{60} = \frac{11}{6}$ hr.

$V_{AVE} = \frac{14.3}{\frac{11}{6}} = \frac{6(14.3)}{11} = 7.8$ MPH

Pg 52

5)



$a = 32$

$V = 32t + C_1$

$C_1 = 0$

$V = 32t$

$s = 16t^2 + C_2$

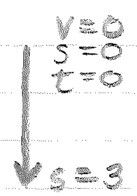
$C_2 = 0$

$s = 16t^2$

$s = 4$

$4 = 16t^2 \qquad t = \frac{1}{2}$

$\underline{V = 16}$



$a = 32$

$V = 32t + C_1 \qquad C_1 = 0$

$V = 32t$

$s = 16t^2 + C_2 \qquad C_2 = 0$

$s = 16t^2$

$s = 3$

$3 = 16t^2$

$\frac{3}{16} = t^2$

$t = \frac{\sqrt{3}}{4}$

$V = 32\frac{\sqrt{3}}{4} = 8\sqrt{3}$

$a_{AVE} = \frac{\Delta V}{\Delta t}$

$\Delta V = 16 + 8\sqrt{3} \qquad \Delta t = .01$

≈ 30

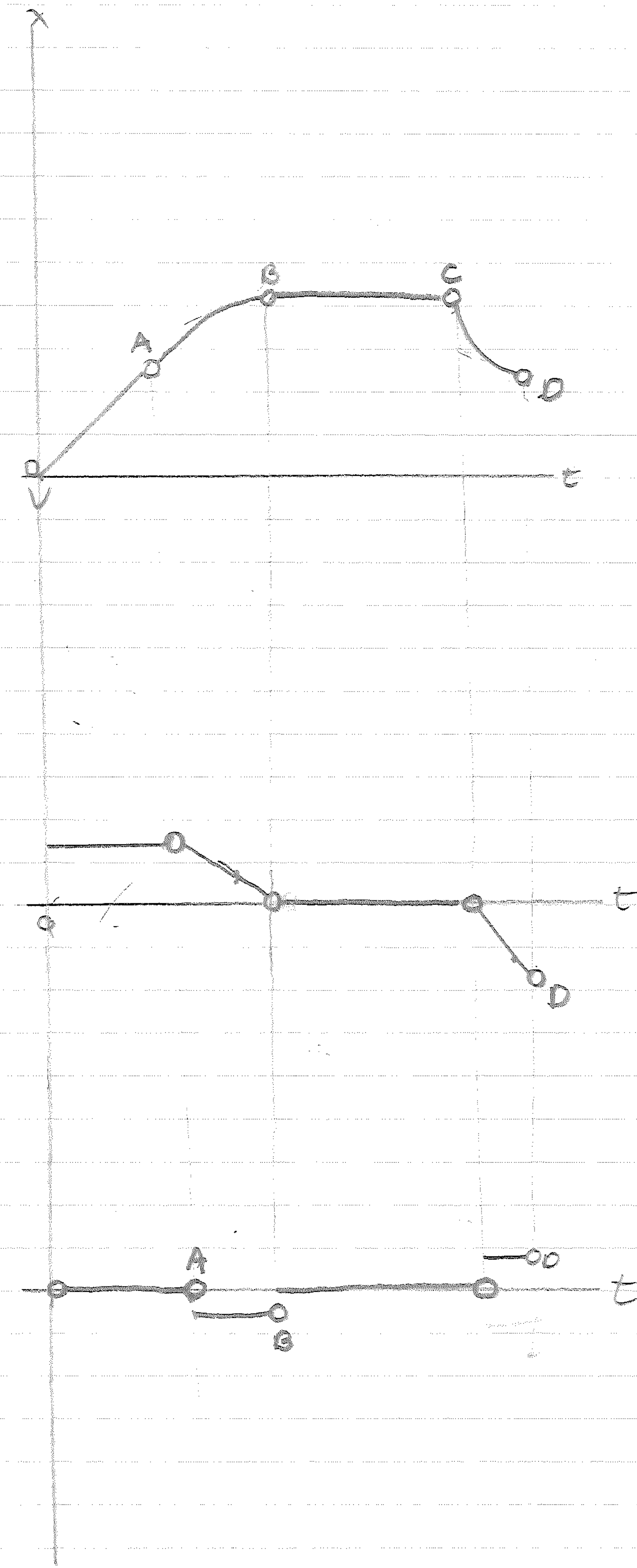
$a_{AVE} = \frac{30}{.01} = 3{,}000 \text{ ft/sec}^2$

Pg 53

6) o-a +
a-b +
b-c 0
c-d −

o-a 0
a-b −
b-c 0
c-d +

no



Pg 53

8) $a_{AVE} = \frac{\Delta V}{\Delta t}$

$V_0 = 0$ $V_f = 200$

$a = k$

$V = kt + c$

at $V=0$, $t=0$

$\therefore c = 0$

$V = kt$

$s = \frac{k}{2}t^2 + C_2$

at $s=0$, $t=0 \therefore C_2 = 0$

$200 = kt$ $2 = \frac{k}{2}t^2$

$t = \frac{200}{k}$ $t^2 = \frac{4}{k}$

$t = \frac{2}{\sqrt{k}}$

$\frac{200}{k} = \frac{2}{\sqrt{k}}$

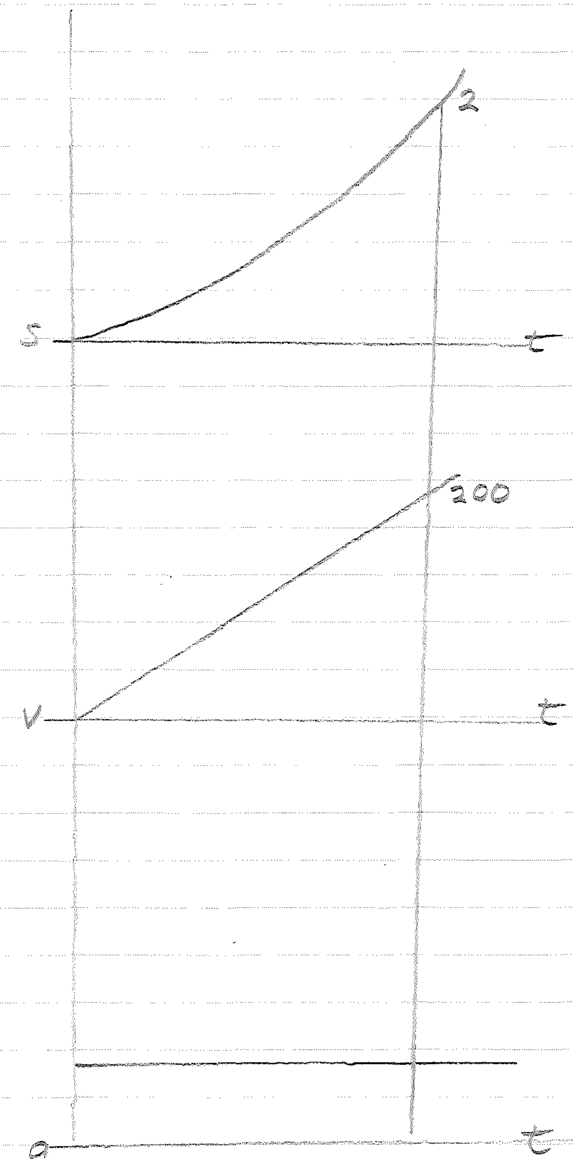
$200\sqrt{k} = 2k$

$100\sqrt{k} = k$

$10,000k = k^2$

$k^2 - 10,000k = 0$

$k = 10,000 \frac{ft}{sec^2}$



Pg 53

9) $\Delta V = 4.0 \times 10^6 - 1.0 \times 10^4$

$= 400 \times 10^4 - 1.0 \times 10^4$

$= 399 \times 10^4 = 3.99 \times 10^6$

$a = k$

$V = kt + c$

at $V = 10^4$, $t = 0$ $\therefore$ $c = 10^4$

$V = kt + 10^4$

$s = \frac{k}{2}t^2 + 10^4 t + c_2$

at $s = 0$; $t = 0$ $\therefore$ $c_2 = 0$

$s = 10^{-2} = \frac{k}{2}t^2 + 10^4 t$ $\quad$ $V = 4 \times 10^6 = kt + 10^4$

$\frac{k}{2}t^2 + 10^4 t - 10^{-2} = 0$ $\quad$ $t = \frac{4 \times 10^6 - 10^4}{k}$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{-10^4 \pm \sqrt{10^8 + 2k(10^{-2})}}{k} = \frac{4 \times 10^6 - 10^4}{k}$

$-10^4 \pm \sqrt{10^8 + 2k(10^{-2})} = 4 \times 10^6 - 10^4$

$\pm \sqrt{10^8 + 2k(10)^{-2}} = 4 \times 10^6$

$10^8 + 2k(10)^{-2} = 16 \times 10^{12}$

$2k10^{-2} = 16 \times 10^{12} - 10^8$

$= 160000 \times 10^8 - 10^8$

$= 159999 \times 10^8$

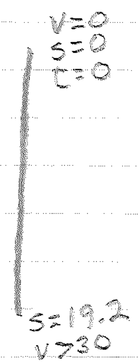
$2k = 159999 \times 10^{10}$

$k = 79999.5 \times 10^{10}$

$\approx 8.00 \times 10^{14}$

Pg 53
§ 8

10) $30 \frac{M}{H} \times \frac{1.47 \frac{ft}{sec}}{} / 1\frac{M}{H} = 44.1 \frac{ft}{sec}$



$a = k$

$V = kt$

$S = \frac{k}{2}t^2$

at $S = 19.2$

$38.4 = kt^2$

$t = \frac{6.20}{(\sqrt{k})}$

$V = k^{\frac{1}{2}}(6.20)$

if $k = 32$

$V = (5.66)(6.20) > 30$

∴ man was speeding.

Pg 53

12)

$a = 32$

$V = 32t + C$

$C = 0$

$V = 32t$

$V=0$, $S=0$, $t=0$

$1.86 \times 10^{5} \frac{M}{S} \times \frac{5280\,ft}{M} = 9.81 \times 10^{8} \frac{ft}{sec}$

$\frac{1}{4}\left(9.81 \times 10^{8} \frac{ft}{sec}\right) = 2.45 \times 10^{8} \frac{ft}{sec}$

$2.45 \times 10^{8} = 32t$

$2.45 \times 10^{6} = 32t$

$t = 7.66 \times 10^{6}$ sec

$S = 16t^{2}$

$= 16\,(7.66 \times 10^{6})^{2}$

$= 939 \times 10^{12}$

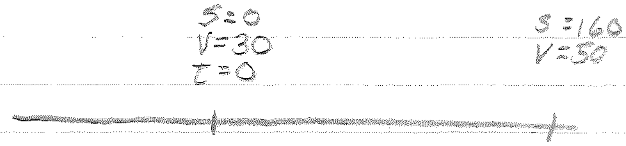
$= 9.39 \times 10^{14}$

5

Pg 53-54

13)

a)



$a = k$

$v = kt + c$

$v = kt + 30$

$s = \frac{k}{2}t^2 + 30t + c_2$

$s = \frac{k}{2}t^2 + 30t$

$160 = \frac{k}{2}t^2 + 30t$ $\qquad$ $50 = kt + 30$

$\frac{k}{2}t^2 + 30t - 160 = 0$ $\qquad$ $kt = 20$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad$ $t = \frac{20}{k}$

$t = \frac{-30 \pm \sqrt{900 + 320k}}{k}$

$-30 \pm \sqrt{900+320k} = 20$

$\pm\sqrt{900+320k} = 50$

$900 + 320k = 2500$

$32k = 160$

$k = 5.0 = a$ ✓

b) $s = \frac{k}{2}t^2 + 30t$

$k = 5 \quad s = 160$

$160 = \frac{5}{2}t^2 + 30t$

$5t^2 + 60t - 320 = 0$

$t^2 + 12t - 64 = 0$

$t = \frac{-12 \pm \sqrt{144 + 256}}{2}$

$= \frac{-12 \pm \sqrt{400}}{2}$

$= \frac{20-12}{2} = 4$ sec ✓

c) $v=0$, $t=0$, $s=0$ ———— $v=30$

$a=5$

$v=5t$

$v=30$

$t = 6$ sec ✓

d) $s=\frac{5}{2}t^2+C_2 \qquad C_2=0$

$s=\frac{5}{2}t^2$

$s=\frac{5}{2}(6)^2=5(18)=$ 90 ft ✓

Pg 54

15) $s=0$, $t=0$ ———— $V=45$, $s=180$, $t=6$

$a = K$

$V = Kt + C_1$

at $V=45$, $t=6$ $\therefore C_1 = 45 - 6K$

$V = Kt + 45 - 6K$

$s = \frac{K}{2}t^2 + (45-6K)t + C_2$

at $s=180$, $t=6$

$180 = \frac{K}{2}(36) + (45-6K)6 + C_2$

$180 = 18K + 270 - 36K + C_2$

$C_2 = 18K - 90$

$s = \frac{K}{2}t^2 + (45-6K)t + (18K-90)$

$s=0$ $t=0$

$18K = 90$

$K = 5$

b) $a = 5\ ^{ft}/_{sec^2}$

$V = Kt + 45 - 6K$

$K=5$ $t=0$

a) $V = 45 - 30 = 15\ ^{FT}/_{SEC}$

$V=0$, $t=0$, $s=0$, $a=5$ ↓ $V=15$

c) $a = 5$

$V = 5t$

$s = \frac{5}{2}t^2$

$V = 15$ $\therefore t = 3$

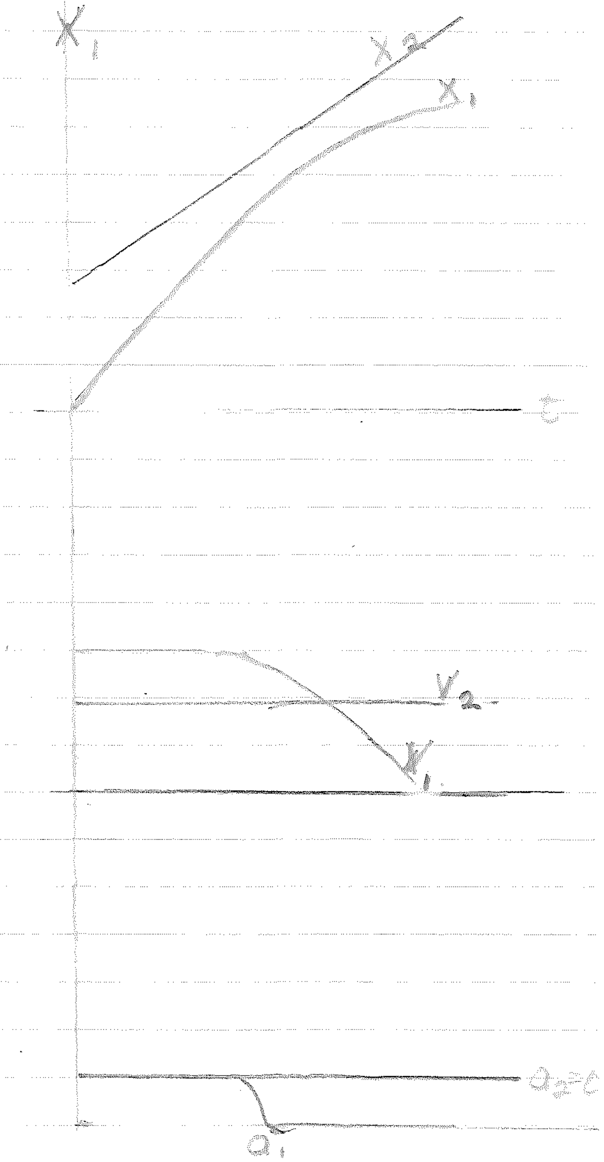
$s = \frac{5}{2}(9) = \frac{45}{2} = 22.5\ ft$

Pg 54

(6)

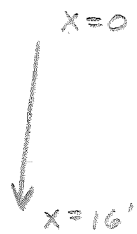
$a = k$

$V = kt \quad v_1$



Pg 54-5

24)



a) $X = \frac{a}{2}t^2 + V_{0x}t + x_0$

$X = \frac{a_x}{2}t^2 = 16$

$32t^2 = 32$

$t = 1 \text{ SEC}$

$V_x = a_x t + V_{0x}$

$= a_x t$

$= 32 \text{ ft/sec}$

$32 \frac{ft}{sec} \times 5 \text{ sec} = 160 \text{ ft}$

b) $V_{AVE} = \frac{\Delta x}{\Delta t} = \frac{176 \text{ FT}}{6 \text{ SEC}} = 29.4 \frac{FT}{SEC}$

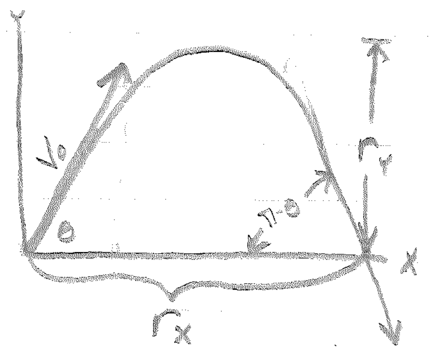
c) $X = \frac{a}{2}t^2 + V_{0x}t + X_0$

$176 = 16(5)^2 + V_{0x}5 + 0$

$176 = 400 + V_{0x}5$

Pg 75

7)



$r_Y = r_X$

$r_Y = \frac{a}{2}t^2$ $\qquad$ $r_X = V_0 \sin\theta\, t$

$r_Y = 16t^2$ $\qquad$ $r_X = V_0 \frac{1}{4}\sqrt{r_Y}\sin\theta$

$t = \frac{1}{4}\sqrt{r_Y}$ $\qquad$ $\sqrt{r_X} = \frac{1}{4}V_0 \sin\theta$

$r_Y = r_X = \frac{1}{16}V_0^2 \sin^2\theta$

8)

$R = \frac{V_0^2 \sin 2\theta}{g}$

$R_1 = \frac{V_0^2 \sin 2(\frac{\pi}{4}+\theta)}{g}$ $\qquad$ $R_2 = \frac{V_0^2 \sin 2(\frac{\pi}{4}-\theta)}{g}$

If $R_2 = R_1 \Rightarrow \frac{2V_0^2 \sin(\frac{\pi}{4}+\theta)\cos(\frac{\pi}{4}+\theta)}{g} = \frac{2V_0^2 \sin(\frac{\pi}{4}-\theta)\cos(\frac{\pi}{4}-\theta)}{g}$

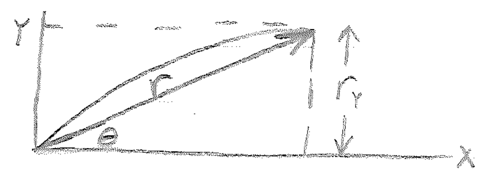
$(\sin\frac{\pi}{4}\cos\theta + \sin\theta\cos\frac{\pi}{4})(\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta) =$

$(\sin\frac{\pi}{4}\cos\theta - \cos\frac{\pi}{4}\sin\theta)(\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta)$

$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

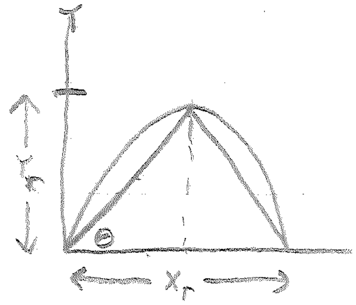
Pg 74

5)



$$a = g$$
$$V_Y = gt - V_{oY}$$
$$V_{oY} = V_o \sin\theta$$
$$r_Y = \frac{g}{2}t^2 + V_o \cos\theta$$
$$\frac{dr_Y}{d\theta} = gt - V_o \sin\theta = 0$$
$$gt = V_o \sin\theta$$
$$t = \frac{V_o \sin\theta}{g}$$
$$Y_{MAX} = \frac{g}{2}\left(\frac{V_o \sin\theta}{g}\right)^2 + V_o \cos\theta$$
$$= \frac{(V_o \sin\theta)^2}{2g}$$

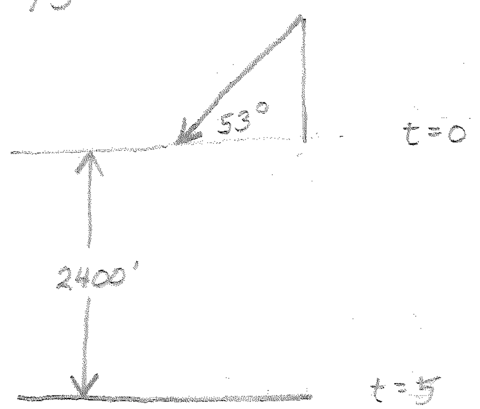
7)



$$Y_r = X_r$$
$$V_r = 2V_o \csc\theta$$
$$X_r = 2V_o t \csc\theta$$
$$a_Y = g$$
$$V_Y = gt$$
$$Y_r = \frac{g}{2}t^2$$

Pg 75

10)

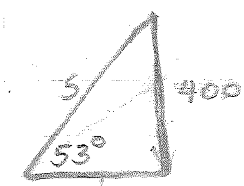


$X_y = \frac{a}{2}t^2 + V_{ox}t$

$2400 = 16(25) + V_{ox}5$

$2000 = 5V_{ox}$

$V_{ox} = 400$



a) $s = 400 \csc 53°$

$s = 502 \text{ ft/sec}$

b) $d_v = s \cos 53°$

$d_v = 302 \text{ ft/sec}$

$d = 302 \frac{ft}{sec} \, 3 \, sec = 1695 \, ft$

c) horizonal

$V_x = 565 \frac{ft}{sec}$

verticle

$V_y = \frac{a}{2}t^2 + V_{ox}t$

$= 16(9) + (752)(3)$

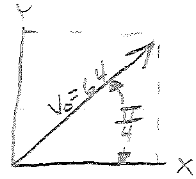
$= 144 + 2256$

$= 3400 \frac{ft}{sec}$

Bob Markes

Pg 75

12)

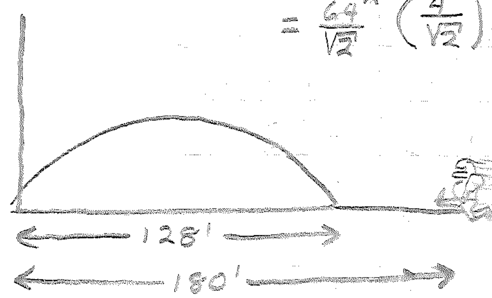


$V_{0y} = V_{0x} = 64 \cos\frac{\pi}{4} = \frac{64}{\sqrt{2}}$

$V=0$ $s=0$
$t=0$
$a=32$
$V=\frac{64}{\sqrt{2}}$

$V = at_1$
$\frac{64}{\sqrt{2}} = 32t$
$t_1 = \frac{2}{\sqrt{2}}$
$t = 2t_1 = \frac{4}{\sqrt{2}}$

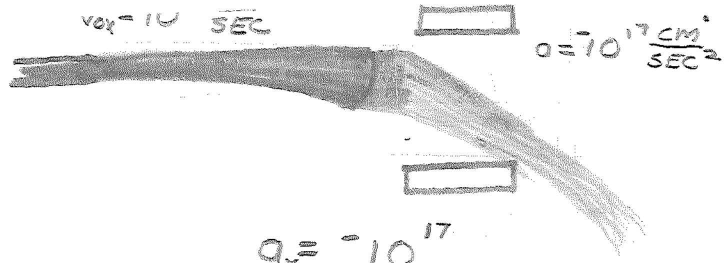
$r_x = V_{0x} t$
$= \frac{64}{\sqrt{2}}\left(\frac{4}{\sqrt{2}}\right) = 128 \text{ ft}$ ✓



$r = 180' - 128' = 52'$
$t = \frac{4}{\sqrt{2}}$
$V_{foot} = \frac{52\sqrt{2}}{4} \cong 18.3 \text{ ft/sec}$ ✓

5

15)



$a_y = -10^{17}$

$S_y = \frac{a}{2}t^2$

$Vt = S_1$

$10^9 t = 2$

$t = 2\times10^{-9}$

a) $S_y = -.5\times10^{17}(2\times10^{-9})^2$

$= -.5\times10^{17}\times4\times10^{-18}$

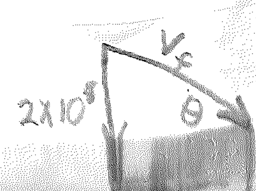
$= -2\times10^{-1} = -.2$ cm

b) $V_y = a_y t$

$= (10^{17})(2\times10^{-9})$

$= 2\times10^{8}\ \frac{cm}{sec}$

$V_x = 10^{9}$ cm/sec



$= \sqrt{10^{18} + \ldots}$

$= \sqrt{1.04\times10^{18}}$

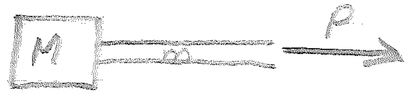
$= 1.02\times10^{9}$ cm/sec

$\tan\theta = \frac{2\times10^8}{10^9} = .20$

$\theta = 11.5^\circ$ S of E

Pg 105 V

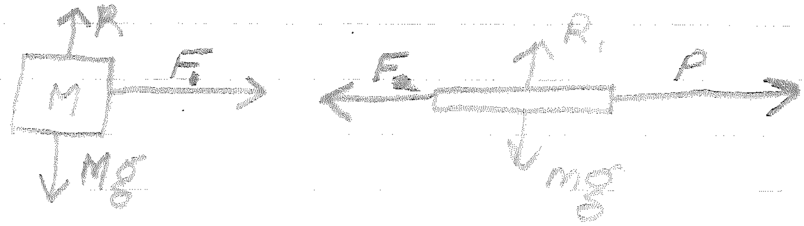
4)



a) $F = ma$

$P = (M + m)\, a$

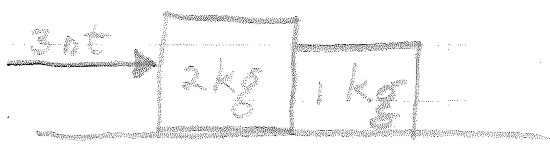
$a = \frac{P}{M+m}$



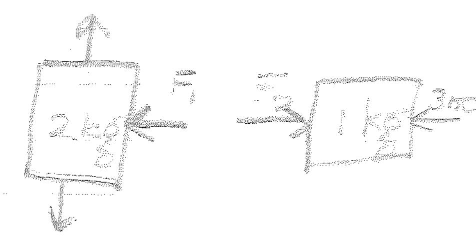
$F_1 = F_2$

$F_1 = M\left(\frac{P}{M+m}\right) = \frac{PM}{M+m}$

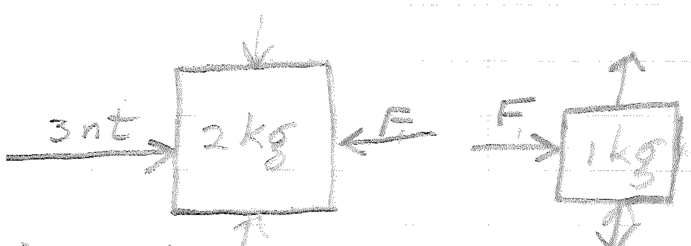
10)



b)



$F_1 = F_2 = ma$
$= 2nt$



a) $F = ma$

$F = ma$

$3nt = 3a$

$a = 1 \frac{m}{sec^2}$

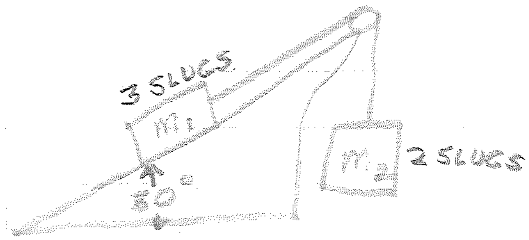
$F_1 = F_2 = ma$

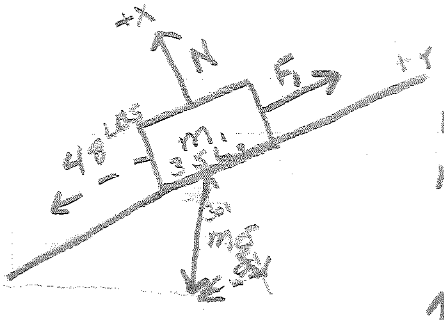
$= 1\,kg \cdot \frac{m}{sec^2}$

$= 1\,nt$

106 V

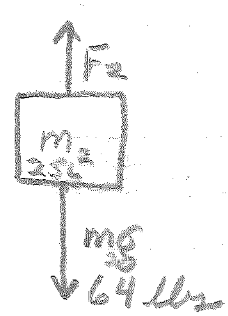
15)





$mg = 3 \text{ SLUGS } 32 \frac{ft}{sec^2} = 96 \text{ lbs}$

$mg_x = mg \sin 30° = 48 \text{ lbs}$



$x^2 = a^2 + b^2 - 2ab\cos x$

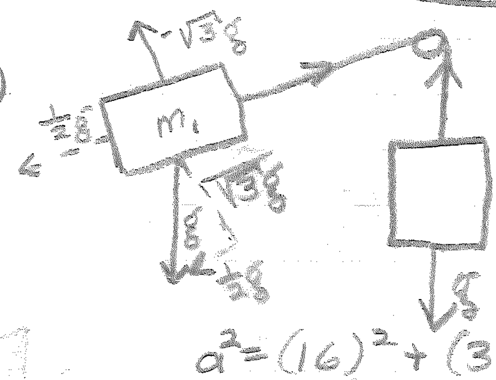
$= 2300 + 4100 - 3040$

$= 6400 - 3040 = 3360$

~~$\Sigma F_1' = \Sigma F_2'$~~

b) $x = 58 \text{ lbs}$

a)



$a^2 = (16)^2 + (32)^2 - 2(16)(32)\frac{\sqrt{3}}{2}$

b) $64 - 48 = 16 \text{ lbs}$

$\frac{16 \text{ lbs}}{5 \text{ slug}} = 3.2 \frac{ft}{sec^2}$

Pg 105 V

5) $F = ma$

$v=0$, $t=0$, $s=0$, $x=0$

↓

$v = 74.5$ ft/sec, $x = 200$ ft, $t =$

$a = ?$

$v = at$ $x = \frac{a}{2}t^2$

$74.5 = at$ $200 = \frac{a}{2}t^2$

$t = \frac{74.5}{a}$ $t^2 = \frac{400}{a}$ $t = \frac{20}{\sqrt{a}}$

$\frac{74.5}{a} = \frac{20}{\sqrt{a}}$

$74.5\sqrt{a} = 20a$

$5550a = 400a^2$

$\frac{555}{40} = a = 13.9$ ft/sec^2

$F = m\,a$ $\frac{3000 \text{ lb}}{32.2 \text{ lb}} \cdot 1 \text{ slug} = 93.3$

$F = 93.3 \text{ slugs} \cdot 13.9 \frac{ft}{sec^2}$ $t = \frac{74.5}{13.9} = 5.4$ sec

$F = 1{,}300$ lbs

6) $1{,}300 \text{ lbs} = 93.3 \text{ slugs } a$

$a = 13.9$ ft/sec^2

$25 \frac{M}{H} \times \frac{1.47 \frac{ft}{sec}}{M/H} = 36.8 \frac{ft}{sec}$

$v = at$

$36.8 = 13.9t$

$t = 2.7$ sec

$x = \frac{a}{2}t^2$

$= (7)(2.7)^2 = 51$ ft.

Pg 105 IV

6) $V=0$, $x=0$ ⟶ $6\times10^6 \frac{m}{sec} = V$, $x = .01\,m$

$V = at \qquad x = \frac{a}{2}t^2$

$6\times10^6 \frac{m}{sec} = at \qquad .01\,m = \frac{a}{2}t^2$

$t = \frac{6\times10^6}{a} \qquad t^2 = \frac{.02}{a}$

$\frac{6\times10^6}{a} = \frac{.141}{\sqrt{a}}$

$6\times10^6\sqrt{a} = .141\,a$

$36\times10^{12}\,a = .02\,a^2$

$a = 1800\times10^{12} = 1.8\times10^{15}$

$F = ma$

$= (9.1\times10^{-31}\,kg)(1.8\times10^{15}\,\frac{m}{sec^2})$

$= 16.4\times10^{-16}\,nt = 1.64\times10^{-15}\,nt$

$V = at$

$6\times10^6 \frac{m}{sec} = (1.8\times10^{15})\,t$

$t = 3.33\times10^{-9}$ sec

$a = 9.8\,\frac{m}{sec^2}$

$F = ma$

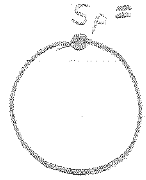
$= 9.1\times10^{-31}\,kg\,(9.8\,\frac{m}{sec^2})$

$= 89.2\times10^{-31} = 8.92\times10^{-30}\,nt$

Yes. Since the magnitude of the force due to gravity is about 10^{-16} times that of the electrical force, it is negligible.

Pg 77

22) $S_p = 2.18 \times 10^6$



$\longleftarrow 5.28 \times 10^{-11} = r$

$$a = \frac{v^2}{r} = \frac{(2.18 \times 10^6)^2}{5.28 \times 10^{-11}} = .902 \times 10^{23} \frac{m}{SEC^2}$$
$$= 9.02 \times 10^{22} \frac{m}{SEC^2}$$

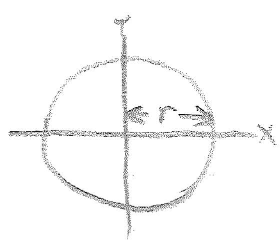
24) $g = 9.8 \frac{m}{SEC^2}$
$r_{\text{OF EARTH}} = 6.37 \times 10^6 m$

$$a = 9.8 \frac{m}{SEC^2} = \frac{V^2}{r}$$
$$9.8 \frac{m}{SEC^2} = \frac{V^2}{6.37 \times 10^6 m}$$
$$V^2 = 62.4 \times 10^6 \frac{m^2}{SEC^2}$$
$$V = 7.90 \times 10^3 \frac{m}{SEC}$$

$$a = 3 \times 10^{-2} \frac{m}{SEC^2} = \frac{V^2}{6.37 \times 10^6 m}$$
$$19.11 \times 10^4 \frac{m^2}{SEC^2} = V^2$$
$$V = 4.37 \times 10^2 \frac{m}{SEC}$$

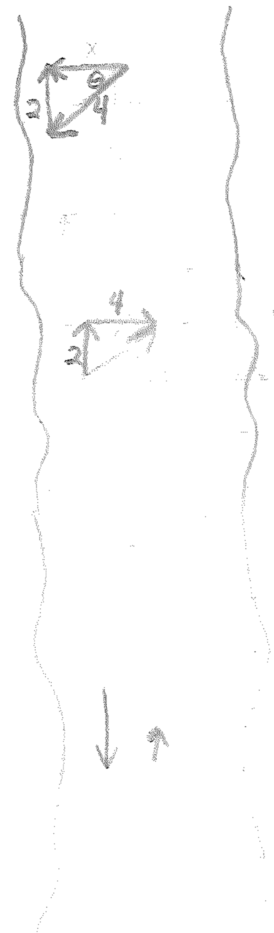
$$4.37 \times 10^2 f = 7.90 \times 10^3$$
$f = 1.81 \times 10 = 18.1$ *times as fast*

29)



$$x^2 j + y^2 j = r^2$$
$$y^2 = r^2 - x^2$$
$$y = (r^2 - x^2)^{\frac{1}{2}}$$
$$\frac{dy}{dx} = V$$
$$U = r^2 - x^2 \qquad y = U^{\frac{1}{2}}$$
$$V = -2x \left[\frac{1}{2}(r^2 - x^2)^{-\frac{1}{2}}\right]$$
$$= -\frac{x}{\sqrt{r^2 - x^2}} = -\frac{x}{y}$$
$$a = \frac{dV}{dt} = \frac{y^2 - \frac{x^2}{y}}{r^2 - x^2} = \frac{-y^2 - xV}{-y^2} = 1 + \frac{xV}{y^2}$$

34) a)



$\sin\theta = \frac{1}{2}$

$\theta = 30^\circ$ S of W

a) Of time is at a min:

$4\frac{M}{H}\,t = 4M$

$t = 1$ HR

b) Of distance traveled is at a min:

$2\sqrt{3}\,t = 4M$

$t = \frac{2}{\sqrt{3}}$ HR $= 1.16$ HR

c)

c) $V_1 = 6$ M/HR

$t_1 = \frac{2M}{6M/HR} = \frac{1}{3}$ HR

$V_2 = 2$ M/HR

$t_2 = 1$ HR

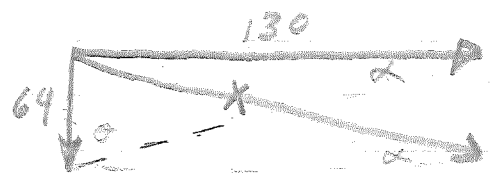
$t_1 + t_2 = \frac{4}{3}$ HR

d) $t = \frac{4}{3}$ HR

Pg 105 IX

7) $F = ma$

$= 2 \text{ slugs} \cdot 32 \frac{ft}{sec^2} = 64 \text{ lbs}$



$\tan\alpha = \frac{64}{130} = .492$

$\alpha = 26.2°$

$|X| = (\csc\alpha)64 = \frac{64}{\sin\alpha} = 145 \text{ lbs}$

$F = ma$

$145 = 2a$

$a = 72.5 \frac{ft}{sec^2}$ } at 26.2° S of E

$V = 72.5 \frac{ft}{sec^2} t$

Pg 105

5)

$F = ma$

$4.5 \times 10^{-15}\,nt = 9.1 \times 10^{-31}\,kg\,(a)$

$a = .495 \times 10^{16} \frac{m}{sec^2}$

$x = \frac{a}{2} t^2$

$Vt = x$

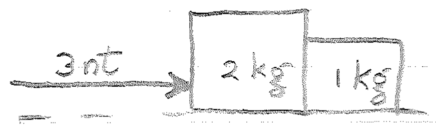
$1.2 \times 10^7 \frac{m}{s} t = 3 \times 10^{-2}\,m$

$t = 2.5 \times 10^{-9}$ sec

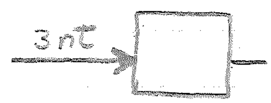
$x = .247 \times 10^{15} (2.5 \times 10^{-9})^2$

$= 15.4 \times 10^{-3}\,m$

Pg 105 V



a)

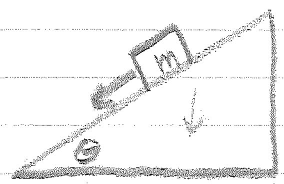


$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$
$$\frac{2\,kg}{1\,kg} = \frac{3-F}{F}$$
$$3-F = 2F$$
$$F = 1\,nt$$

b)
$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$
$$\frac{2}{1} = \frac{F}{3-F}$$
$$6-2F = F$$
$$6 = 3F$$
$$F = 2\,nt$$

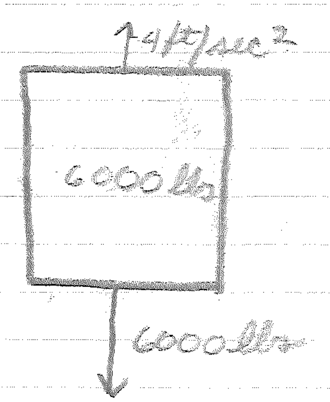
Pg 106

19)



a) $a = g \sin\theta$
b) $a = g \sin\theta$
c) $A = (g - a) \sin\theta$
d) $A = (g + a) \sin\theta$
e) 0

20)



a) $F = ma$

$= \left(\frac{6000}{32.2}\right) 4 = 746$ lbs

6746 lbs

b) 5254 lbs

Pg 126 VI

2) $\mu_k = .25$

$F_f = \mu_k N$

$ma = \mu_k mg$

$a = \mu_k g$

$= (.25)(32.2) = 8.05 \frac{ft}{sec^2}$

$30 \frac{mi}{hr} \times 1.47 = 44.1 \frac{ft}{sec}$

$V = at$

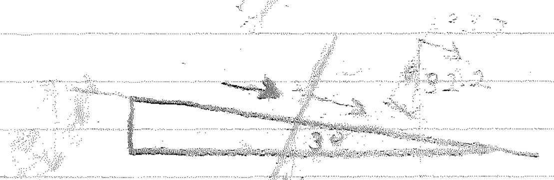
$44.1 \frac{ft}{sec} = 8.05 \frac{ft}{sec^2} t$

$t = 5.48 sec$

3) $X = \frac{a}{2} t^2$

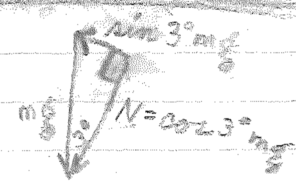
$1400 = a(61)^2$ $\quad$ $1400 = a(42)^2$

$a = .38 \frac{ft}{sec^2}$ $\quad$ $a = .79 \frac{ft}{sec^2}$



$f_k = \mu_k N$

$\Sigma F = f_f + N - WT = 0$



$f_k = \mu_k N$

5) $a = \frac{a}{2}t^2$ $v = at$ $x = \frac{a}{2}t^2$

$20 = at$ $100 = at^2$

$t = \frac{20}{a}$ $t = \sqrt{\frac{100}{a}}$

$\frac{400}{a^2} = \frac{100}{a}$

$a = 4$

$F = ma$

$= \left(\frac{.25}{32}\right)4 = .031$ lbs

$f_k = \mu_k N$

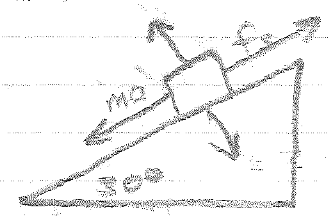
$.031 = \mu_k .25$

$\mu_k = .124$

4) $x = \frac{a}{2}t^2$

$8 = a16$

$a = .50 \frac{m}{sec^2}$



$f_k = \mu_k N$

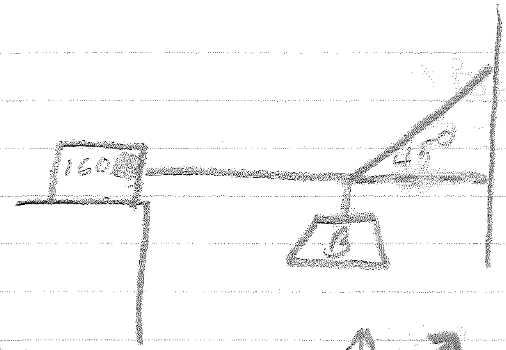
$ma = \mu_k mg$

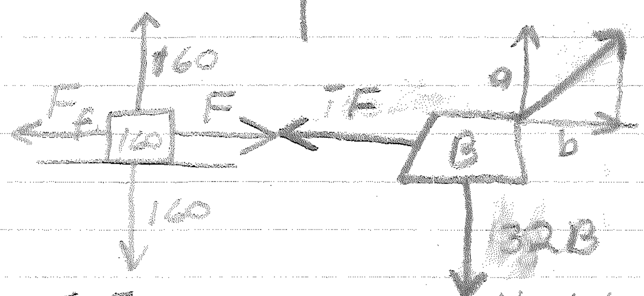
$a = \mu_k g$

$.50 = \mu_k 9.8$

$\mu_k = .52$

9)



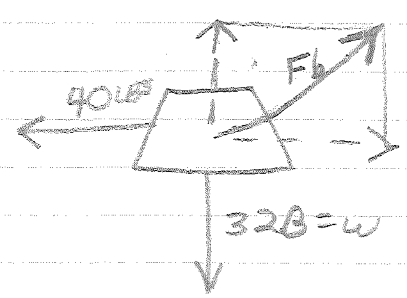


$\Sigma F_x = ma_x$, $N - W = 0$

$F - F_f = 0$, $N = W$

$F = F_f$, $N = 160$ lbs

$F_f = \mu_s N = .25(160) = 40$ lbs



$\Sigma F = 0$

$= F_b \cos 45° + F_b \sin 45° - 40 \text{ lbs} - w$

$w = F_b(1.42) - 40$

$w = F_b \cos 45°$

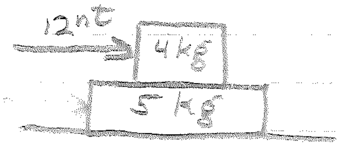
$F_b = \frac{w}{.707} = \frac{w+40}{1.42}$

$2w = w + 40$

$w = 40$ lbs

Pg 127 III

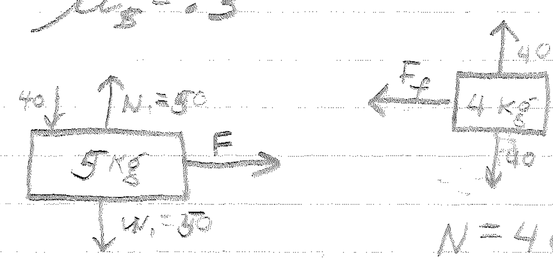
10)



$F_f = \mu_s N$

$12nt = \mu_s \, 40nt$

$\mu_s = .3$



$N = 40$

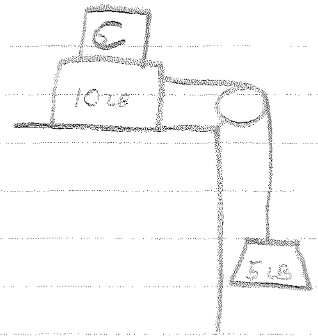
$F = ma$

$F = \mu N$

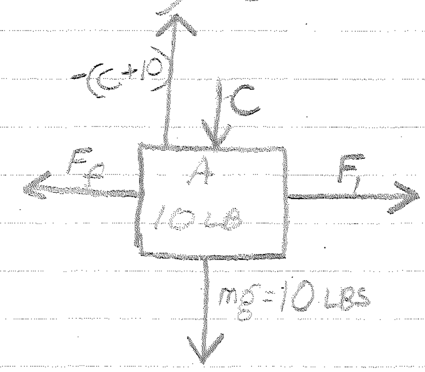
$N = 90nt$

$F = (.3)(90) = 30nt$

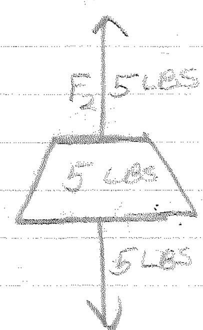
11)



$\mu_s = .20$



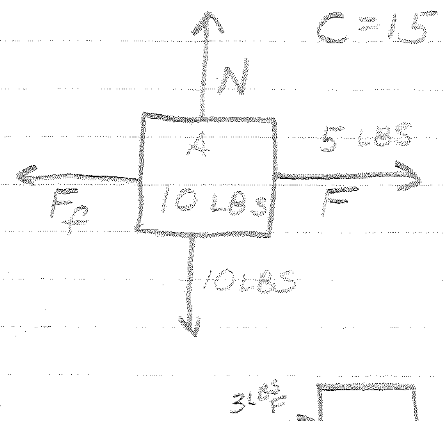
$F_f = F_1 = F_2$



$F_f = 5\ LBS = (.20)(C+10)$

$25 = C + 10$

$C = 15\ LBS$

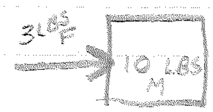


$F_f = \mu N$

$= (.2)(10) = 2$ lbs

$F - F_f = 3$ lbs

$F = ma$

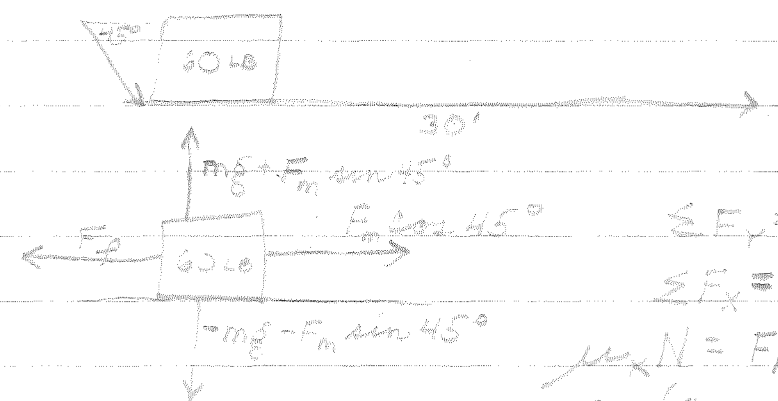


$3\ LBS\ M = \frac{15}{32}$ slugs a

$a = 6.4\ \frac{ft}{sec^2}$

Pg 146 VII

2)



$\Sigma F_y = 0$

$\Sigma F_x = F_m \cos 45° - F_f = 0$

$\mu_k N = F_m \cos 45°$

$.20 \left(60 + \frac{F_m}{\sqrt{2}}\right) = \frac{F_m}{\sqrt{2}}$

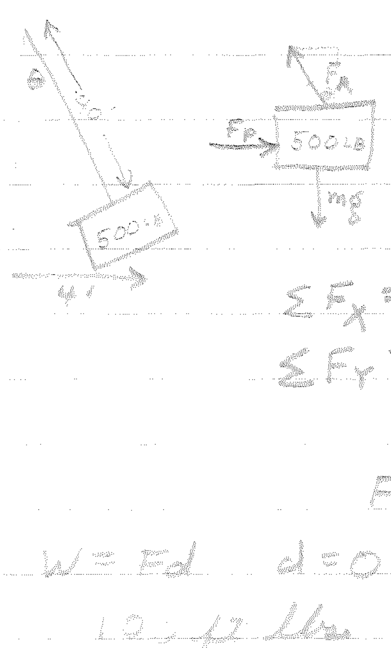
$12 + \frac{.20 (F_m)}{\sqrt{2}} = \frac{F_m}{\sqrt{2}}$

$12 = \frac{.80\, F_m}{\sqrt{2}}$

$F_m = \frac{12\sqrt{2}}{.80}$

$W = F_m d = \frac{12\sqrt{2}\; 30}{.80} = 530$ ft-lbs

3)



$\Sigma F = 0 = \vec{F}_R + \vec{F}_P + \vec{mg}$

$\tan\theta = .10$

$\theta = 5.7°$

$\Sigma F_x = 0 = F_P - F_R \sin\theta$

$\Sigma F_Y = 0 = F_R \cos\theta - mg$

$F_R = \frac{mg}{\cos\theta}$

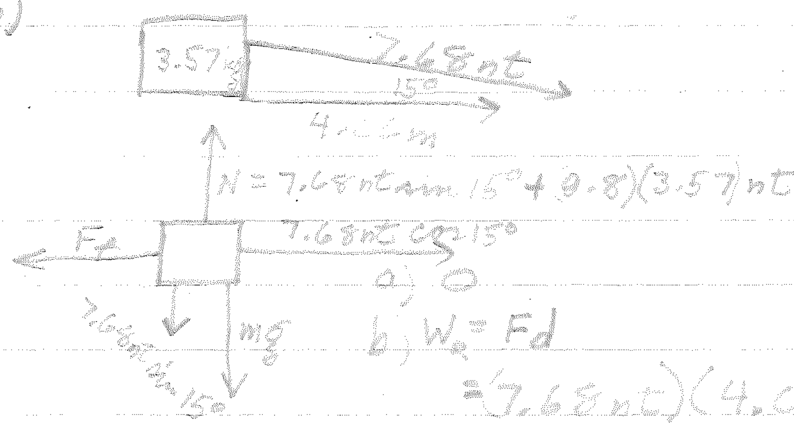
$F_P = mg \tan\theta = 50$ lbs

b) No $W = Fd$ $d = 0 \Rightarrow W = 0$

c) YES 12 ft lbs

d) NO

5)



a) 0

b) $W_a = Fd$

$= (7.68\,nt)(4.06\,m) = 30.2\,nt$

c) $W_f = -30.2\,nt$

d) $F_f = \mu_k N$

$7.68 \cos 15° = \mu_k ([illegible] + 35.0)$

$\mu_k = \frac{7.68 \cos 15°}{7.68 \sin 15° + 35.0}$

$= \frac{7.43}{37.5 + 2.} = .225$

6) a) 6j

b) $F = \frac{9}{x^2}$

$W = \int_1^3 9x^{-2}$

$= -9x^{-1}$

$= -\frac{9}{3} + \frac{9}{1} = 6j$

9) I_1

V_{1m} [m] $\qquad V_{1B}$ [B]

$$\tfrac{1}{2}K_B = K_m \qquad \tfrac{1}{2}m_m = m_B$$

$$\tfrac{1}{2}m_B V_{1B}^2 = m_m V_{1m}^2$$

$$\tfrac{1}{4}m_m V_{1B}^2 = m_m V_{1m}^2$$

I_2 $V_{2m} = V_{1m}+1$ [m] $\qquad V_{2B} = V_B$ [B]

$$m_m (V_{1m}+1)^2 = m_B (V_{1B})^2$$

$$m_m (V_{1m}+1)^2 = \tfrac{1}{2}m_m (V_{1B})^2$$

$$4V_{1m}^2 = V_{1B}^2 \qquad \sqrt{2}V_{1m} + \sqrt{2} = V_{1B}$$

$$2V_{1m} = V_{1B}$$

$$2V_{1m} = \sqrt{2}V_{1m} + \sqrt{2}$$

$$.6V_{1m} = \sqrt{2}$$

$$V_{1m} = 2.36\,\tfrac{m}{sec} \qquad V_{1B} = 4.72\,\tfrac{m}{sec}$$

12) $F = ma$

$$m = .03\,kg$$

$$x = \tfrac{a}{2}t^2 \qquad V = at$$

$$X = \frac{V^2}{2a} \qquad t = \frac{V}{a}$$

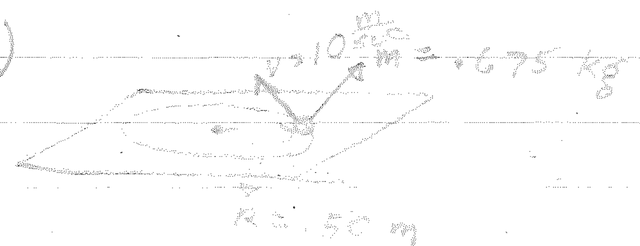
$$.12\,m = \frac{25\times10^4}{2a}$$

$$a = 1.04\times10^6$$

$$F = (1.04\times10^6)(.03)\,nt$$

$$= 3.13\times10^4\,nt$$

15)



a) $F = ma$

$= m\frac{V^2}{R} = (.675)\frac{(100)}{.5}$

$= 135.0 \text{ nt}$

b) $W = Fd$

$F = ma$

$(4.63)(135) = (.675)a_B$

$a_B = 926 \frac{m}{sec^2} = \frac{V^2}{.3}$

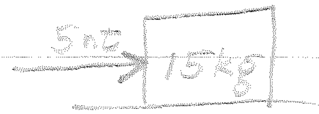
$V_B = 16.2 \frac{m}{sec}$

$W = \frac{1}{2}mV_B^2 - \frac{1}{2}mV_A^2$

$= \frac{1}{2}(.675)(278 - 100)$

$W = 60 J$

20)



$F = ma$

$5 = 15a$

$a = \frac{1}{3} \frac{m}{sec^2}$

a) $d_1 = \frac{a}{2} t^2$

$= \frac{1}{6} m$

$W = Fd$

$= 5\left(\frac{1}{6}\right) j$

$= .834 j$

b) $d_2 = \frac{1}{6}(4)$

$= \frac{2}{3}$

$W = (5)\left(\frac{2}{3}\right) j$

$= 3.33 j$

c) $d_3 = \frac{1}{6}(9) = \frac{3}{2}$

$W = (5)\left(\frac{3}{2}\right)$

$= 7.5 j$

d) $\vec{P} = \vec{F} \cdot \vec{V}$

$d_4 = \frac{1}{6}(16)$

$W = \frac{8}{3} \frac{5}{1} j = 12.5 j$

$V = at$

$= \frac{4}{3} = 1.25 m/sec$

$F = 5 nt$

$\cos \vec{F} \vec{V} = 0$

$P = 6.67 \frac{joules}{sec}$

21) a) $K = \frac{1}{2} m V^2$

$= \frac{10^5 \text{ lbs}}{2\,(32)} (4 \times 10^3 \times 1.47)^2$

$= .54 \times 10^{11} \text{ nt} = 5.4 \times 10^{10} \text{ nt}$

b) $P_{AVE} = \frac{W}{T}$

$V = at$

$4000(1.47) = a\,(60)$

$a = 98.3 \frac{ft}{sec^2}$

$F = \frac{10^5}{32}\, 98.3$

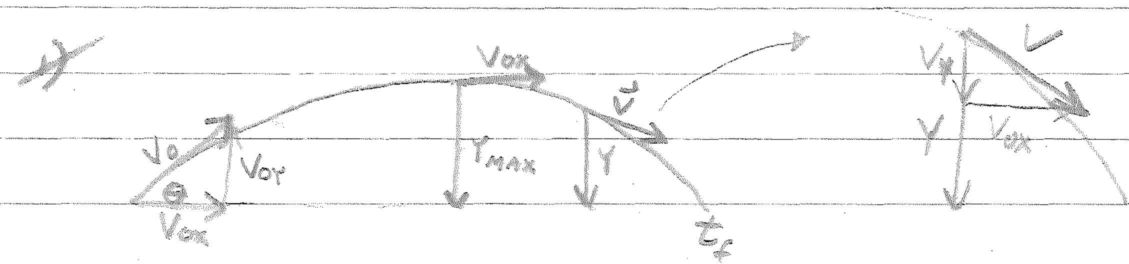
$= 3.07 \times 10^5$

$P = 3.07 \times 10^5 \, (4 \times 10^3)(1.47) \cos 0°$

$= 18 \times 10^8 \frac{ft\ lb}{sec} \times \frac{HP}{550 \frac{ft\ lb}{sec}} = 3.3 \times 10^6 \text{ HP}$

Pg 175 VIII

1)



at Y, $\vec{V} = \vec{V}_{0x} + \vec{V}_Y$ $\vec{V}_{0x} = \vec{V}_0 \cos\theta$

$V_Y = \frac{g}{2}\left(\frac{t_f}{2}\right)^2$

$= \frac{g}{8} t_f^2$

at Y, $V_Y = \frac{g}{2} t^2 - V_{0Y} t$

$\vec{V}_{0Y} = \vec{V}_0 \sin\theta$

$\vec{V} = \vec{V}_0 \cos\theta + \frac{g}{2} t^2 - \vec{V}_0 \sin\theta$

$=$

2) $V = \sqrt{2gh}$

$= \sqrt{2(32)4}$

$= 16 \frac{ft}{sec}$

3) $V_1 = \sqrt{2gl}$ $V_d = \sqrt{4g(l-d)}$

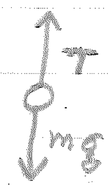
$2gl = 4gl - 4gd$

$2l = 4d$

$d = 2l$

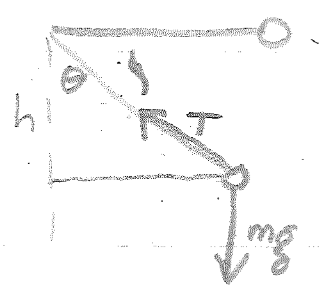
5) a)

$$V = \sqrt{2gh}$$
$$= 2\sqrt{gl}$$



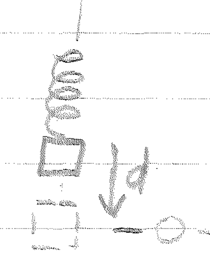
$$\Sigma F_y = T - mg$$
$$T = ma$$
$$= m\frac{v^2}{r}$$
$$= m\frac{4gl}{l}$$
$$= 4mg$$

b)



$$T = mg$$
$$m\frac{v^2}{r} = mg$$
$$\frac{2gh}{l} = g$$
$$l = 2h$$
$$\cos\theta = .5$$
$$\theta = 60°$$

7)



2d

9) a) $V_p = V_0$

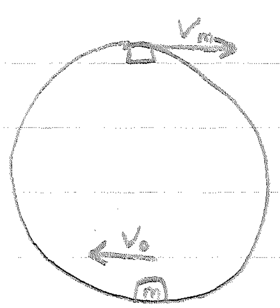
b) $\vec{V}_x = \vec{V}_0$

$V = gt$ $\quad \frac{h}{2} = \frac{g}{2} t^2$

$V_y = g\sqrt{\frac{h}{g}}$

$V_c = \sqrt{V_0^2 + h/g^2}$

11)



$P_0 = P_f$

$mV_0 = mV_m$

$W = (U_m + K_m) - (U_0 + K_0)$

$-gm2R = 2Rgm + \frac{1}{2} mV_m^2 - 0 - \frac{1}{2} mV_0^2$

$\frac{1}{2} V_m^2 = -4gR + \frac{1}{2} V_0^2$

$V_m = \sqrt{V_0^2 - 5gR}$

$\frac{dV_m}{dV_0} = (2V_0) \frac{1}{2\sqrt{V_0^2 - 8gR}} = 0$

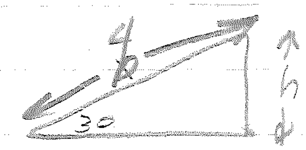
$V_0 = 0$

$V_{m(min)} = \sqrt{5gR}$

13) $F = -kx$

$100\,nt = -k$

$k = -100 \frac{nt}{ft}$



a) $10\,kg\; a = -100(2)$

$a = 20 \frac{m}{sec^2}$

$W = (U_s + K_s) - (U_o + K_o)$

$= (-50(\frac{1}{2})^2 + 0) - (mgh + 0)$

$(U_A + K_A) = (U_s + K_s)$

$mgh + 0 = \frac{1}{2} k x^2 + 0$

$(10)(9.8) h = \frac{1}{2}(100)(4)$

$h = 2 \qquad P = 4\,m$

$(U_4 + K_4) = (U_5 + K_5)$

~~$mgh =$~~

$V = (\frac{g}{2}) t \qquad 2 = \frac{g}{4} t^2$

$V = \frac{g}{2} \sqrt{\frac{8}{g}} \qquad t = \sqrt{\frac{8}{g}}$

$= \sqrt{2}$

25) $k = 2\ \frac{nt}{m}$ $\qquad F_f = 2.5\ nt$

$F = 8$

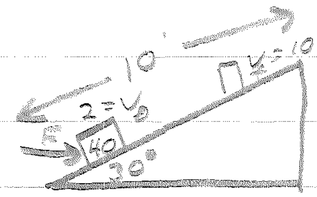
$$W + (U_A + K_A) = U_B + K_B$$

$$^{-}(2.5)(4) + \left(0 + \frac{1}{2} m V_A^2\right) = \frac{1}{2} k x^2 + 0$$

$$^{-}10 + \frac{1}{2} V_A^2 = 16$$

$$V_A = \sqrt{52} = 7.2\ \frac{m}{s}$$

27)



$$W = (U_f + K_f) - (U_0 + K_0)$$

$$= \left(mgh + \frac{1}{2} m V_f^2\right) - \left(0 + \frac{1}{2} m V_0^2\right)$$

$$= \frac{40}{32}(32)5 + \left(\frac{1}{2}\right)\frac{40}{32}(100) - \frac{1}{2}\frac{40}{32}1600$$

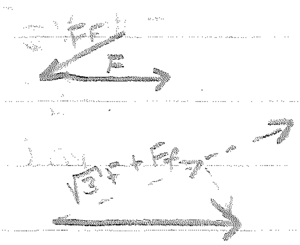
$$= 200 + 63 - 1000$$

$$= 737\ \text{ft lb}$$

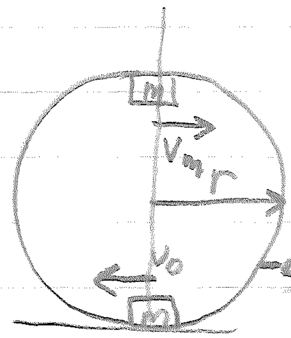
b) $\mu_k = .15$

$$F_f = \mu_k N$$

$$= (.15)(40) = 6$$



11)



$$W = (U_m + K_m) - (U_0 + K_0)$$
$$-mgh = \left(mgh + \tfrac{1}{2} m V_m^2\right) - \left(\tfrac{1}{2} m V_0^2\right)$$
$$-4g\,2R = \tfrac{1}{2} V_m^2 - \tfrac{1}{2} V_0^2$$
$$V_m = \sqrt{8gR - V_0^2}$$
$$\frac{dV_m}{dV_0} = -2V_0 \Big/ \sqrt{8gR - V_0^2} = 0$$
$$V_0 = 0 \Rightarrow V_{m_{min}} = 2\sqrt{2gR}$$

9) a) $V_0 = V_B$

$$U_B + K_B = U_C + K_C$$
$$2mgh + \tfrac{1}{2} m V_B^2 = \tfrac{1}{2} mgh + \tfrac{1}{2} V_C^2$$
$$V_C^2 = gh + V_B^2$$
$$V_C = \sqrt{V_0^2 + gh}$$

b) $U_C + K_C = U_D + K_D$

$$\tfrac{1}{2} mgh + \tfrac{1}{2} m(V_0^2 + gh) = 0 + \tfrac{1}{2} m V_D^2$$
$$V_D^2 = 2gh + V_0^2$$
$$V_D = \sqrt{2gh + V_0^2}$$

$V = at$ $\qquad L = \frac{a}{2} t^2$

$t = \frac{v}{a}$ $\qquad L = \frac{V^2}{2a}$

$$a = \frac{V^2}{2L} = \frac{2gh + V_0^2}{2L}$$

Pg 205 IX

1) m_1 ——— x ——— m_2

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$X_{m_1} = \frac{0 + m_2 x}{m_1 + m_2}$$

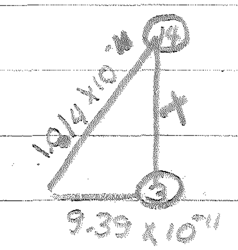
$$X_{m_2} = \frac{m_1 x + 0}{m_1 + m_2}$$

$$\frac{X_{m_1}}{X_{m_2}} = \frac{m_2}{m_1}$$

3) C (12) ← 1.13×10^{-10} m → O (16)

$$X_{cm} = \frac{0 + 16 \cdot 1.13 \times 10^{-10}}{28}$$

$$= .646 \times 10^{-10} \text{ m}$$

5) 

$$x^2 = (10.1^2 \times 10^{-22}) - (9.39^2 \times 10^{-22})$$

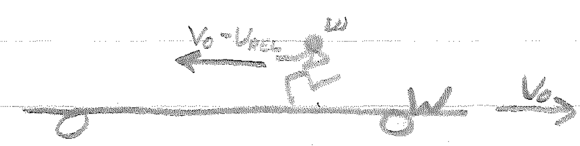
$$= (103 - 88) \times 10^{-22}$$

$$= 15 \times 10^{-22}$$

$$x = 3.87 \times 10^{-11}$$

$$X_{cm} = \frac{0 + 3(3.87) \times 10^{-11}}{17} = .684 \times 10^{-11} \text{ m}$$

19)



$P_0 = P_f$

$M \frac{dv}{dt} = F_{EXT} + V_{REL} \frac{dM}{dt}$

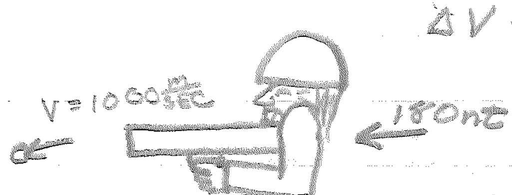
$0 +$

$M \Delta v = V_{REL} \Delta M$

$(W + w)(V_f - V_0) = -V_{REL} (+w)$

$\Delta V = -V_{REL} \left(1 + \frac{w}{W} \right)$

21)



$F = 180nt = V_{REL} \frac{dM}{dt}$

$= 1000 \frac{m}{sec}$

$M \frac{dv}{dt} = V \frac{dM}{dt}$

$.050 kg \frac{dv}{dt} = 1000 \frac{m}{sec} \frac{dM}{dt}$

$.050 (1000)$

$\frac{dM}{dt} =$

$\frac{dM}{dt} = .180$

$dM = .180 \frac{m}{sec} dt$

$= .180 \frac{m}{sec} 60 \frac{sec}{min}$

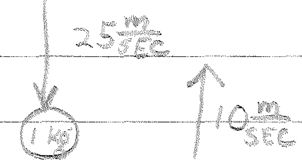
$V \frac{dM}{dt} = 180nt$

$\frac{dM}{dt} = .180 \; 60 / .050$

$= 216 \frac{bul}{min}$

Pg 234

2)



a) $J = p_f - p_i$

$= (1)(10 - 25)$

$= -15 \frac{m\,kg}{sec}$

b) $J = F\Delta t$

$F = \frac{J}{\Delta t} = \frac{-15}{.02} = -750\ nt$

3) $J = \frac{4}{3}\left(\frac{3}{2} \times 10^{-3}\ sec\right)(2{,}200 \times 10^{3}\ nt)$

$= 4.40\ nt\,sec = F\Delta t = P_f - P_i \qquad P_i = 0$

$4.40 = .50\ V$

$V = 8.8\ \frac{m}{sec}$

4)

a) $F\Delta t = P_f - P_i \qquad P_i = 0$

$ma\Delta t = P_f$

$at = V \qquad m\frac{V}{\Delta t}\Delta t = P_f$

$a = \frac{V}{t} = \qquad mV = P_f = (.025)(50)$

$= 1.250\ \frac{kg\,m}{sec}$

c) $F\Delta t = J$

$\frac{1.25\ kg\,m}{.01\ sec^2} = F = 125\ nt$

d) $F = ma$

$a = \frac{125}{.025} = 5000\ \frac{m}{sec^2}$

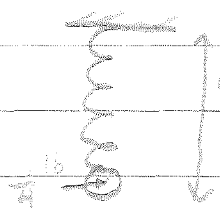
$X = \frac{a}{2}t^2 = 2500(0.0001) = .25\ m$

$W = Fd = (5000)\ .25 = 1250\ J$

b) $\frac{m}{t} = \mu$ $m = \mu t$

$F = ma$

Pg 255

6) 

$kd = 32\,m$

$= \frac{8}{2} = 4$

$A = 2 \quad B = 0$

$\therefore Y = A \sin \sqrt{\frac{k}{m}}\, t$

$Y = 2 \sin 4t$

Pg 280

2) $Y = \sin^{-1} \frac{x}{3}$

$u = \frac{x}{3}$

$\dot{Y} = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$

$= \frac{1}{3\sqrt{1 - \frac{x^2}{9}}}$

22) $Y = x \tan^{-1} x$

$\dot{Y} = x(\tan^{-1} x)^{\cdot} + (\tan^{-1} x)\dot{x}$

$= x\left(\frac{1}{1+x^2}\right) + \tan^{-1} x$

$= \frac{x}{1+x^2} + \tan^{-1} x$

$\ddot{Y} = \frac{(1+x^2) - 2x}{(1+x^2)^2} + \frac{1}{1+x^2}$

19) $Y = (x^2+1)\tan^{-1} x - x$

$\dot{Y} = (x^2+1)\left(\frac{1}{1+x^2}\right) + 2x \tan^{-1} x - 1$

$= 2x \tan^{-1} x$

Pg 306

8) a) $Y = \ln X^2$

$Y' = \frac{1}{X^2} 2X = \frac{2}{X}$

f) $Y = X \log X$

$Y' = X \frac{1}{X} + \log X$

$Y' = \log X + 1$

g) $Y = X \log X - X$

$Y' = \log X + 1 - 1 = \log X$

h) $Y = \frac{\log X}{X}$

$= -X^{-2} \log X + \frac{1}{X^2}$

$= -\frac{\log X}{X^2} + \frac{1}{X^2}$

$= \frac{1}{X^2}(1 - \log X)$

i) $Y = \log(\log X)$

$U = \log X \quad Y = \log U$

$Y' = \frac{1}{X} \frac{1}{\log X} = \frac{1}{X \log X}$

10) a) $Y' = \frac{1}{3X}$

$U = \log X \quad \frac{dU}{dX} = \frac{1}{X}$

$Y' = \frac{1}{3U}\frac{dU}{dX} \quad Y = \frac{\log X}{3} + C$

b) $Y' = \frac{2X}{X^2+1}$

$U = \log(X^2+1) \quad \frac{dU}{dX} = \frac{2X}{X^2+1}$

$Y = \log(X^2+1) + C$

e) $Y' = \frac{X-3}{X^2-6X+10}$

$U = \log(X^2-6X+10) \quad \frac{dU}{dX} = \frac{2X-6}{X^2-6X+10}$

$Y = \frac{1}{2}U\frac{dU}{dX}$

$= \frac{1}{2}\log(X^2-6X+10) + C$

f) $\dot{Y} = \frac{\cos x}{\sin x}$

$U = \log|\sin x| \quad \frac{dU}{dx} = \frac{\cos x}{\sin x}$

$Y = U\frac{dU}{dx} = \log|\sin x) + C$

g) $\dot{Y} = \frac{\sin x}{\cos x}$

$U = \log|\cos x| \quad \frac{dU}{dx} = \frac{-\sin}{\cos}$

$\dot{Y} = -U\frac{dU}{dx}$

$Y = -\log|\cos x| + C$

j) $\dot{Y} = \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

$U = \log|\sin x| \quad \frac{dU}{dx} = \frac{\cos x}{\sin x}$

$\dot{Y} = U\frac{dU}{dx}$

$Y = \log|\sin x| + C$

k) $\dot{Y} = \frac{\sec^2 x}{\tan x}$

$U = \log|\tan x| \quad \frac{dU}{dx} = \frac{\sec^2 x}{\tan x}$

$Y = \log|\tan x| + C$

11) $\dot{A} = \frac{1}{x}$

$U = \log x \quad \frac{dU}{dx} = \frac{1}{x}$

$\int \dot{A} = \log x$

$\int_1^{10} \log x = \log 10 - \log 1 = 2.3$

$3^3 =$

13) $Y = \frac{\log x}{x-1}$

$\dot{Y} = \frac{1}{x^2 - x} - \frac{\log x}{x^2}$

$= \frac{1}{x(x-1)} - \frac{\log x}{x^2}$

$= \frac{x - (x-1)\log x}{x(x-1)}$

Pg 310

1) a) $Y = e^{x^2}$

$U = x^2 \quad \frac{du}{dx} = 2x$

$Y' = e^{x^2}(2x)$

c) $Y = e^{-\frac{1}{x}}$

$U = -\frac{1}{x} \quad \frac{du}{dx} = \frac{1}{x^2}$

$Y' = \frac{1}{x^2} e^{-\frac{1}{x}}$

f) $Y = x^2 e^{-x}$

$U = -x \quad \frac{du}{dx} = -1$

$Y' = -x^2 e^{-x} + 2x e^{-x}$

$= e^{-x}(2x - x^2)$

h) $Y = (e^x)^2 = e^{2x}$

$U = 2x \quad \frac{du}{dx} = 2$

$\frac{dy}{dx} = 2e^{2x}$

j) $Y = \log \frac{e^x}{1+e^x}$

$= \log e^x + \log \frac{1}{1+e^x}$

$= \frac{1}{e^x} +$

$U = \frac{1}{1+e^x} = 1 + e^{-x}$

$\frac{du}{dx} = -e^{-x}$

j) $y = \ln \frac{e^x}{1+e^x}$

$= \ln e^x + \ln (1+e^x)^{-1}$

$d = e^x \quad u = \ln d \quad d = 1+e^{-x} \quad u = \ln d$

$\acute{d} = e^x \quad \acute{u} = \frac{\acute{d}}{d} \quad \acute{d} = -e^{-x} \quad \acute{u} = \frac{\acute{d}}{d}$

$= \frac{e^x}{e^x} = 1 \qquad = \frac{-e^{-x}}{1+e^{-x}}$

$\acute{y} = 1 - \frac{e^{-x}}{1+e^{-x}} =$

$= \frac{1+e^{-x}-e^{-x}}{1+e^{-x}} = \frac{1}{1+e^{-x}} =$

4) a) $\acute{y} = e^{-x}$

$u = -x \quad \frac{du}{dx} = -1$

$\acute{y} = -e^u \frac{du}{dx} -$

$y = -e^{-x} + C$

c) $\acute{y} = e^{\sin x} \cos x$

$u = \sin x \quad \acute{u} = \cos x$

$\acute{y} = e^u \frac{du}{dx} -$

$y = e^{\sin x} + C$

9) a) $\acute{y} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$u = \log(e^x + e^{-x}) \qquad \acute{u} = \frac{e^x - e^{-x}}{e^x - e}$

$y = \log(e^x + e^{-x}) + C$

c) $\dot{y} = \frac{e^{2x}}{1+e^{2x}}$

$u = \log(1+e^{2x}) \quad \frac{du}{dx} = \frac{2e^{2x}}{1+e^{2x}}$

$\dot{y} = u \frac{1}{2} \frac{du}{dx}$

$y = \frac{1}{2} \log(1+e^{2x}) + C$

10) $y = De^{kx}$

$\dot{y} = kxDe^{kx}$

b) $\dot{y} = \frac{e^{2x}-1}{e^{2x}+1}$

$u = \log(e^{2x}+1) \quad \dot{u} = \frac{2e^{2x}}{e^{2x}+1}$

Pg 369

1) a) $Y = \int \sqrt{1+3x}\, dx$

$3x = u^2 - 1 \quad u = \sqrt{3x+1}$

$x = \frac{u^2-1}{3} \quad \frac{dx}{du} = \frac{2u}{3}$

$Y = \int u\, dx \quad dx = \frac{2u}{3} du$

$Y = \int \frac{2u^2}{3} du = \frac{2u^3}{9}$

$Y = \frac{2(3x+1)^{3/2}}{9} + C$

c) $Y = \int \sin^2 x \cos x\, dx$

$Y = \int \sin^2 x \sqrt{1-\sin^2 x}\, dx$

$Y = \quad x = \quad \sin^2 x$

Pg 357

1) a) $Y = X^{2x}$

$\log Y = 2x \log x$

$\frac{Y'}{Y} = \frac{2x}{x} + 2 \log x$

$Y' = x^{2x}(2 + 2 \log x)$

c) $Y = \sqrt{\frac{2x+5}{x^2+7}}$

$\log Y = \frac{1}{2}\log(2x+5) - \frac{1}{2}\log(x^2+7)$

$\frac{Y'}{Y} = \frac{2}{2(2x+5)} - \frac{2x}{2(x^2+7)}$

$Y' = \left(\frac{1}{(2x+5)} - \frac{x}{(x^2+7)}\right)\sqrt{\frac{2x+5}{2x+7}}$

Pg 362

1) a) $Y = x^3$

$\frac{dY}{dx} = 3x^2$

$dY = 3x^2 dx$

c) $Y = x^{\frac{3}{2}}$

$\frac{dY}{dx} = \frac{3}{2} x^{\frac{1}{2}}$

$dY = \frac{3}{2} x^{\frac{1}{2}} dx$

e) $Y = \cos x$

$\frac{dY}{dx} = -\sin x$

$dY = -\sin x \, dx$

h) $Y = e^{2x}$

$\frac{dY}{dx} = 2e^{2x}$

$dY = 2e^{2x} dx$

2) $\sqrt{101}$

$Y = \sqrt{x}$

$\frac{dY}{dx} = \frac{1}{2(\sqrt{x})}$

$dY = \frac{1}{2\sqrt{x}} dx$

$= \frac{1}{20}$

$Y = 10.05$

3) $Y = \sin x$

$\frac{dY}{dx} = \cos x$

$dY = \cos x \, dx$

$x = \frac{\pi}{3}$ $dx = .018$

$\pi = 180°$

$\frac{\pi}{180} = 1°$

$180 \overline{)3.14}$ $.018$

180

1340

1340

$dY = (.5)(.018)$

$= .009$

$\sin 59° = \frac{\sqrt{3}}{2} - .009$

Pg 364

1) a) $Y = \int \sin x \, dx$

$Y = -\cos x + C$

Pg 369

1) a) $Y = \int \sqrt{1+3x} \, dx$

$U = 1 + 3x \qquad \frac{du}{dx} = 3$

$Y = \int U^{\frac{1}{2}} \, du$

$= \frac{2}{3}(1+3x)^{\frac{3}{2}} \left(\frac{1}{3}\right)$

$= \frac{2}{9}(1+3x)^{\frac{3}{2}} + C$

b) $Y = \int (x^2+5)^{\frac{1}{2}} x \, dx$

$U = x^2 + 5 \qquad \frac{du}{dx} = 2x$

$Y = \int U^{\frac{1}{2}} \, du$

$= \frac{4}{3}(x^2+5)^{\frac{3}{2}} + C$

c) $Y = \int \sin^2 x \cos x \, dx$

$U = \sin x \qquad \frac{du}{dx} = \cos x$

$Y = \int U^2 \frac{du}{dx} \, du$

$= \frac{1}{3} \sin^3 x + C$

h) $Y = \int (x^3-4)^3 x^2 \, dx$

$U = x^3 - 4 \qquad \frac{du}{dx} = 3x^2$

$Y = \int U^3 \frac{du}{dx} \, dx$

$= \frac{1}{4}(x^3-4)^4 \, x^2 \, \frac{1}{3x^2}$

$= \frac{1}{12}(x^3-4)^4 + C$

l) $Y = \int (1-x^2)^{\frac{1}{2}} dx$

$x = \sin\theta \qquad \frac{dx}{d\theta} = \cos\theta$

$Y = \int \cos\theta \, dx \qquad dx = \cos\theta \, d\theta$

$Y = \int \cos^2\theta \, d\theta$

$\frac{dY}{d\theta} = \cos^2\theta = \frac{1+\cos 2\theta}{2} = \frac{1}{2} + \frac{\cos 2\theta}{2}$

$Y = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$

$\theta = \sin^{-1} x$

$Y = \frac{1}{2}\sin^{-1} x + \frac{1}{4}\sin(2\sin^{-1} x)$

$U = 2\sin^{-1} x$

$\sin \frac{U}{2} = x$

$\frac{1}{4}\sin U$

n) $Y = \int \frac{x+1}{\sqrt{1-x^2}} dx$

$= \int \frac{x+1}{\sqrt{x+1}\sqrt{1-x}} dx = \int \frac{\sqrt{1+x}}{\sqrt{1-x}}$

$x = \sin\theta$

$Y = \int \left(\frac{\sin\theta - 1}{\cos\theta}\right) dx = \int (\tan\theta - \sec\theta) dx$

$dx = \cos\theta \, d\theta \qquad \tan \quad \frac{\sin}{}$

$Y = \int \sin\theta - 1 \, d\theta$

$Y = -\cos\theta - \theta$

$\theta = \sin^{-1} x$

$Y = \cos(\sin^{-1} x) - \sin^{-1} x + C$

W (net forces) $= K_B - K_A$
$A \to B$

W (non conservative forces) $= (K_B + U_B) - (K_A + U_A)$
$A \to B$

If $W_{A\to B}$ (non cons forces) $= 0$ then $U_A + K_A = U_B + K_B$

Power = rate of doing work ; Power (average) =

<u>Center of mass theorem</u> (for a system)

$$\Sigma m_i \, \vec{a}_c = \Sigma \vec{F}_{ext}$$

$$x_c = \frac{\Sigma m_i x_i}{\Sigma m_i} \quad ; \quad y_c = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

<u>Momentum Theorem (for system)</u>

$$\frac{d\vec{P}}{dt} = \Sigma \vec{F}_{ext} \qquad \text{If } \Sigma \vec{F}_{ext} = 0 \text{ then}$$

<u>Prob 1</u>

A 10 lb block is pushed across a smooth ... force of 3 lbs. If the block starts from rest at A, how much work is done by the 3 lb force as block moves from A to B. 30 ft lbs What is the average rate (power) the 3 lb force does work during t...

3 lb

<u>Prob 2</u>

A block weighing 2 lbs is forced agai...

Physics I Test I J

Name Bob

Score (

14

I (i) Two vectors $\vec{A}$ and $\vec{B}$ lie in the xy plane as indicated. Each has a magnitude of 10 units. A third vector $\vec{C} = -3\hat{j} + 4k$. Circle the correct statements of those that follow. The vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors pointing respectively along the x, y and z axes.

(a) $A_x = 10$

(b) $B_y = -6$

c) $|\vec{C}| = 5$ ✓

(d) $\vec{A} + \vec{B} = 2\hat{i} - 6\hat{j}$

(e) $3\vec{C} + \vec{A} = 10\hat{i} - 9j + 12\hat{k}$

(f) $\vec{A} \cdot \vec{C} = 50$ ✗

12 (g) $\vec{C} \times \vec{A} = 0$ ✗

(h) $\vec{A} \times \vec{B} = -60\hat{k}$

(i) $\vec{A} \cdot (\vec{B} + \vec{C}) = 150$ ✗

(j) $\vec{B} \cdot \vec{C} = 18$

(ii) Sketch on the figure the vector $\vec{B} - \vec{A}$

(iii) Calculate the angle between $\vec{B}$ and $\vec{C}$.

$W_{A\to B}$ (net force) = $K_B - K_A$

$W_{A\to B}$ (non conservative forces) = $(K_B + U_B) - (K_A + U_A)$

If $W_{A\to B}$ (non con force) = 0 then $U_A + K_A = U_B + K_B$

If $\vec{F}$ = const + ...

$W_{A\to B} = |F|\, d \cos(\vec{F}, \vec{d})$

Power = rate of doing work ; Power (average) = $\frac{W}{time}$; Inst. Power = $\frac{dW}{dt} = \vec{F}\cdot\vec{v}$

Center of mass theorem (for a system)

$$\Sigma m_i \; \vec{a}_c = \Sigma \vec{F}_{ext}$$

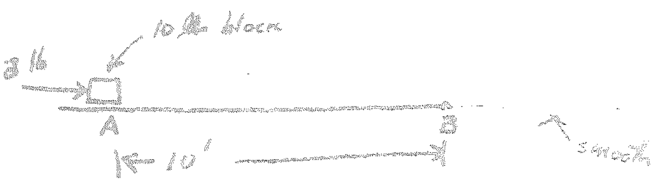
$$x_c = \frac{\Sigma m_i x_i}{\Sigma m_i} \; ; \; y_c = \frac{\Sigma m_i y_i}{\Sigma m_i}$$

Momentum Theorem (for system)

$$\frac{d\vec{P}}{dt} = \Sigma \vec{F}_{ext} \quad \text{If } \Sigma \vec{F}_{ext} = 0 \text{ then } \vec{P} = \text{constant}$$

3.61

A 10 lb block is pushed across a smooth floor by an applied force of 3 lbs. If the block starts from rest at A, how much work is done by the 3 lb force as block moves from A to B. 30 ft lbs What is the average rate (power) the 3 lb force does work during the time required to go from A to B?



$$\frac{30}{\sqrt{\frac{20}{9.6}}}$$

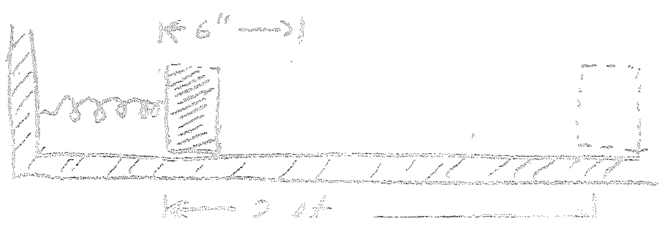
Prob 2

A block weighing 2 lbs is forced against a horizontal spring of negligible mass, compressing the spring ½ ft. When released, the block moves on a horizontal table top a distance x = 2 ft before coming to rest. The spring constant k = 8 lbs/ft. What is the coefficient of friction μ between the block and the table

$$W_{A\to B}\binom{non}{cons} = (K_B + U_B) - (K_A + U_A)$$

$$F_f(2)(-1) = (0+0) - (0 + \tfrac{1}{2}kx^2)$$

$$F_f = N\mu_K$$



55%

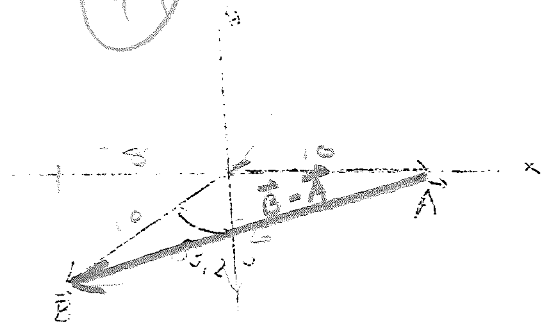
Physics I Test I January 17, 1969

Name Bob Marks

Score 44

14

I (i) Two vectors $\vec{A}$ and $\vec{B}$ lie in the xy plane as indicated. Each has a magnitude of 10 units. A third vector $\vec{C} = -3\hat{j} + 4k$. Circle the correct statements of those that follow. The vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors pointing respectively along the x, y and z axes.



$\sin 30° = .50$

$\cos 30° = .87$

$\sin 53.2° = .80$

$\cos 53.2° = .60$

a) $A_x = 10$

b) $B_y = -6$

c) $|\vec{C}| = 5$ ✓

d) $\vec{A} + \vec{B} = 2\hat{i} - 6\hat{j}$

e) $3\vec{C} + \vec{A} = 10\hat{i} - 9j + 12\hat{k}$

f) $\vec{A} \cdot \vec{C} = 50$ ✗

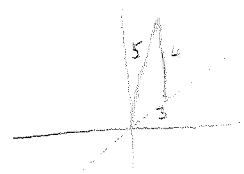
g) $\vec{C} \times \vec{A} = 0$ ✗

h) $\vec{A} \times \vec{B} = -60\hat{k}$

i) $\vec{A} \cdot (\vec{B} + \vec{C}) = 150$ ✗

j) $\vec{B} \cdot \vec{C} = 18$

12

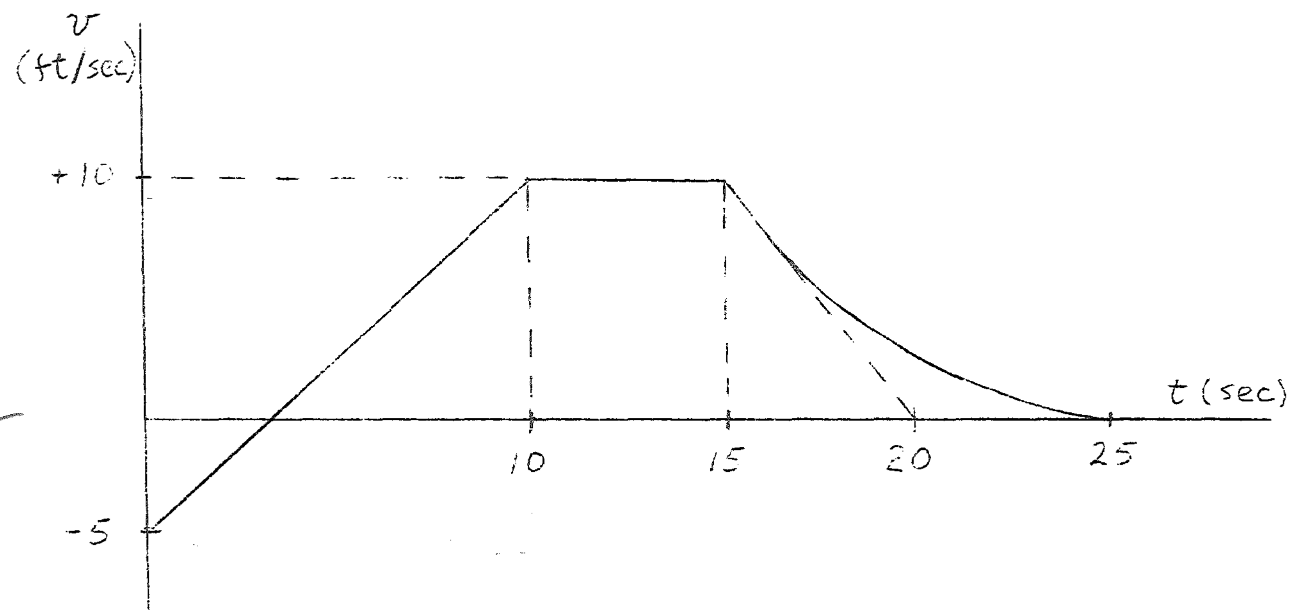


(ii) Sketch on the figure the vector $\vec{B} - \vec{A}$. ✓2

(iii) Calculate the angle between $\vec{B}$ and $\vec{C}$. 0

$44 \cdot \frac{4}{3} = \frac{176}{3} = 55$

2) An object moving in a straight line along the x axis starts from the origin at time t=o. The figure below shows how its instantaneous velocity depends on time, motion to the right being represented by positive values of v and motion to the left by negative values.



19

Determine:

(6) (a) the magnitude and direction of the displacement of the object over the interval from t = o to t = 15 seconds.

$a = \frac{15}{10} = \frac{3}{2}$ ft/sec^2 $X_{10-15} = 10\frac{ft}{sec} \times 5 sec = 50 ft$

$X_{0-10} = \frac{a}{2}t^2 + V_0 t$

$= 3(10)^2 - 5(10)$

$= 300 - 50$

$= 250 ft$

$X_{0-15} = 300$ ft east

5

(6) (b) the magnitude and direction of the average velocity over the interval from t=o to t=15 seconds.

$V_{AVE} = \frac{\Delta X}{\Delta t} = \frac{300 ft}{15 sec} = 20 \frac{ft}{sec}$ east 6

method O.K.

(6) (c) The magnitude and direction of the average acceleration over the interval from t=15 to t= 25 seconds.

$a_{ave} = \frac{\Delta V}{\Delta t} = \frac{-10 \text{ ft/sec}}{10 \text{ sec}} = -1 \frac{\text{ft}}{\text{sec}^2}$ ✓

~~(assuming constant acc)~~

6

(6) (d) the magnitude and direction of the instantaneous acceleration immediately after time t=15 seconds.

$a_{15T} = a_{ave}$ during constant acceler. ∴

$a_{15T} = -1 \frac{\text{ft}}{\text{sec}^2}$

2

3.) From a high cliff a man shoots a body A straight up and a body B straight down with the same initial speeds V_0 = 29.4 m/sec.

11 (6) (a) At what time is the speed of body B twice that of body A?

$V_A = a_{x_1} t + V_{0x_1}$ $V_B = a_x t + V_{0x}$ ✓

$V_A = 9.8t - 29.4$ ✓ $V_B = 9.8t + 29.4$

$2V_A = 19.6t - 58.6$ $2V_B = 19.6t + 58.6$

$2V_A = V_B$ ✓

6

t= 9.0 sec

$19.6t - 58.6 = 9.8t + 29.4$

$9.8t = 88.0$

$t = 9.0$ sec

(6) (b) What is the distance of separation between the bodies at that time?

$X_A = \frac{a}{2}t^2 + V_{ox_A}t$ $\quad X_B = \frac{a}{2}t^2 + V_{ox_A}$

$V_{ox_A} = -29.4$ $\quad V_{ox_B} = 29.4$

$a = 9.8$

$t = 9.0$ sec

$X_a = 4.6(9.0)^2 - 29.4(9.0)$ $\quad X_B = 4.6(90)^2 + 29.4(90)$

$= 3.72 \times 10^2 - .265 \times 10^2$ $\quad = 3.72 \times 10^2 + .265 \times 10^2$

$= 7.07 \times 10^2$ $\quad = 6.37 \times 10^2$

math

$X_a + X_b = 7.44 \times 10^2 m$

y = ~~7.44×10^2~~ m

5

A shell is fired straight upward and travels a distance of 543.9m during the third second. Neglect air friction. Assume all motion is upward during the third second.

(6) (c) What is the total flight time of the shell?

~~$X_x = \frac{a}{2}t^2 + V_{ox}t$~~

~~$543.9 m = 4.9(3)^2 + V_{ox}(3)$~~

$587.9 = V_{ox}(3)$

$V_{ox} = 196$ m/sec

$a = 9.8$

$V = 9.8t$

$X = 4.9t^2$

$196 = 9.8t$

$t = 200$ sec

~~$\frac{1}{2}X = 4.9(2 \times 10^2)^2 = 2.4 \times 10^5$~~

~~$X = 1.92 \times 10^6$~~

T = ~~400~~ sec

(6) (d) What was the initial speed of the shell as it left the ground?

0

V_o = ~~196 m/s~~

(68)

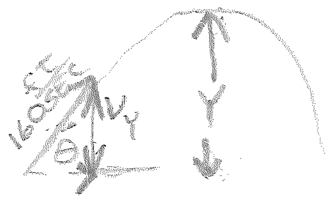
Physics I Test 2 February 6, 1969

Name BOB MARKS

To receive credit on any test question it is necessary to indicate clearly how you arrived at your answer.

(26)

I. (A) A projectile is fired with an initial velocity of 160 ft/sec reaches a maximum height of 200 ft. What angle did its initial velocity vector make with the ground?



$Y = \frac{a}{2}t^2$ $V_{Y_0} = 160 \sin\theta$

$Y = 16t^2$ (15)

$200 = 16t^2$

$t^2 = \frac{200}{16}$ $\frac{14.1}{4} = 3.53$ sec

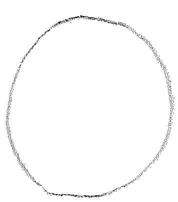
$V_{Y_0} = at$

$= (3.53)(32) = 160 \sin\theta$

$\sin\theta = .705$ $\theta = 45°$ ✓ 15

(B) A 2000 lb. car is moving around a circular race track at a constant speed of 90 ft/sec. The radius of the track is 500 ft.

What is the acceleration of the car? (Magnitude and direction)



$a = \frac{v^2}{r}$ ✓ 8

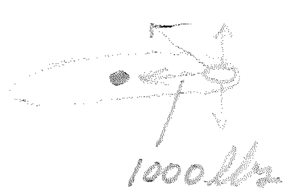
$= \frac{(90)^2}{500} = \frac{8100}{500} = 16.2 \frac{ft}{sec^2}$ (8)

toward the center

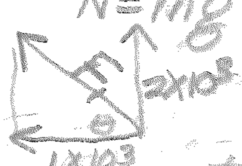
What is the frictional force exerted by the ground on the car. (Magnitude and direction)

$F = ma$

$= \frac{2000}{32}(16) = 1000$ lbs (7) $\frac{3}{}$



$N = mg = \frac{2000}{32} 32 = 2000$ lbs



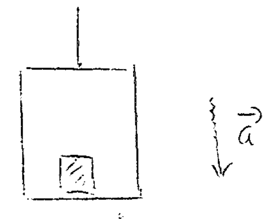
$F_1 = \sqrt{5 \times 10^6}$?

$= 2.23$ lbs ?

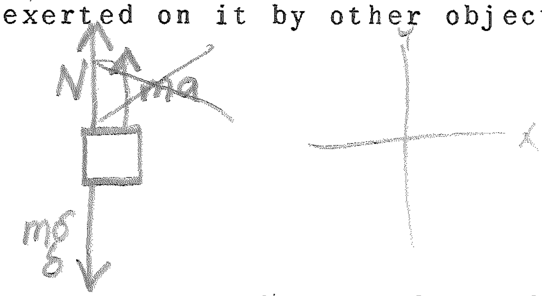
$\tan\theta = 2$

What is F.? $\theta = 63.5°$ N of W

II. The figure shows a box of mass m = 3 slugs sitting on the floor of an elevator which is accelerating downward, speeding up 2 ft./second each second.



(a) Draw a free body diagram below showing the box and the real forces exerted on it by other objects.



5

(b) Determine the magnitude of each of the forces identified in part (a).

10

$\Sigma F = m\vec{g} + m\vec{a} + \vec{N} = 0$

$m\vec{g} = 3 \text{ slugs} \left(32 \frac{ft}{sec^2}\right) = 96$ lbs down

~~$ma = 3 \text{ slugs} \left(2 \frac{ft}{sec^2}\right) = 6$ lbs up~~

$N = |m\vec{g}| - |m\vec{a}| = 96 - 6 = 90$ lbs up

15

(c) According to Newton's third law, for every force there is an associated reaction force. What is the magnitude and direction of the reaction force associated with each force of part (a), and upon what object does the reaction force act?

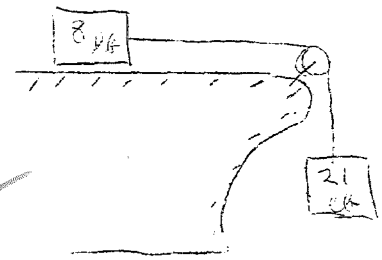
Force of string in +Y direction acting on block

Force of gravity in −Y direction acting on block

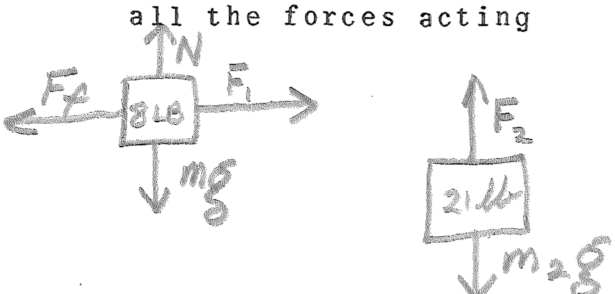
Normal force - since $\Sigma F = 0$ the elevator floor must be exerting a force up equal to the sum of the other two vectors

0

III. An 8 lb. block and a 21 lb. block are tied together by a string running over a massless frictionless pulley as indicated in the figure. Assume that any additional weight added to the 21 lb. block would make the system move.



(a) Draw a free body diagram for each block, showing all the forces acting



5

(5)

(b) If the table top and the 8 lb. block are both made of oak calculate the coefficient of static friction for oak on oak.

$F_1 = F_2 = m_2 g = F_f = (21$ lbs

(15)

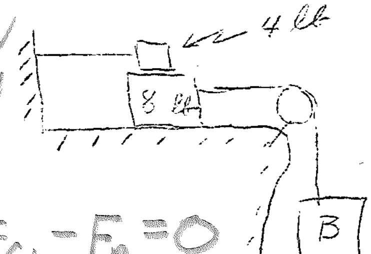
$F_f = \mu_s N$

$21 \text{ lbs} = \mu_s \, 8 \text{ lbs}$

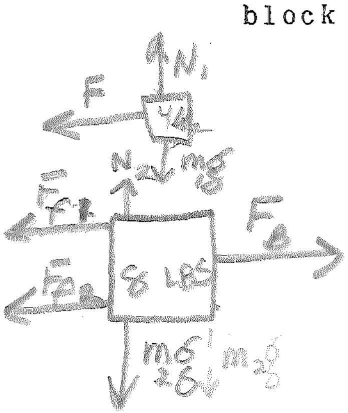
$\mu_s = 2.62$ 15

(27)

(c) An additional 4 lb. block made of oak is placed on the 8 lb. block and a new block B replaces the 21 lb. block. The 4 lb. block is tied to a vertical post as shown. What is the maximum weight of block B if the system is to remain at rest?



(15)



$F_f = \mu_s N$

$F_{f1} = 2.62(12) = 31.4$

$F_{f2} = 2.62(12)$

$\sum F_x = ma_x = F_{f2} + F_{f1} - F_B = 0$

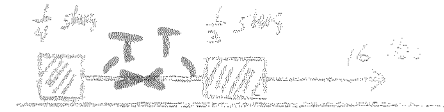
$2.62(24) = 32B$

$B = $ ~~[illegible]~~ lbs

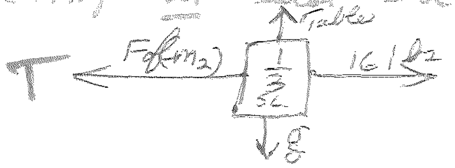
4/10

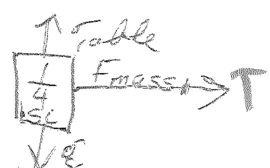
BOB MARKS
NAME

Two blocks tied together by a string rest on a horizontal smooth surface. The two blocks have masses of $\frac{1}{4}$ slug and $\frac{1}{3}$ slug respectively. A 16 lb force is exerted on the heavier mass to the right as indicated in the figure.



a) Draw free body diagrams for each block showing all the forces acting on each block.





5

b) Calculate the tension in the string

$F = ma$
$16 = \frac{1}{3}\,sl\,a$
$a = 48\ \frac{ft}{sec^2}$

$F = (m_1 + m_2)a$
~~$= \left(\frac{7}{12}\right)48 = 28\ lbs$~~

28 lbs

~~$F = (m_1 + m_2)a$~~
~~$\left(\frac{7}{12}\right)$~~

1

~~$F = m_1a + m_2a$~~
~~$F = 16\ lbs$~~

[illegible]

Gravitation

A. Law of gravitation $F = G \frac{m_1 m_2}{r^2}$

for particles or spherical mass distributions

B. Acceleration, due to gravity, of object of mass m near one of mass M $\quad a = g = \frac{F}{m} = G \frac{M}{r^2}$

C. Object in circular orbit about object of mass M (e.g. earth)

$$a = \frac{v^2}{r} = \frac{F}{m} = G \frac{M}{r^2}$$

Orbit time $T = \frac{2\pi r}{v}$

D. Gravitational potential energy of particle (or spherical mass distribution) of mass m_2 at distance r from one of mass m_1

$$U = -G \frac{m_1 m_2}{r}$$

II. Fluid Statics

A. Definition of pressure p, mass density ρ, and the relative density (specific gravity) of a substance.

B. Change in pressure dp in going upward infinitesimal distance dy in a fluid of density ρ (not necessarily constant)

$$dp = -\rho g \, dy$$

C. Incompressible fluid, ρ is the same everywhere in the fluid. The pressure p at a level which is distance h below level 0 is,

$$p = p_0 + \rho g h$$

D. Archimedes principle

Buoyant Force = weight of displaced fluid

III. Fluid Dynamics

A. Definitions of steady, irrotational, incompressible, nonviscous flow.

B. Steady, irrotational flow with no "sources" or "sinks" of mass between ① and ② in stream of flow

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad \text{(Equation of continuity)}$$

C. Steady, rotational, incompressible flow –

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 + \frac{W_f}{V} = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

where (W_f/V) = work done by friction (negative) per unit volume of fluid transferred from ① to ②

D. Steady, irrotational, incompressible, nonviscous flow –

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

(Bernoulli's Equation)

IV. Temperature

A. Ideal gas, Absolute, or Kelvin Temperature scale

$$\frac{T}{273.16} = \lim_{P_{tr} \to 0}\left(\frac{P}{P_{tr}}\right)$$ where 273.16 °K is the triple point of water and the pressures are constant volume gas thermometer pressures measured at temperature T and the triple point $T_{tr} = 273.16$ °K.

B. Celsius and Fahrenheit Temperature Scales

$$T_c = T - 273.15 \qquad T_F = \frac{9}{5}T_c + 32$$

C. Temperature Expansion

(1) linear expansion of a solid ⟶ coefficient of linear expansion α is the fractional change in length of the object (any linear dimension) per unit temperature change ⟶ $\alpha = \frac{\Delta L}{L\,\Delta T}$

(2) area expansion of a solid ⟶ fractional change in area per degree temperature change $\frac{\Delta A}{A\,\Delta T} = 2\alpha$

(3) volume expansion of solid or liquid ⟶ coefficient of volume expansion β is the fractional change in the volume per degree temperature change

$$\beta = \frac{\Delta V}{V\,\Delta T} = 3\alpha$$

PHYSICS II

3

BOB MARKS
NAME

1 STATE IN WORDS THE WORK ENERGY THEORUM

The total Energy of a system is equal to its potential energy plus its kinetic energy. Work, then, is [illegible] [illegible] this energy over a distance.

6

2 STATE IN WORDS THE PRINCIPLE OF CONSERVATION OF TOTAL MECHANICAL ENERGY

conditions?

The total [illegible] energy of the universe is in the form of kinetic or potential energy. The mechanical energy can not be destroyed or created but is conserved in the form of kinetic or potential energy

2

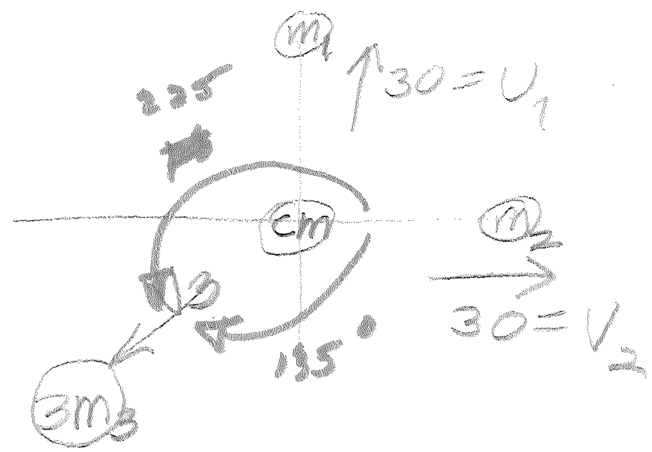
3 Define the potential energy U_A of a mass m at a point A in a gravitational field.

$U_A = mg(Y_A)$

1

BOB MARKS

... at rest ... explodes ... having equal mass, fly off perpendicular to one another with the same speed of 30 m/sec 10. The third piece has three times the mass of each other piece. What is the direction and magnitude of its velocity immediately after the explosion?



Direction = 135° from m_1 and m_2

$$\frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = 0$$

(cm remains at rest)

$m_1 \vec{x}_1 + m_2 \vec{x}_2 + m_3 \vec{x}_3 = 0$

$|\vec{x}_1| = |\vec{x}_2|$

$\vec{x}_1 + \vec{x}_2 + 3\vec{x}_3 = 0$

$\vec{x}_1 + \vec{x}_2 = -3\vec{x}_3$

$\frac{\sqrt{2}}{3}|x| = x_3$ V are constant

$\frac{\sqrt{2}}{3} 30 \frac{m}{sec} = 10\sqrt{2} \frac{m}{sec}$

For a Single Particle

Kinetic energy $K = \frac{1}{2} m v^2$

Potential energy $U_A = W_{A \to \text{Ref Pt.}}(\text{conser. forces})$

Special cases

$U(\text{grav}) = mgy$

$U(\text{elastic}) = \frac{1}{2} k x^2$

linear momentum $\vec{p} = m\vec{v}$

angular momentum about a point $\vec{l} = \vec{r} \times m\vec{v}$

THEORUMS

$W_{A \to B}\left(\begin{matrix}\text{resultant force} \\ \text{or} \\ \text{all forces}\end{matrix}\right) = K_B - K_A$

$W_{A \to B}(\text{non conservative forces}) = (K_B + U_B) - (K_A + U_A)$

If $W_{A \to B}(\text{non conservative forces}) = 0$ then $K_B + U_B = K_A + U_A$	Principle of Conservation Total (mechanical) energy

$\Sigma \vec{F} = m\vec{a}$

Misc.

If $\vec{a} = \text{const}$

$x = \frac{1}{2} a_x t^2 + v_{0x} t + x_0$

$v = v_{0x} + a_x t$

$2a_x(x - x_0) = v_x^2 - v_{0x}^2$

Misc

Particle moving in circle

comp of $\vec{a}$ toward center $= \frac{v^2}{R}$

comp. of $\vec{a}$ tangential $= \frac{d|v|}{dt} = \alpha R$

$v = \omega R$

For a System of Particles

DEFINITIONS

Kinetic energy $K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \cdots$

Potential energy $U = U_1 + U_2 + U_3 + \cdots$

Special cases

$U_{\text{grav}} = m_1 g y_1 + m_2 g y_2 + m_3 g y_3 + \cdots$

$= (m_1 + m_2 + m_3 + \cdots) g\, y_c$

linear momentum $\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots$

angular momentum about a point $\vec{L} = \vec{r}_1 \times m_1\vec{v}_1 + \vec{r}_2 \times m_2\vec{v}_2 + \vec{r}_3 \times m_3\vec{v}_3 + \cdots$

center of mass $\vec{R}_c = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3}$

THEORUMS

$W_{\text{conf } A \to \text{conf } B}\left(\begin{matrix}\text{all forces acting} \\ \text{on all particles}\end{matrix}\right) = K_{\text{conf } B} - K_{\text{conf } A}$

where $K_{\text{conf } B}$ = kinetic energy of all particles when they are in configuration B

$W_{\text{conf}A \to \text{conf } B}(\text{non cons. forces}) = (K + U)_{\text{conf } B} - (K + U)_{\text{conf } A}$

If $W_{\text{conf } A \to \text{conf } B}(\text{non cons. forces}) = 0$ then $(K + U)_{\text{conf } B} = (K + U)_{\text{conf } A}$	conserv. mech energy

$\dfrac{d\vec{P}}{dt} = \Sigma \vec{F}(\text{external})$

If $\Sigma \vec{F}(\text{external}) = 0$ then $\vec{P} = \text{constant}$	Conservation of linear momentum

$\dfrac{d\vec{L}}{dt} = \Sigma \overrightarrow{\text{Torques}}$ (due to external forces)

If $\Sigma \overrightarrow{\text{Torques}}(\text{external forces}) = 0$ then $\vec{L} = \text{constant}$	conserv. ang. momentum

$\Sigma m_i\, \vec{a}_c = \Sigma \vec{F}(\text{external})$ Motion of center of mass

RIGID BODY

Potential Energy in Gravitational Field

$$U = M g y_c$$

$M \equiv$ mass of body

$y_c \equiv$ y coor. of center of mass of body

Rotating about a fixed axis with angular velocity ω

$$K = \frac{1}{2} I \omega^2$$

$K \equiv$ kinetic energy

$I \equiv$ moment of inertia of body about the axis of rotation

$$L = I \omega$$

$L \equiv$ component of angular momentum along axis of rotation.

If body is symmetrical about axis of rotation, then L is the total angular momentum & points along axis of rotation

$$\Sigma \tau = I \alpha$$

$\Sigma \tau \equiv$ components of torques (due to external forces) along axis of rotation.

If body is rolling without slipping along a surface

$$K = \frac{1}{2} I_c \omega^2 + \frac{1}{2} M v_c^2$$

$I_c \equiv$ moment of inertia about axis thru center of mass

$v_c \equiv$ velocity of center of mass.

$$\Sigma \tau = I_c \alpha$$

$\Sigma \tau \equiv$ components of torques along axis thru c of mass

$$v_c = \omega R$$

$v_c \equiv$ velocity of c of mass

$R =$ radius of rolling object

$$M \vec{a}_c = \Sigma \vec{F}(ext)$$

$a_c \equiv$ acc. of c of mass

(1) Newton's law of universal gravitation

$g = \frac{GM}{R^2}$

(2) Kepler's laws
- 2nd (law of areas)
- 3rd (harmonic law) $T^2 \propto a^3$

$T = 2\pi\sqrt{a^3/g}$

(3) Pressure $P = \frac{dF}{dA}$; F = normal force

(4) Equation for hydrostatic equilibrium $\frac{dP}{dy} = -\rho g$

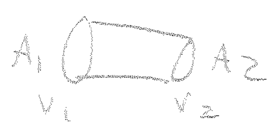
from which we get $P = P_0 + \rho g h$; h = depth

(5) Pressure in atmosphere $\frac{dP}{dy} = -\rho g$, $\frac{\rho}{\rho_0} = \frac{P}{P_0}$

where $a = \frac{g\rho_0}{P_0} = 0.116$

(6) Pascal's and Archimedes' principles

(7) Eq of continuity



$A_1 V_1 = A_2 V_2$

(8) Bernoulli's Equation

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

(9) Zeroth of thermodynamics

Temperature scale $T \propto X$, X = thermal property

(10) Temperature expansion

$\Delta L = \alpha L \Delta T$

$\Delta A = 2\alpha A \Delta T$ } for isotropic substance

$\Delta V = 3\alpha V \Delta T$

Volume expansion coeff $\beta = \left(\frac{1}{V}\frac{\Delta V}{\Delta T}\right)$

8

Bob Marks

$\frac{1}{2} m V^2 = mgh$ $9.8 = g$

$\frac{1}{2}(V^2) = (10)(2.5)$

$= \sqrt{50} = \sqrt{49} = 7$

$mV_1 + mV_2 = V(m_1 + m_2)$

$2(\sqrt{50}) + 0 = V(4)$

$V = \frac{\sqrt{50}}{2} = 3.5$

$W_f = (U_f + K_f) - (U_0 + K_0)$

$= 0 + \frac{1}{2}(4)(50) - 0 - 0$

$W_f = 100$ nt m

$K = \frac{1}{2} m V^2$

Bob Marks

7

A wheel $2\frac{1}{2}$ ft in diameter is rotating about a fixed axis with an initial angular velocity of 2 rev/sec. The angular acceleration is 3 rev/sec^2, $\alpha = 3\frac{REV}{SEC}$

a) What is the angular velocity after 6 sec.

$$\omega = \alpha t + \omega_0$$
$$= 3(6) + 2$$
$$= 18 + 2 = 20 \frac{REV}{SEC}$$ ✓

b) Then what angle has the wheel turned in the time

$$\theta = \frac{\alpha}{2}t^2 + \omega_0 t + \theta_0$$
$$= \frac{3}{2}(36) + 12 + 0$$
$$= 54 + 12 = 66\,REV \times \frac{2\pi\,RAD}{REV} = 132\pi\ RADIANS$$ ✓

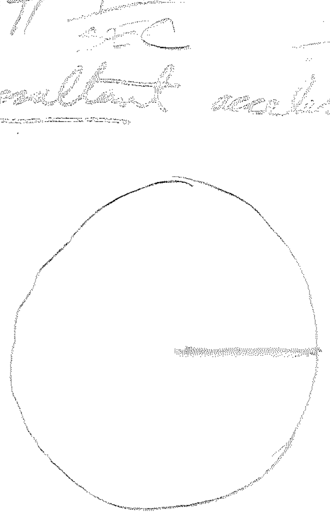
c) What is the tangential velocity of a point on the rim of the wheel at t = 6 sec

$$|\vec{V}| = r\omega \quad 1.25 \qquad \omega = \alpha t + \omega_0$$
$$= (40\pi)(2.5)$$
$$= 90\pi \frac{FT}{SEC}$$

-1

d) What is the resultant acceleration of a point on the wheel at t = 6 sec

$$a_T = r\alpha$$
$$= (2.5)(3)$$
$$= 1.5\pi \frac{ft}{sec}$$



O

$132\pi / 2$

-2

$a_r = ?$ $A_{TOT} = ?$

(58)

MARKS

(23)

$M = .3$ kg
$\ell = 2$ m

$I\alpha = \tau = r \times F$

$(\frac{1}{12} m \ell^2 + 2 m r^2)\alpha = r F \sin\theta$ ✓

90.0
36.8
53.2

$\theta = 53.2°$

~~$\alpha = \omega t$~~

$\omega = \alpha t + \omega_0$

$\alpha = \frac{-\omega}{t}$

$= \frac{40}{5} = 8 \frac{rad}{sec^2}$

$F = \left[\frac{(.3)(2)^2}{12} + 2(.2)(.5)^2\right]\frac{8}{.8} = .F$

✓ $\left[\frac{1.2}{12} + .10\right]10$

$= [.2]10 =$ (5) nt

9

$W = \Delta K = \frac{1}{2} I \omega^2$

$= \frac{1}{2}[.2][40]^2$

$= (.1)(1600)$

$= 160 \frac{Kg\,m^2}{sec^2}$

6

If at the end of 5 seconds force F is removed and the masses M slide out to the ends of the rod, what will the angular velocity be from then on?

~~$U_A + K_A = U_B + K_B$~~

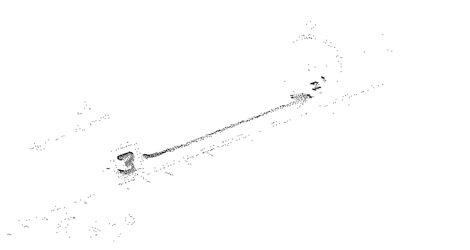
$U_A + K_A = U_B + K_B$ X

$(40)^2 \frac{1}{2}\left(\frac{ml^2}{12} + 2m_2(.5)^2\right) = \frac{1}{2}\left(\frac{ml^2}{12} + 2m_2(1)^2\right)\omega_f^2$

0

$$\omega_f = \sqrt{\frac{(40)^2 \frac{1}{2}\left[\frac{(.3)(2)^2}{12} + 2(.2)(.5)^2\right]}{\frac{1}{2}\left(\frac{.3(2)^2}{12} + 2(.2)(1)^2\right)}}$$ sec

A rope is wound several times around a cylindrical drum of radius R = 2 ft and attached to a [illegible] 3 slug on an inclined plane [illegible]. If it is observed that the [illegible] up 6 ft sec each second as [illegible] down the plane, determine the moment of inertia of the drum. Assume friction negligible.



$a_T R = a$ X

$\alpha ?, a_T = \frac{a}{R} = \frac{6}{2} = 3$ ft/sec²

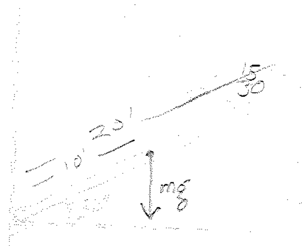
$F \times R = I\alpha$ ✓

$\left(\frac{3}{32}\right)(2) = I\,3$

$I = \frac{2}{32} = \frac{1}{16}$ SLUG FT²

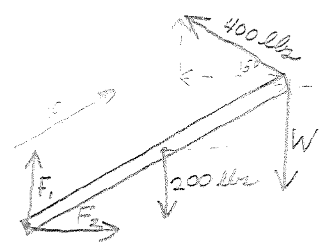
8

. . . [illegible] the center of gravity . . . [illegible] the midpoint . . . [illegible] the tension of the cable is 100 lbs



(30)

A. Isolate the boom and sketch in all the forces acting on it. Label each force: give its numerical value if known, or a symbol if unknown.



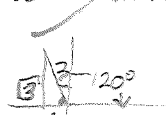
9

B. In terms of the forces and the given angles and lengths, write down the three equations expressing the fact that the boom is in equilibrium.

$$\Sigma F_x = ma_x = 0 = F_2 - 400\cos 15°$$

$$\Sigma F_y = ma_y = 0 = F_1 + 400\sin 15° - 200 - W$$

$$\Sigma \tau_z = 0 = \cancel{F \times d} = (-200)(10)\sin 120° + (400)(20)(\sin 45°) - (20)(W)\sin 120°$$



C. Find the value of the load W

$$(-2\times10^3)\frac{\sqrt{3}}{2} + (8\times10^3)\frac{1}{\sqrt{2}} = 20\frac{\sqrt{3}}{2}W = 10\sqrt{3}\,W$$

$$+\ 5.64\times10^3 - 1.73\times10^3 = 17.3W$$

$$W = \frac{3.91\times10^3}{17.3} = \frac{39.1\times10^2}{17.3} = 2.26 \text{ lbs}$$

5 on this page

$V = gt + v_0$ $\qquad$ $h = gt^2$

$= gt$

$V_{(MAX)} = gt = Y\,\omega \sin(\omega t + \alpha)$

$X = Y \cos(\omega t + \alpha)$

at X,

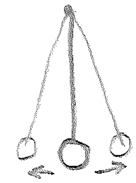
$\omega = \sqrt{\frac{K}{M}}$

$X_{MAX} = A$

$V_{MAX} = gt = Y\sqrt{\frac{K}{M}}\left(\cos\left(\sqrt{\frac{K}{M}} + \alpha\right)\right)$

Describe motions of the following three pendulums. Set up equations of motion and try to solve it.

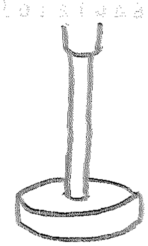
a. Simple pendulum



Mass on a rod, executing simple harmonic motion thru small angles

$T = 2\pi\sqrt{\frac{I}{K}}$

b. Torsional pendulum



a uniform disk rotating in simple harmonic motion

5

$\tau = \frac{-K}{I}\theta$

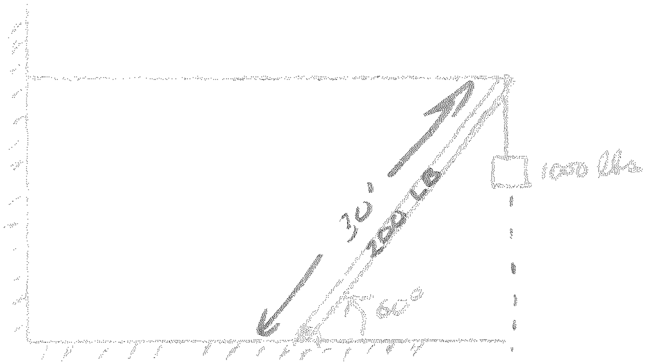
[redacted]

$\theta = \theta_{MAX} \cos(\omega t + \alpha$

c. Physical pendulum

BOB MARKS
NAME

In the figure below assume boom is uniform and weighs 200 lbs. It is 30 ft long. The guy wire is horizontal.



(a) ISOLATE THE BOOM AND DRAW IN ALL FORCES ACTING ON IT

(b) WRITE DOWN THE THREE EQUATIONS

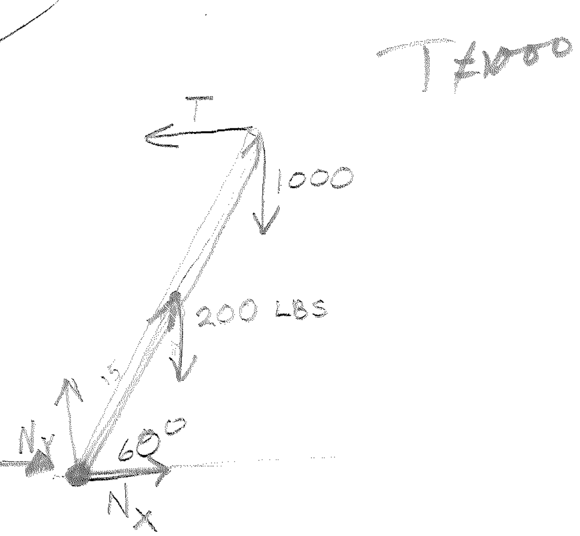
$\Sigma F_x = 0 = N_x - \cancel{1000}\ T$

$\Sigma F_y = 0 = N_y - 1200$ ✓

$\Sigma T_3 = 0 = (15)(200)\sin 30° + (30)(1000)\sin 30° - 30T\cos 30°$ ✓

INDICATE THE POINT about which you are calculating torques.

$T \neq 1000$

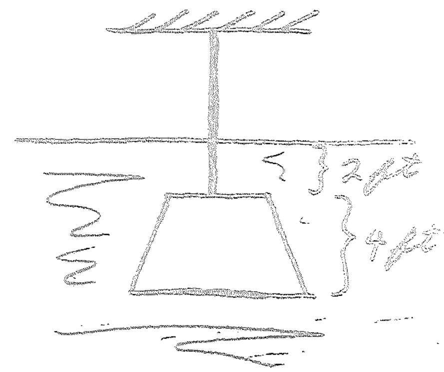


MARKS

Physics II – Test II (Make-up)

① A satellite is in a circular orbit of radius 5000 miles about the earth and makes one complete orbit in 7100 seconds (about 2 hours). If the satellite is then "flown" to the moon and goes into an orbit around it of radius 2000 miles, how long does it take to orbit the moon once? The mass of the moon is 1/80 th that of the earth.

② An object in the shape of a truncated cone weighs 1000 lb. in vacuum, has a volume of 12 ft^3, and is suspended by a rope in an open tank containing a liquid of density 2 slugs / ft^3 as shown. Atmospheric pressure is 2120 lb / ft^2.

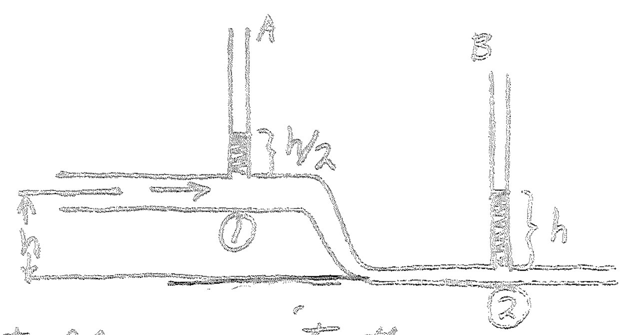


(a) What is the tension in the rope?

(b) If the area of the top surface is $A_1 = 2\ ft^2$ and the area of the bottom is $A_2 = 4\ ft^2$ find the forces F_1 and F_2 exerted by the liquid on the top and bottom surfaces respectively.

(c) Does the net upward force $(F_2 - F_1)$ exerted by the liquid on the lower and upper surfaces of the cone equal the buoyant force exerted by the fluid on the cone? Should it? Explain briefly.

③ The figure shows a fluid flowing in a pipe having a 10 inch diameter at point ① and a 5 inch diameter at point ②. The pressures at points 1 and 2 cause some fluid to flow up into the standpipes A and B which are open to the atmosphere at the



top (note that the three heights h in the figure represent the same distance). If the flow speed at point 1 is $v_1 = 3$ ft/second, what is height h assuming the flow to be frictionless?

(4) A. A steel [$\alpha = 12 \times 10^{-6}\ (^\circ C)^{-1}$] rod is 110 cm long. What must be the initial length of an aluminum rod [$\alpha = 24 \times 10^{-6}\ (^\circ C)^{-1}$] which would expand by the same amount as the iron rod for the same temperature change?

B. Explain the ideal gas temperature scale. (How is it arrived at?)

MARKS

① $T^2 = \frac{4\pi^2}{GM} r^3$

$(2\,HR)^2 = \frac{4\pi^2}{GM_E}(R_E + 5000)^3$ $\qquad T_M^2 = \frac{4\pi^2}{GM_m}(R_M + 2000)^3$

$G = \frac{4\pi^2 (R_E + 5000)^3}{M_E (4\,HR^2)} = \frac{4\pi^2 (R_M + 2000)^3\, 80}{M_E\, T_M^2}$

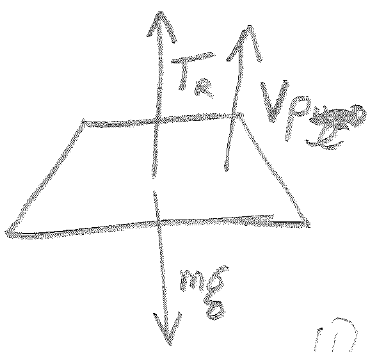
$T_M^2 = \frac{(2.\times10^3)^3\, 80\, (4)}{(5.0\times10^3)^3}$ OK

$= \frac{8\times10^9 (3.2\times10^2)}{1.25\times10^9 \times 10^2}$

$= 5.06$ HR ✓ 15

1 15
2 21
3 18
4 25
79
84

②



a) $\Sigma F_y = 0 = T_R + V\rho_e - mg$

$mg = T_R + V\rho_e$

$1000\,lbs = T_R + 12\,ft^3 \frac{2\,slugs}{ft^3} \frac{32.2\,lbs}{slug}$

$1000\,lbs = T_R + 774$

$T_R = 226\,lbs$ ✓

10

b) $p_1 = p_0 + \rho g h$

$= \frac{2\,slug}{ft^3}\; 2\,ft\; \frac{32.2\,lbs}{slug}$

$= 2120 + 2^2(32.2)\,g$ (2) what is this?

$= 2120 + 129g$

$p_1 A_1 = F = 2(2120 + 129g)\,lbs$

$p_2 = 2120 + 2(32.2)\,g\,6$ what is g?

$= 2120 + 387g$

$p_2 A_2 = F_2 = 4(2120 + 387g)\,lbs$

4

c) No. Although the resultant buoyant force "lies on the y axis," the forces acting on the sides have y components which

4 (A) STEEL $\Delta L_1 = \alpha_1 L_1 \Delta T_1$ ALUM $\Delta L_2 = \alpha_2 L_2 \Delta T_2$ MARKS

$\Delta L = (110)(12 \times 10^{-6}) \Delta T$ $\Delta L = (24 \times 10^{-6}) L_2 \Delta T$

$\frac{\Delta L}{\Delta T} = (110)(\overset{.5}{\cancel{12}} \times \cancel{10^{-6}}) = (\cancel{24} \times \cancel{10^{-6}}) L_2$

$L_2 = 55 cm$ (15)

B) ~~Fahr~~

The ideal gas temperature scale is arrived at by plotting ΔV verses ΔT, which, ~~except for phase changes~~, is for the most part, linear. Where the line passes through $V=0$ is set at absolute zero. (An ideal gas can have zero volume for its molecules have no volume.)

~~It was calibrated~~

It was calibrated in degrees of the same magnitude of for runner scales. Centigrade degrees used for absolute scale Kelvin, Farenheit for Rankin

(10)

22-1) $F = ma$

$W = F \times d$

$= ma \times d$

$= (6.00\,kg)(9.8\,\frac{m}{SEC^2})(50.0\,m)$

$= 2940\,j$

$\frac{4186\,j}{kg}\Delta T = \frac{2940\,j\ \ ^\circ C}{.600\,kg}$

$\Delta T = 1.17^\circ C$

22-5) HT. LOST BY METAL = HT GAINED BY (H_2O + CONT)

$m_M C_M \Delta T_M = (m_m C_m + m_W C_W)\Delta T_c$

$(4\,LB)\,C_m\,(285^\circ F) = (8\,LB\,C_m + 30\,LB\,\frac{BTU}{LB^\circ F})\,5^\circ F$

$1140\,C_m = -40\,C_m - 150\,\frac{BTU}{LB^\circ F}$

$m_m C_m \Delta T_M = (m_m C_m + m_W C_W)\Delta T_c$

$(4)\,C_m(285) = (8\,C_m + (30)\frac{BTU}{LB^\circ F})\,5$

$1140\,C_m = 40\,C_m + 150\,\frac{BTU}{LB^\circ K}$

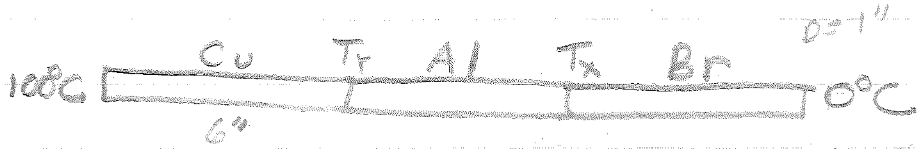
22-13) $.40\,hp = 220\,\frac{FT\ LB}{SEC}$

$W = (220)(160\,SEC) = 25600\,FT.\,LB \times \left(\frac{777.9\,BTU}{FT.\,LB}\right)^{-1} = 34\,BTU$

$.75 \times 34 = 25.5\,BTU$

1 lb block

22-17) $K_{Cu} = 2K_{Al} = 4K_{BR}$



$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$\frac{\Delta Q}{\Delta t} = \frac{k_{Cu}A(373° - T_Y)}{6''} = \frac{k_{AL}A(T_Y - T_X)}{6''} = \frac{k_{BR}A(T_X - 273)}{6''}$$

$(373 - T_Y) = 2(T_Y - T_X) = 4(T_X - 273)$

$373 - T_Y = 2T_Y - T_X = 4T_X - 1092$

$3T_Y = 373 + T_X$

$T_X = 3T_Y - 373$

$2T_Y - 3T_Y + 373 = 4T_X - 1092$

$373 - T_Y = 2T_Y - T_X$ $\quad$ $4T_X - 1092 = 2T_Y - T_X$

$1665 - 15T_Y = -5T_X$ $\quad$ $5T_X = 2T_Y + 1092$

$1092 + 2T_Y = 5T_X$

$2757 = 13T_Y$

$T_Y = 212°K$

22-19) $k = 2\times10^{-4}\ \frac{KCAL}{m\ sec\ °C}$

$\frac{dT}{dx} = .07\ \frac{°C}{m}$

$\frac{\Delta Q}{\Delta t} = -kA\frac{dT}{dx}$

~~$\Delta Q\ (-2\times10^{-4})\frac{KCAL}{m\ sec\ °C}$~~

$= (-2\times10^{-4})(1)(.07) = -1.4\times10^{-5}\ \frac{KCAL}{m^2 sec}$

22-23) $\Delta U = Q - W$

$-30 = -36 - W$

$W = 6\ cal$

b) ~~$\Delta U - Q = -30$~~ ~~$-13 - Q = -30$~~ ~~$Q =$~~ ~~$\Delta U = Q - W$~~ ~~$30 =$~~

$-30 = Q + 13$

$Q = -43$

c) $U_f - 10 = 30$

$U_f = 40\ cal$

23-1) $\rho_A = 1.293$ $\quad PV = nRT$

$\rho_O = 1.429$ $\quad \frac{n}{V} = \frac{P}{RT}$

$\rho_N = 1.251$ $\quad \left(\frac{n}{V}\right) =$

$1.293 \frac{kg}{m^3}$

$1.429 \frac{kg}{m^3}$ $\quad \underline{1 \cdot n}$

23-3) $P_1 = P_2 + \rho g h$

$= 1.01 \times 10^5 + \left(\frac{10^3\ kg}{m^3}\right)\left(9.8 \frac{m}{sec^2}\right)(40\ m)$

$= 1.01 \times 10^5 + 3.91 \times 10^5$

$P_1 = 4.92 \times 10^5$

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$V_2 = \frac{P_1 V_1 T_2}{T_1 P_2}$

$= \frac{(4.92 \times 10^5)(20\ cm^3)(293)}{(277)(1.01 \times 10^5)}$

$= 103\ cm^3$

23-5) $P_0 V_0 = P_f V_f$

$P_0 (50\ cm^3) = P_f (44\ cm^3)$ $\quad P_0 (30\ cm^3) = P_f (27\ cm^3)$

$P_f = \frac{P_0\, 30}{27}$

$P_0 (50\ cm^3) = \frac{P_0\, 30\, (44)}{27}$

$P_0 V_0 = nRT.$

24-1) $\bar{l} = \frac{1}{\pi\sqrt{2}\,n d^2}$

$d = \sqrt{\frac{1}{\pi\sqrt{2}\,n\,\bar{l}}}$

$d^2 = \frac{6.03\times10^{9}}{(1.41)(3.14)(2.7\times10^{9})(.80\times10^{-})}$

24-3) $P = 10^{-3}\,mm \times \frac{1\,atm}{760} = 1.31\times10^{-3}\,atm$

$P_1V_1 = P_2V_2$

$(1\,atm)(22.4) = (1.31\times10^{-3})V_2$

$V_2 = 1.71\times10^{3}\,l$

$1.71\times10^{3}\,l \times \frac{6.02\times10^{24}\,molec}{22.4\,l} =$

$4.60\times10^{27}\,molec$

$3.5\times10^{10}\,molec/cm^3$

$\bar{l} = \frac{1}{\pi\sqrt{2}\;3.5\times10^{1}}$

24-9) $V_{RMS} = \sqrt{\frac{3P}{\rho}}$

$= \sqrt{\frac{7.91\times10^{-3}}{4\times10^{19}}}$

$PV = nRT$

$P = \frac{n}{V}RT$

$= \frac{4\times10^{19}\,mole}{6\times10^{-23}\,cm^3}\;\frac{1000\,cm^3}{l}\;\frac{.082\,l\,atm}{mol\,^\circ K}\;4\times10^{2}$

$= \frac{16\times}{6\times10^{4}} = 2.67\times10^{-4}\,atm$

24-12) $W = \Delta K$

$K_1 =$

22-1) $F = ma$

$W = F \times d$

$= ma \times d$

$= (6.00\,kg)(9.8\,\frac{m}{SEC^2})(50.0\,m)$

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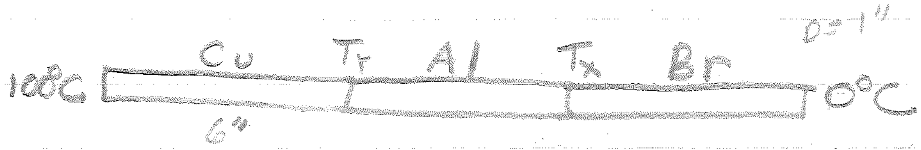

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$$T_X = 3T_Y - 373$$

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$\frac{\Delta Q}{\Delta t} = -kA \frac{dT}{dx}$

~~$\Delta Q = (2 \times 10^{-4}) \frac{KCAL}{m \, sec \, °C}$~~

$= (-2 \times 10^{-4})(1)(.07) = -1.4 \times 10^{-5} \frac{KCAL}{m^2 SEC}$

22-23) $\Delta U = Q - W$

$= 30 = 36 - W$

$W = 6 \text{ cal}$

b) ~~$\Delta U - Q = -30$~~ ~~$\Delta U = Q - W$~~

~~$-13 - Q = -30$~~ ~~$30 =$~~

~~$Q =$~~

$-30 = Q + 13$

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$1.293 \frac{kg}{m^3}$

$1.429 \frac{kg}{m^3}$ $\quad \frac{1}{n}$

23-3) $P_1 = P_2 + \rho g h$

$= 1.01 \times 10^5 + \left(\frac{10^3\ kg}{m^3}\right)\left(9.8 \frac{m}{sec^2}\right)(40\ m)$

$= 1.01 \times 10^5 + 3.91 \times 10^5$

$P_1 = 4.92 \times 10^5$

$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$V_2 = \frac{P_1 V_1 T_2}{T_1 P_2}$

$= \frac{(4.92 \times 10^5)(20\ cm^3)(293)}{(277)(1.01 \times 10^5)}$

$= 103\ cm^3$

23-5) $P_0 V_0 = P_f V_f$

$P_0 (50\ cm^3) = P_f (44\ cm^3)$ $\quad P_0 (30\ cm^3) = P_f (27\ cm^3)$

$P_f = \frac{P_0\, 30}{27}$

$P_0 (50\ cm^3) = \frac{P_0\, 30\, (44)}{27}$

$P_0 V_0 = nRT$

24-1) $\bar{l} = \frac{1}{\pi\sqrt{2}\, n d^2}$

$d = \sqrt{\frac{1}{\pi\sqrt{2}\, n \bar{l}}}$

$d^2 = \frac{6.03 \times 10^{9}}{(1.41)(3.14)(2.7 \times 10^{9})(.80 \times 10^{-5})}$

24-3) $P = 10^{-3}\,mm \times \frac{1\,atm}{760} = 1.31 \times 10^{-3}\,atm$

$P_1V_1 = P_2V_2$

$(1\,atm)(22.4) = (1.31 \times 10^{-3})V_2$

$V_2 = 1.71 \times 10^{3}\,l$

$1.71 \times 10^{3}\,l \times \frac{6.02 \times 10^{24}\,molec}{22.4\,l} =$

$4.60 \times 10^{27}\,molec$

$3.5 \times 10^{10}\,molec/cm^3$

$\bar{l} = \frac{1}{\pi\sqrt{2}\; 3.5 \times 10^{1}}$

24-9) $V_{RMS} = \sqrt{\frac{3P}{\rho}}$

$= \sqrt{\frac{7.91 \times 10^{-5}}{4 \times 10^{19}}}$

$PV = nRT$

$P = \frac{n}{V} RT$

$= \frac{4 \times 10^{19}\,mole}{6 \times 10^{-5}\,cm^3} \; \frac{1000\,cm^3}{l} \; \frac{.082\,l\,atm}{mol\,°K} \; 4 \times 10^{2}$

$= \frac{16 \times}{6 \times 10^{4}} = 2.67 \times 10^{-4}\,atm$

24-17) $W = \Delta K$

$K_1 =$

PHYS. III

PHYSICS III

FORMULAS

I CHAPT. 26 · CHARGE & MATTER

A. COULOMB'S LAW

1) $F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$

2) ϵ_0 = PERMITTIVITY CONSTANT = $8.85 \times 10^{-12} \frac{coul^2}{nt\, m^2}$

3) $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \frac{nt\, m^2}{coul^2}$

B. $q = it$ (DEF)

1) q in coul

2) i in amps

3) t in sec.

II, CHAPT. 27 - THE ELECTRIC FIELD

A) $\vec{E} = \frac{\vec{F}}{q_0} = \frac{q}{4\pi\epsilon_0 r^2}$

1) q_0 = small test body

2) $E = \frac{dF}{dq_0}$

B) Dipole

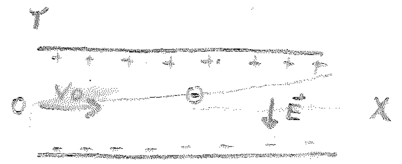
1)


2) p = dipole moment = $2aq$ if $r \gg a$

3) $E = \frac{p}{4\pi\epsilon_0 r^3}$

C) $\vec{F} = \vec{E}q$ $a = F/m$

D) ELECTRON TRAJECTORY

1)


2) $Y = \frac{eE}{2mV_0^2}X^2$

E) $\vec{\tau} = \vec{p} \times \vec{E}$ } $p = 2aq$ = dipole moment

F) $U = -\vec{p} \cdot \vec{E}$

III. CHAPT. 28 - GAUSS'S LAW

A) Φ = FLUX = Hypothetical surface in a field

1) Φ_V = flow field flux

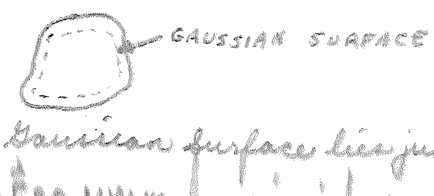
2) Φ_E = electric field flux (# of lines of force cutting thru surface)

B) $\Phi_E = \oint E \cdot dS \cong \Sigma E \cdot \Delta S$

($\oint \rightarrow \int$ over a surface)

C) $\epsilon_0 \Phi = \epsilon_0 \oint E \cdot dS = q$

D) AN INSULATED CONDUCTOR

1)


2) Gaussian surface lies just below surface

3) $\vec{E} = 0$ everywhere inside cond. → $\vec{E}$ is totally on surface

E) $E = \frac{qr}{4\pi\epsilon_0 R^3}$ for sphere ($r \geq R$)

F) $E = \frac{\lambda}{2\pi\epsilon_0 r}$ for infinite rod (λ = linear charge density in $\frac{coul}{m}$)

G) $E = \frac{\sigma}{2\epsilon_0}$ for infinite sheet ($\frac{\sigma}{\epsilon_0}$ for one side)

IV. CHAPT 29 - ELECTRICAL POTENTIAL

A) $V_{BA} = V_B - V_A = \frac{W_{AB}}{q_0}$ = electrical potential difference:

1) Work done moving q_0 from A to B

2) V is path-independent

B) Locus of points having same elec. potential = equipotential surface

C) $W_{AB} = Fd = q_0 E d \Rightarrow V_B - V_A = \frac{W_{AB}}{q_0} = Ed$

D) $\frac{V}{M} = \frac{NT}{COUL}$

E) $W_{AB} = \int_A^B F \cdot dl = -q_0 \int_A^B E \cdot dl \Rightarrow$

$$V_B - V_A = \frac{W_{AB}}{q_0} = -\int_A^B E \cdot dl$$

F) $V = \frac{q}{4\pi\varepsilon_0 r}$ for point charges

G) A GROUP OF PT. CHARGES

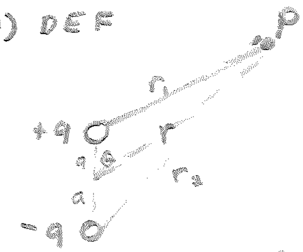
1) $V = \sum_n V_n = \frac{1}{4\pi\varepsilon_0} \sum_n \frac{q_n}{r_n} \Rightarrow$

2) $V = \int dV = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} \Rightarrow$

3) if $r = K$ then $V = \frac{\Sigma q}{4\pi\varepsilon_0 r}$

H) DIPOLE

4) DEF



2) $V = \frac{q}{4\pi\varepsilon_0} \frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

I) Electrical Potential Energy

$$U(=W) = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

J) $E_l = \frac{-dV}{dl_{max}}$ or in X, Y, Z

K) For 2 charged spheres

1) $\frac{q_1}{q_2} = \frac{R_1}{R_2} = \frac{\sigma_2}{\sigma_1}$

L) $K.E. = qV$

1) K in Joules

2) q in coul

3) V in Volts

M) elec. Volt = 1 quantum energy / 1 volt = 1.60×10^{-19} J

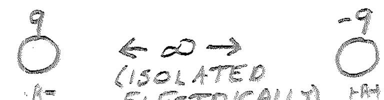
V. CHAPTER 30 - CAPACITORS AND DIELECTRICS

A) CAPACITANCE

1) POTENTIAL (V) (of a charged conducting sphere)

$$V = \frac{q}{4\pi\varepsilon_0 R}$$

2)



a) $q = 2\pi\varepsilon_0 R V = CV$ (c = capacitance)

b) OR $C = \frac{q}{V} = 4\pi\varepsilon_0 R$

3) UNITS IN MKS

a) FARADAY (CAPAC.) $= \frac{\text{COUL}}{\text{VOLT}} = f$

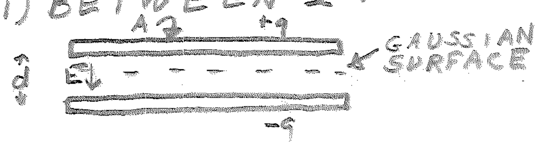
b) μf = microfarad $= 10^{-6} f$

c) $\mu\mu f$ = MICROMICROFARAD $= 10^{-12} f$

4) Defn: two nearby conductors (of any shape) with equal, but opposite charges is a capacitor

B) CODUCTING CAPACITANCE

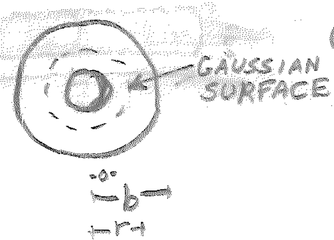
1) BETWEEN 2 PLATES



a) $\varepsilon_0 \Phi_E = \varepsilon_0 E A = q$

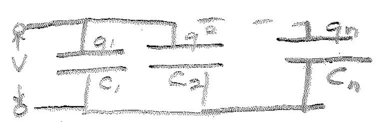
b) $C = \frac{q}{V} = \frac{\varepsilon_0 E A}{E d} = \frac{\varepsilon_0 A}{d}$

2)



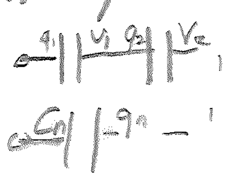
$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

3) capacitors in //:



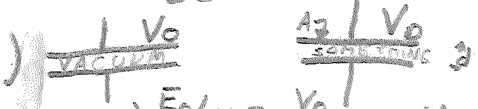
$$\Sigma C = C_1 + C_2 \ldots C_n = \frac{q_1}{V_1} + \frac{q_2}{V_2} \ldots \frac{q_n}{V_n}$$

4) capacitors in series:



$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \ldots \frac{1}{C_n} = \frac{V_1}{q_1} + \frac{V_2}{q_2} \ldots \frac{V_n}{q_n}$$

C) PARALLEL-PLATE CAPACITOR WITH DIELECTRIC

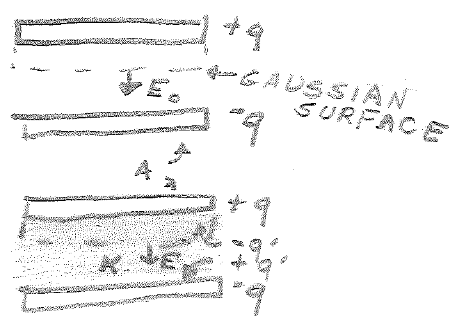
1) 

a) $E_0/E = \frac{V_0}{V_d} = K$

b) $V_d = V_0/K$

c) $C = \frac{K\varepsilon_0 A}{d}$ FOR // PLATES

d) $C = K\varepsilon_0 L \Rightarrow L$ depends on the geometry of the conductors with unit of length

2) 

a) $E = \frac{q - q'}{\varepsilon_0 A} = \frac{E_0}{K} = \frac{q}{K\varepsilon_0 A}$

b) $q' = q\left(1 - \frac{1}{K}\right)$

D) THREE ELECTRIC VECTORS

1) ELECTRIC POLARIZATION

a) $P = \frac{q'}{A}$ (q' = induced surface charges)

b) $\frac{q}{A} = \varepsilon_0 E + P$

2) ELECTRIC DISPLACEMENT (D)

a) $\vec{D} = \varepsilon_0 \vec{E} + P = K\varepsilon_0 \vec{E}$

b) $D = \frac{q}{A}$

c) $\vec{P} = \varepsilon_0 (K-1)\vec{E}$

d) $\oint \vec{D} \cdot d\vec{S} = q$

E) ENERY STORAGE IN AN ELECTRIC FIELD

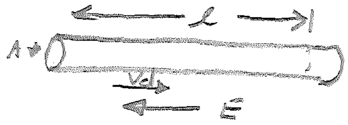
a) $W = \int_0^q \frac{q'}{C} dq' = \frac{q^2}{2C} = \frac{1}{2}CV^2$

b) u (ENERY DENSITY) $= \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{K\varepsilon_0}{2}\left(\frac{V}{d}\right)^2 = \frac{1}{2}K\varepsilon_0 E^2$

F) U (

VI. CHAPTER 31 CURRENT & RESISTANCE

A) CURRENT (i) & SUCH



a) $i = q/t = dq/dt$

d) CURRENT DENSITY $= J = \frac{i}{A}$

e) $i = \int J \cdot dS$

d) $q = nA\ell e$

1) n = # elec. (conduction elec)
2) $A\ell$ = VOL OF WIRE

e) $V_d = \frac{i}{nAe} = \frac{J}{ne}$

B) RESISTANCE, RESISTIVITY, & CONDUCTIVITY

a) $R = \frac{V}{i}$

b) resistivity $= \rho = \frac{E}{J}$

c) $\alpha = \frac{d\rho}{\rho\, dT}$ (α = ?) $\frac{E}{J} = \frac{VA}{i\ell}$ = Temp. coef. of resistivity

d) $R = \rho \frac{\ell}{A} = \frac{E\ell}{JA}$

C) OHMS LAW ($V = iR$) & SUCH

1) $\frac{dq}{dt} = i = -\sigma A \frac{dV}{dx}$ (σ = material's conductivity)

2) $V_d = a\left(\frac{\lambda}{\bar{V}}\right) = \frac{eE\lambda}{m\bar{V}}$ (λ = mean free path) → ave. distance between electron collisions

$a = \frac{eE}{m}$

3) $\rho = \frac{m\bar{V}}{ne^2\lambda}$

D) ENERGY TRANSFERS IN AN ELECTRIC CIRCUIT

1) P(rate of energy transfer) $= \frac{dU}{dt} = iV_{ab} = i^2R = \frac{V^2}{R}$

2) $1 \frac{\text{VOLT}}{\text{AMP}} = 1 \frac{\text{JOULE}}{\text{SEC}}$

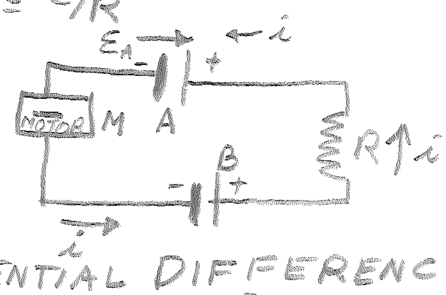
VII. ELECTROMOTIVE FORCE & CIRCUIT (CHAPT. 32)

A) ELECTRMOTIVE FORCE = emf $= \mathcal{E} = dW/dq$

B) CALCULATING CURRENT

1) $i = \mathcal{E}/R$

2)



$\mathcal{E} = iR$

C) POTENTIAL DIFFERENCES

$$V_{ab} = \mathcal{E}\frac{R}{R+r}$$ (R & r are resistances)

D) MULTI-LOOPED CIRCUITS

(KIRKOFF'S LAWS. SEE E. SCI. I NOTES

E) RC CIRCUITS

1) $\mathcal{E} = iR + \frac{q}{C} = R\frac{dq}{dt} + \frac{q}{C}$

2) $q = C\mathcal{E}(1 - e^{-t/RC})$

3) $i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{\frac{-t}{RC}}$

4) $iR = \frac{-q}{C}$

5) $q_0 = q_0 e^{-t/RC}$

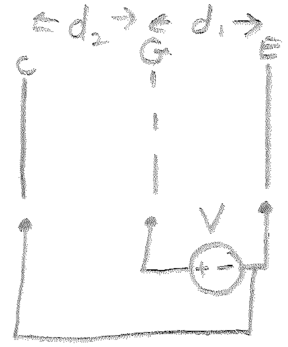
6) $i = (-q_0/RC)\, e^{-t/RC} = \frac{-\mathcal{E}}{R}e^{\frac{-t}{RC}}$

BOB MARKS

1

2

5)



$$\Delta K_{EG} = \frac{1}{2} m_e v_G^2 = F_{EG} d_1$$

$$F_{EG} = \frac{m_e v_G^2}{2d_1}$$

$$a_{EG} = \frac{v_G^2}{2d_1}$$

SINCE a_{EG} IS CONSTANT & $v_0 = 0$

$$\frac{v_G}{t} = \frac{v_G^2}{2d_1} \Rightarrow v_G = \frac{2d_1}{t}$$

$$\Delta K_{GC} = \frac{1}{2} m_e (v_G^2 - v_C^2) = F_{GC} d_2$$

$$F_{GC} = \frac{m_e (v_G^2 - v_C^2)}{2d_2}$$

$$a_{GC} = \frac{v_G^2 - v_C^2}{2d_2}$$

LABEL YOUR ANSWERS PLAINER

SINCE a_{GC} IS CONSTANT

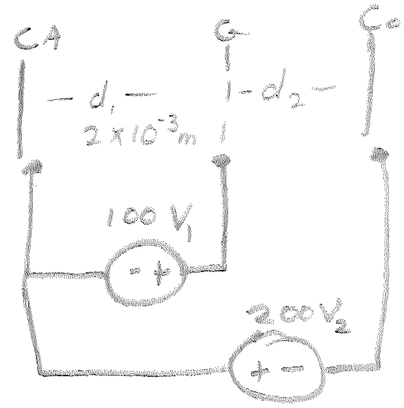
$$\frac{v_C - v_G}{t_2} = \frac{v_G^2 - v_C^2}{2d_2}$$

$$\frac{-1}{t_2} = \frac{(v_G + v_C)}{2d_2}$$

$$v_G = \frac{2d_2}{t_2} - v_G$$

$$= \frac{2d_2}{t_2} - \frac{2d_1}{t_1}$$

12)



a) $F_{CAG}\, d_1 = Q_e V$

$\vec{F} = \frac{Q_e V_1}{d_1} = \frac{(1.6\times10^{-9})\,N\cdot C^2/m^2 \cdot 100\,V}{2\times10^{-3}\,m} = 8.0\times10^{-15}$ N

(TOWARD GRID)

b) $m_e a = \frac{F}{d_1} = \frac{q_e V}{d_1}$ $\qquad m_e a_e = F_e$

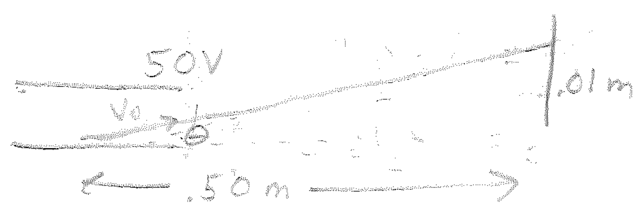
$dv = \frac{q_e V}{d_1 m_e}\, dt$ $\qquad F_e d_1 = \frac{1}{2} m_e v_G^2$

$v = \frac{q_e V}{d_1 m_e}$ $\qquad \frac{2F_e d_1}{v_G} = \frac{F_e}{d_1}$ $\qquad \frac{2F_e d_1}{v_G^2} a_e = F_e$

$=$

$v_G^2 = 2\, d_1 a_e$

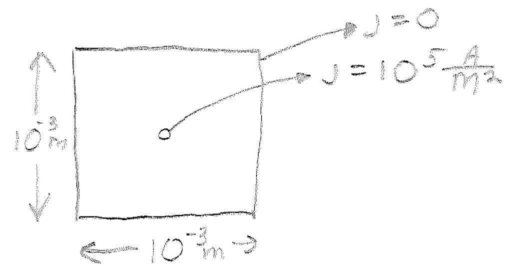
19)

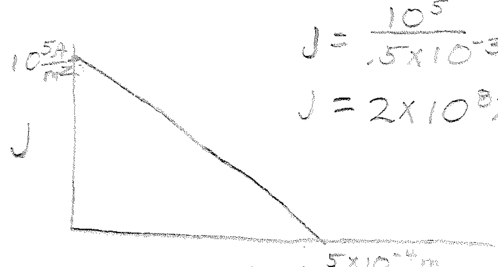


BOB MARKS

(3)

(13) 1





x = distance from center to outer edge's midpoint

$J = \frac{10^5}{.5\times10^{-3}} x$

$J = 2\times10^8 x$

$A = 4x^2$

$dA = 8x\,dx$

$J\,dA = 2(10)^8 x\, 8x\,dx$

$= 16(10^8)x^2 dx$

$\int J\,dA = \frac{16}{3} 10^8 x^3$

$j = \iint J\,dA = (\frac{4}{3})(10^8)x^4 \Big|_0^{5\times10^{-4}}$

$j = (\frac{4}{3})(10^8)(6.25\times10^{-16})$

$= 8.35\times10^{-8}$ A

$= 8.35\times10^{-6}$ A

(15) 2

a) $\frac{dx}{dt} = 3\times10^7 \frac{m}{sec}$

$\frac{q}{x} = \frac{10^{11}e}{m}\frac{(-1.62\times10^{-19})C}{e} = -1.62\times10^{-8}\frac{C}{m}$

$q = -1.62\times10^{-8}x$

$\frac{dq}{dx} = -1.62\times10^{-8}$

$j = \frac{dq}{dt} = \frac{dq}{dx}\frac{dx}{dt} = (-1.62\times10^{-8})(3\times10^7) = -.486$ A ✓

b) $W = \frac{1}{2}mv^2 = eV$

$V = \frac{mv^2}{2e} = \frac{(9.11\times10^{-31})(3\times10^7)^2}{-3.2\times10^{-19}} = 25.6\times10^2$

$= 2560$ V ✓

c) $P = jV$

$= (.486A)(2560V) = 1240$ W $= 1.24$ Kw

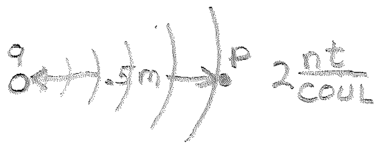
(1.24) KW $(2$ HR$) = 2.48$ KwHr ✓

R. MARKS

CHAPT. 27; Pg 681-4

(9)

5)



$E = \frac{q}{4\pi\epsilon_0 r^2}$

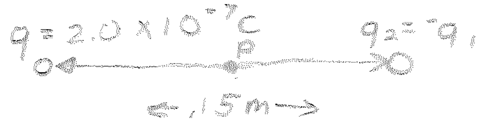
$q = 4\pi\epsilon_0 r^2 E$

$= 4\pi(8.85\times10^{-12} \frac{c^2}{m^2 nt})(.25m^2)(2\frac{nt}{c})$

$= 55.6 \times 10^{-12}\,C$

$= 5.56 \times 10^{-11}\,Coul$

6)



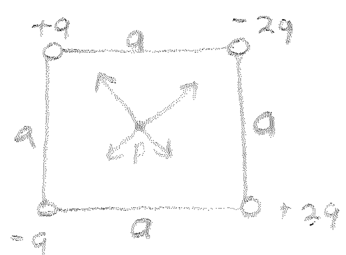
$E_1 = \frac{q}{4\pi\epsilon_0 r^2}$

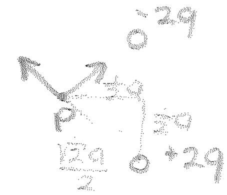
$= \frac{2\times10^{-7}\,c^2 \quad nt\,m^2}{4\pi(8.85\times10^{-12}\,c^2)(7.5)^2\times10^{-4}\,m^2}$

$= 3.19\times10^{-6}\,nt$

$E_T = 2E_1 = 6.38\times10^{-6}\,\frac{nt}{c}$

(crossed out)





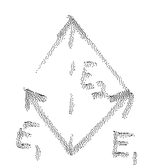
$E_{2q} = \frac{2q \quad 2}{4\pi\epsilon_0 a^2}$

$E_{1q} = \frac{q}{2\pi\epsilon_0 a^2}$

$E_1 = \frac{q}{\pi\epsilon_0 a^2}$

$E_4 = \frac{-\sqrt{2}\,q}{2\pi\epsilon_0 a^2}$

$E_2 = \frac{\sqrt{2}\,q}{4\pi\epsilon_0 a^2}$



$E_T = E_2 + E_1 = \frac{2\sqrt{2}\,q - \sqrt{2}\,q}{2\pi\epsilon_0 a^2}$

$= \frac{\sqrt{2}\,q}{2\pi\epsilon_0 a^2}$

$= \frac{\sqrt{2}\times10^{-8}\,C^2 \quad nt\,m^2}{2\pi(8.85\times10^{-12}\,C^2)25\times10^{-4}}$

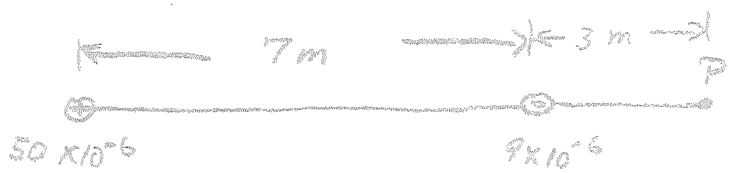
$= 51$

OV

Bob Marlow

Physics III Quiz (4)

Two charges $+50 \times 10^{-6}$ coulombs and -9×10^{-6} coulombs are located as indicated below.



A) Find the electric field $\vec{E}$ at P due to these two charges (magnitude & direction of $\vec{E}$)

$$E_1 = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$E = E_1 + E_2 = \frac{-9 \times 10^{-6}}{4\pi\varepsilon_0 9} + \frac{50 \times 10^{-6}}{4\pi\varepsilon_0 100}$$

4

$$= 9 \times 10^{9} \times 10^{-6} + 9 \times$$

$$= 9 \times 10^{9}(-10^{-6} + .50 \times 10^{-6})$$

$$= 9 \times 10^{3} \frac{nt}{coul} \quad 9 \times 10^{9}(1.5 \times 10^{-6})$$

$$E_p = 13.5 \times 10^{3} \frac{nt}{coul} \text{ to right}$$

B) Find the potential at P due to these two charges

$$V = \frac{q\, 2a \cos\theta}{4\pi\varepsilon_0 r^2}$$

$\frac{v}{m}$ V

0

$$F = Eq = 13.5 \times 10^{3} \frac{nt}{coul}$$

$$\Sigma V = V_1 + V_2 = (9 \times 10^{9})\left(\frac{50 \times 10^{-6}}{100}\right) +$$

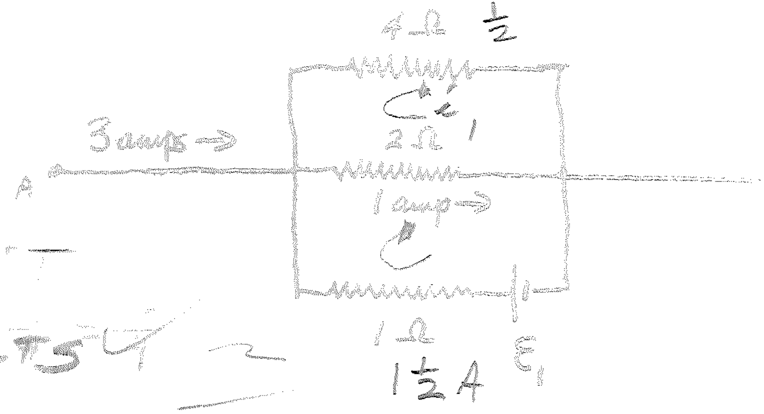
$$V = QEL$$

$$V = \left[(50 \times 10^{-6})\left(\frac{50 \times 10^{-6}}{4\pi\varepsilon_0 9}\right)(10) + (9 \times 10^{-6})\left(\frac{9 \times 10^{-6}}{4\pi\varepsilon_0 9}\right)(3)\right] \text{ VOLTS}$$

BOB MARKS
NAME

(5)

The drawing opposite shows only part of a complete circuit. A current of 1 ampere is flowing in the 2 Ω resistor



A) What is the potential difference between the two ends of the 2 Ω resistor?

$V = iR$

$V = 2$ VOLTS

B) At what rate is energy being dissipated in the 2 Ω resistor?

~~$V_A - V_B$~~ 1 AMP = 2 ~~joules~~ watts

C) What is the current in the 4 Ω resistor?

$-2 + 4 i_1 = 0 \quad i_1 = \frac{1}{2}$ A

D) What is the emf $\mathcal{E}_1$ of the battery shown in the diagram.

~~JV = E~~ ~~$-2V + 1\frac{1}{2}V = \mathcal{E}_T$~~ ~~$\mathcal{E}_1 = -\frac{1}{2}V$~~

$\mathcal{E}_1 = \left(\frac{3}{2}\right)(R)$

~~$\frac{1}{R}$~~ $= \frac{1}{4} + \frac{1}{2} + 1 \Rightarrow R = \frac{4}{7}$

$\mathcal{E}_1 = \frac{12}{14} = \frac{6}{7}$ VOLTS

E) At what rate is energy being supplied to the battery shown in the diagram.

$JV = E$

$(1\frac{1}{2}V)$

$\frac{dE}{dt}$

Physics 111 Lab quiz
Fall, 1969

Name Bob MARKS
Section: Day of week FRIDAY
(Circle one) a.m. p.m.

Instructions – Show method, assumptions and approximations made.
Answer three of the five items below.

1. A quantity x is observed $N(x)$ times, as shown:

x	$N(x)$	x	$N(x)$	x	$N(x)$
1	1	7	8	13	0
2	3	8	9	14	2
3	2	9	8	15	1
4	3	10	4	16	0
5	5	11	6	17	0
6	8	12	4		3
	22		39		

For this distribution compute $\bar{x}$ and the standard deviation. Give your own careful interpretation of these two quantities.

2. (a) In the pendulum experiment did you observe a significant dependence of frequency of a system upon amplitude?
Would you expect to find such a dependence if your precision of measurements were increased?
If so, to what would the dependence be due?

(b) Deduce the dependence (or lack of dependence) of the period of a simple pendulum upon its weight – from Newton's laws.

3. (a) What is the effect, if any, of increased damping on the amplitude of a mechanical oscillator at resonant frequency?

Explain in terms of mechanisms of energy transfer.

(b) Given an oscillator with natural frequency $\omega = \sqrt{k/m}$, how does damping effect the observed frequency, if at all?

4. Given $G \cong 6.7 \times 10^{-11}$ n-m²/kg², *estimate* the variation of "g" between the banks of the Wabash (550 ft above sea level) and the top of Rose Poly's proposed new water tower (635 ft above sea level). Hints – earth mass is 6×10^{24} kg; your weight may be expressed as (mg) or Gmm'/r^2.

Could our "free fall" apparatus provide an instrument to detect the above variation? Explain.

5. ~~Deduce~~ the orthogonality of ~~lines of electric~~ field and ~~surfaces of equipotential~~.

4VE - 60

59

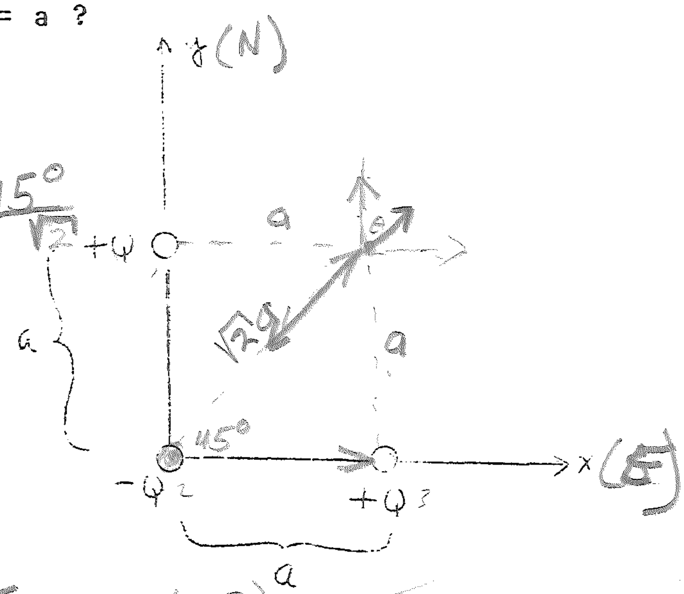
Physics III Test 1 October 24, 1969

Name ROBERT MARKS

BE SURE TO SHOW HOW YOU ARRIVE AT YOUR ANSWER

I.

a) In the figure below what is the magnitude and direction of E at the point x = y = a ?



$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E_y = E_x = \frac{Q}{4\pi\varepsilon_0 a^2} - \frac{Q \sin 45^\circ}{4\pi\varepsilon_0 2a^2}$$

$$= \frac{Q}{4\pi\varepsilon_0 a^2} - \frac{Q}{8\pi\varepsilon_0 a^2\sqrt{2}}$$

$$= \frac{(2\sqrt{2}-1)Q}{8\pi\varepsilon_0 a^2\sqrt{2}}$$

$$E = \frac{(2\sqrt{2}-1)Q}{2(4\pi\varepsilon_0 a^2)}$$ 45° E of N (↗)

Take $Q = 3 \times 10^{-10}$ coul., $a = \sqrt{2}$ m., $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\ \frac{\text{nt-m}^2}{\text{coul}^2}$

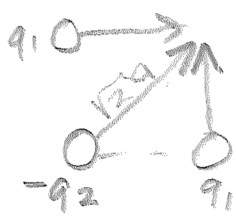
$$E = \frac{(1.83)(3 \times 10^{-10})(9 \times 10^{9})}{(2)(2)} =$$

20

$$E = 1.23\ \frac{V}{m} = 1.23\ \frac{nt}{coul}$$

24

b) What value of the negative charge will give a field of E = 0 at x = y = a for positive charges as given above?



$$E_y = E_x = 0 = \frac{q_1}{4\pi\varepsilon_0 a^2} ? \frac{q_2}{4\pi\varepsilon_0 2a^2}$$

equate $\vec{E}_- = \vec{E}_+$

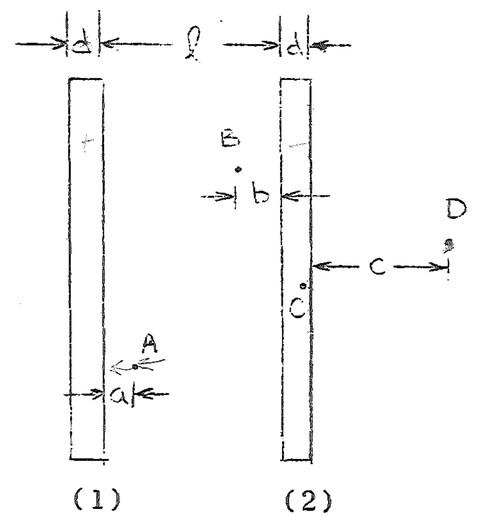
4

$$\frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{q_2}{4\pi\varepsilon_0 2a^2}$$

$$q_2 = 2q_1 \Rightarrow$$

$Q_- = 2\sqrt{2}\, Q_+$

II. Two large metal plates face each other as shown in the figure. The surface charge densities are + 4.0×10^{-6} coul/m^2 and $- 4.0 \times 10^{-6}$ coul/m^2 on plates (1) and (2) respectively. (Circle the answer which is correct. All answers for E have the dimensions nt/coul.)



a = 0.25 cm d = 0.35 cm
b = 0.35 cm l = 1.0 cm
c = 2.0 cm

$$\frac{1}{4\pi E_0} = 9 \times 10^9 \ \frac{\text{nt-m}^2}{\text{coul}^2}$$

$$E_0 = 8.85 \times 10^{-12} \ \text{coul}^2/\text{nt-m}^2$$

a) The value of E at point A is

1) 2.26×10^{-7}
2) 9.04×10^{-7}
3) 4.52×10^{-7}
4) 1.44×10^{-5}
5) 0

b) The value of E at point B is

1) 2.26×10^{-7}
2) 9.04×10^{-7}
3) 4.52×10^{-7}
4) 1.03×10^{-5}
5) 0

c) The value of E at point C is

1) 2.26×10^{-7}
2) 9.04×10^{-7}
3) 4.52×10^{-7}
4) 6.54×10^{-5}
5) 0

USE BACK OF PAGE 1 FOR WORK AREA

d) The value of E at point D is

1) 2.26×10^{-7}
2) 9.04×10^{-7}
3) 4.52×10^{-7}
4) 1.80×10^{-6}
5) 0

(10)

III. Two charges one + and one - are located with respect to a point P as indicated in fig. 1.

a) Calculate the potential at point P due to these two charges (12)

$V = Ed = V_A + V_B$

$V_A = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{(26\times10^{-6})(9\times10^{9})}{13^2} = 1.38\times10^3 \frac{V}{m}\ 13\,m = 18.0\times10^3\ V$

$V_B = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{(5\times10^{-6})(9\times10^{9})}{1^2} = 45\times10^3 \frac{V}{m}(1\,m) = 45.0\times10^3\ V$

$V = 45.8\times10^3\ V$

b) A third charge of 1×10^{-6} coulombs is placed at P. This charge is now moved by some external agent from P to the point P1. At each point of its path (dotted line) it is acted on by a resultant force due to the other two charges. How much work is done by this resultant force as the 1×10^{-6} coulomb charge is moved from P to P1 (12)

$V = W_{AB}/q_0$

$W_{AB} = Vq_0$

$V_{P_1} = -\frac{(26\times10^{-6})(9\times10^{9})}{13}$

$= -1.80\times10^3\ V$

$V_{P_2} = \frac{(5\times10^{-6})(9\times10^{9})}{5}$

$= 9\times10^3\ V$

$V = 27\times10^3\ V$

$W_{AB} = (27\times10^3)(10^{-6})$

$W = 27\times10^{-3}$ V coul

3

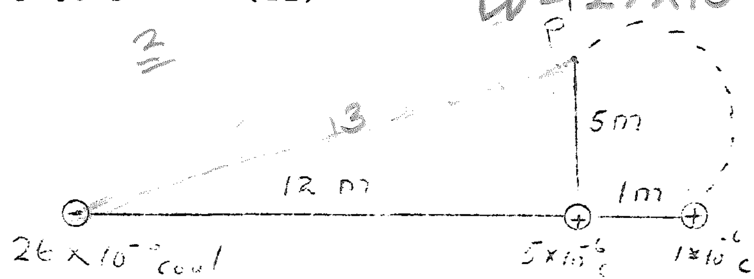


Fig 2

c) What is the potential energy of the system consisting of the two charges shown in Fig. 1. (11)

$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$

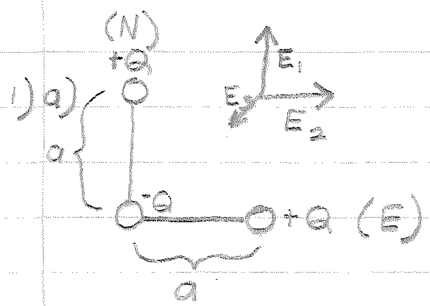
$= \frac{(26)(5)(10^{-12})(9\times10^{9})}{12}$

7

$= 97.5\times10^{-3}$ V-coul.

ANS. TEST 1

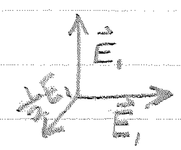
1) a)



$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$|\vec{E}_1| = |\vec{E}_2| = \frac{Q}{4\pi\varepsilon_0 a^2}$$

$$|\vec{E}_3| = \frac{Q}{4\pi\varepsilon_0(\sqrt{2}a)^2} = \frac{Q}{8\pi\varepsilon_0 a^2} = \frac{1}{2}|\vec{E}_1|$$



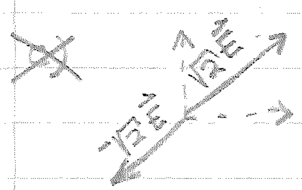
$$|\vec{E}_1 + \vec{E}_2| = \sqrt{2}\,|\vec{E}_1|$$

$$|\vec{E}_3 + \sqrt{2}E_1| = \left(\sqrt{2} - \frac{1}{2}\right)|\vec{E}_1|$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{\left(\sqrt{2} - \frac{1}{2}\right)Q}{4\pi\varepsilon_0 a^2}$$ AT 45° N OF E

b) $\left(\sqrt{2} - \frac{1}{2}\right) = .91$

$$\vec{E} = \frac{(.91)(3\times10^{-10})(9\times10^9)}{2} = 1.23 \frac{nt}{coul}$$



$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{(3\times10^{-10})(9\times10^9)}{2}$$

$$= 1.35$$

$$\sqrt{2}E = 1.91 \frac{nt}{coul}$$

$$1.91 = \frac{Q_1}{4\pi\varepsilon_0(\sqrt{2}a)^2}$$

$$Q_1 = \frac{(1.91)(2)(2)}{(9\times10^9)} = .849\times10^{-9}$$

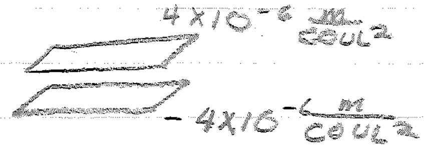
$$= 8.49\times10^{-10}$$

c) $E_y = E_x = 0$

$^{+}E_x = \frac{Q}{4\pi\varepsilon_0 a^2} = {}^{+}E_y \qquad \vec{E}_{+} = \frac{\sqrt{2}\,Q}{4\pi\varepsilon_0 a^2}$

$^{-}E_x = \frac{Q_2}{4\pi\varepsilon_0 2a^2} = \frac{\sqrt{2}\,Q}{4\pi\varepsilon_0 a^2}$

$Q_2 = {}^{-}2\sqrt{2}\,Q$

II) 

a) #3
b) #3
c) #5
d) #5

III) $V = Ed$

$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}$

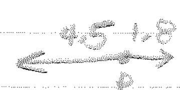
$V_1 = \frac{q_1 d}{4\pi\varepsilon_0 r^2}$

$= \frac{(26\times10^{-6})(9\times10^{9})}{13}$

$= 1.8\times10^{4}\ V$

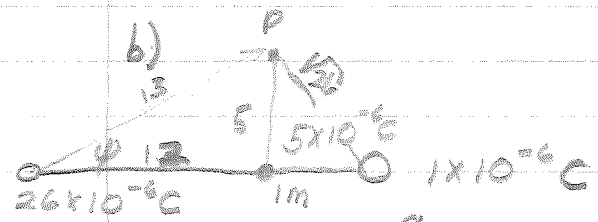
$V_2 = \frac{(5\times10^{-6})(9\times10^{9})}{1}$

$= 45\times10^{3}$



$V = {}^{-}2.70\times10^{4}\ V$

b)



$$V_{1x} = \frac{q_1}{4\pi\varepsilon_0 r}\cos\varphi$$

$$= \frac{(26\times10^{-6})(9\times10^{9})}{13}\left(\frac{12}{13}\right) = 16.1\times10^{3}\ V$$

$$V_{2x} = 0 \quad V_{3x} = \frac{(1\times10^{-6})(9\times10^{9})}{\sqrt{26}}\frac{1}{\sqrt{26}} = .346\times10^{3}\ V$$

$$V_{1y} = \frac{(26\times10^{-6})(9\times10^{9})}{13}\left(\frac{5}{12}\right) = 7.5\times10^{3}\ V$$

$$V_{2y} = \frac{(5\times10^{-6})(9\times10^{9})}{5} = 9\times10^{3}\ V$$

$$V_{3y} = \frac{(10^{-6})(9\times10^{9})}{\sqrt{26}}\ 5 = 8.82\times10^{3}\ V$$

$V_x = 16.5\ V$ $\qquad V_y = 25.3\ V$

c) $U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$

$$= \frac{(26\times10^{-6})(5\times10^{-6})(9\times10^{9})}{12}$$

$$= 975\times10^{-3}\ V\,\text{coul}$$

(60)

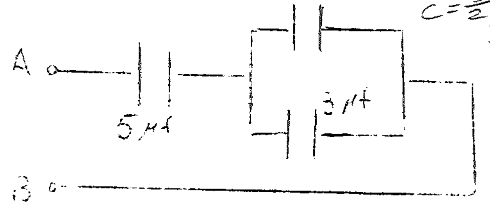
November 14, 1969 Name BOB MARKS

Physics III Test 2

(21)

In problem I circle correct answers.

I. A. The equivalent capacitance of the arrangement of capacitors shown in the figure opposite is in microfarads.



1) 10. 2) 11. 3) 6.2 4) 3.875

5) 2.50 6) 4.43 7) 30

A parallel plate capacitor has a dielectric completely filling the space between the plates. The area of one surface of each plate is 0.01 m^2. The dielectric constant of the dielectric is 2.0. When a 50 volt battery is connected to the two plates, the total free charge on each plate is found to be 30 x 10^{-6} coulombs. (Note ϵ_0 = 8.85 x 10^{-12} $coul^2/n.m^2$)

B. The capacitance of this parallel plate capacitor is

1) 0.6 μf 2) 1.2 μf 3) 150 μf 4) 75 μf 5) 300 μf

6) 16.6 μf

C. The energy stored in this capacitor is (in joules)

1) 1.5 x 10^{-3} 2) 7.5 x 10^{-4} 3) 2.65 x 10^{-2} 4) 1.47 x 10^{-15}

5) 4.42 x 10^{-10} 6) 8.43 x 10^{-2} 7) 0.6 x 10^{-5}

D Which of the following is not true.

1) The polarization $\vec{P}$ is defined as the electric dipole moment per unit volume.

2) For most materials $\vec{P}$ is proportional to the electric field $\vec{E}$

3) The polarization $\vec{P}$ is zero at every point in a vacuum.

4) For a given electric field, the polarization will be greater, the greater the dielectric constant of the material.

5) The polarization vector is sometimes referred to as the electric displacement vector.

E The capacitance of the parallel plate capacitor shown opposite is

1) $\dfrac{K \epsilon_0 S}{d}$

2) $\dfrac{\epsilon_0 S}{Kd + a + b + c}$

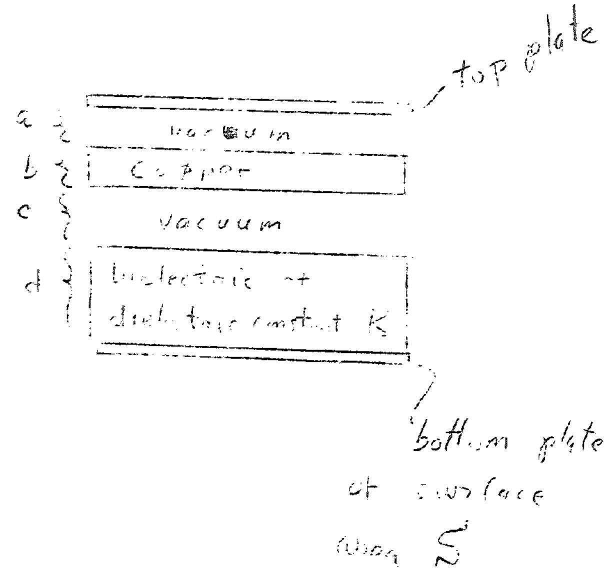
3) $\dfrac{\epsilon_0 S}{\frac{d}{K} + a+b+c}$

4) $\dfrac{\epsilon_0 S}{\frac{d}{K} + a + c}$

5) $\dfrac{\epsilon_0 S b}{a+c+\frac{K}{d}}$

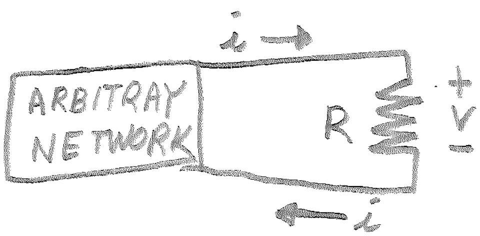
6) $\dfrac{\epsilon_0 S (a+c)}{\frac{a+c}{K} + d}$

7) $\dfrac{K \epsilon_0 S}{a+c+d}$



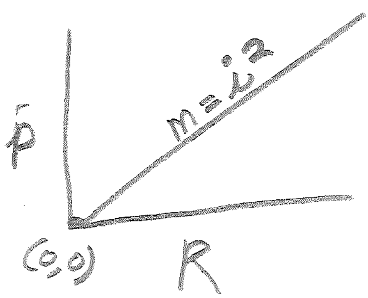
2) The power dissipated by a resistor is given by $P = i^2R$ or $P = V^2/R$. How does P change if R is increased or decreased? Clearly justify your answer. Explain all apparent inconsistancies or ambiguities.

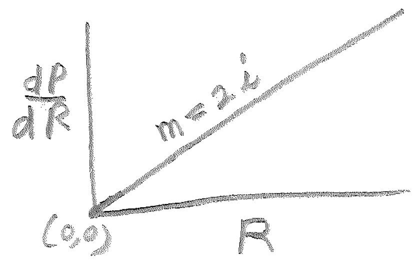
1)



$P = i^2 R$

$\frac{dP(i)}{dR} = \cancel{2 i R} = i^2$



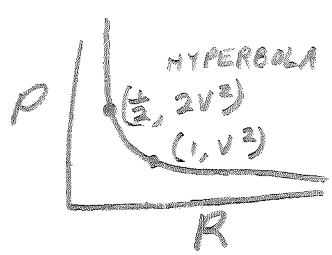


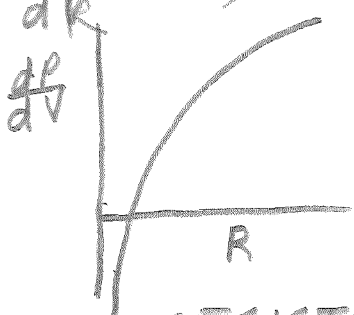
29

$P \propto R \Rightarrow P = KR \Rightarrow K = i^2$

ERGO, UNDER A CONSTANT CURRENT i THE POWER IS PROPORTIONAL TO THE RESISTANCE IDEALLY

2) $P = V^2/R$ $\quad \frac{dP(v)}{dR} = \cancel{V^2 \ln R} = -\frac{V^2}{R^2}$





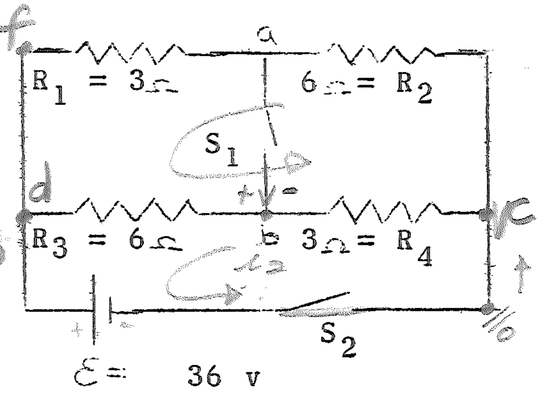
THE POWER IN A RESISTOR IS INVERSLY PROPORTIONAL TO THE RESISTANCE UNDER CONSTANT VOLTAGE $P \propto \frac{1}{R}$ OR $PR = K \Rightarrow K = V^2$

III. For the circuit shown in the figure find the following

A. What is the potential difference $V_a - V_b = V_{ab}$ when the switch S_1 is left open and S_2 is closed?

10

(8 points)



$6(i_1 - i_2) + 3i_1 + 6i_1 + 3(i_1 - i_2) = V_b$

$9(i_1 - i_2) + 9i_1 = 0$

$18i_1 - 9i_2 = V_b$

$6(i_2 - i_1) + 3(i_2 - i_1) + 36 = -V_b$

$9i_2 - 9i_1 = -(36 + V_b)$

$V_c + \frac{V_c - 36}{3} + \frac{V_c - 36}{9} = 0$

$36 + \frac{36 - V_c}{6} + \frac{36 - V_c}{9} = 0$

$V_{ab} = 36 - V_b$

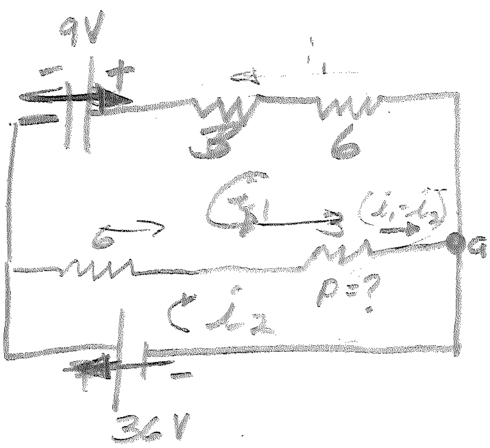
0

V_{ab} =

3 equations with 3 unknowns
solve for V_b, then V_{ab}

B. If S_1 is left open, and R_1 is replaced by a battery with its positive terminal connected to a and with an emf of 9 v, what is the power dissapation in R_4 after S_2 is closed?

(12 points)



8

P_4 =

$9 + 9i_1 + 9(i_1 - i_2) = 0$

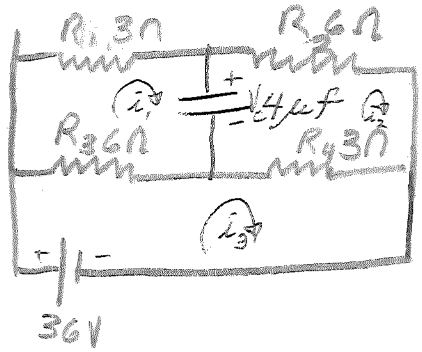
$9(i_2 - i_1) = 36$

$V_a = \frac{-9}{i_1} + 36$?

solve for $(i_1 - i_2)$ and i, and multiply by V_a

C. If switch S_1 is replaced by a capacitor $C = 4\ \mu f$, and the resistors R_1, R_2, R_3, and R_4 are in their original positions, what is the value of the charge q which appears on C sometime after switch S_2 has been closed?

(5 points)



$C = \frac{q}{V}$

2

$q = V_c(4\mu f)$

$3i_1 + V_c + 36(i_1 - i_3) = 0$

$6i_2 - V_c + 3(i_2 - i_3) = 0$

$-36 + 36(i_3 - i_1) + 3(i_3 - i_2) = 0$

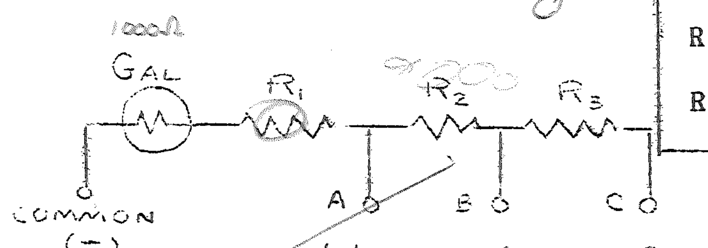
solve for V_c

$4\mu f = \frac{q}{V_c}$

IV. The internal wiring of a multirange voltmeter is shown in the figure. The galvanometer has a resistance of 1000 Ω and will give a full scale deflection when a current of 1×10^{-3} amp passes through it. If the connections at A, B, and C are for full-scale voltages of 1 volt, 10 volts, and 100 volts, respectively, what are the values for R_1, R_2, and R_3? (A full-scale voltage will give a full-scale deflection on the galvanometer.)

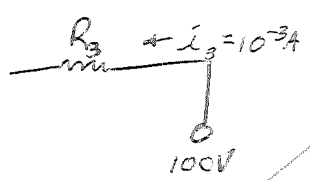
(10 points)

0



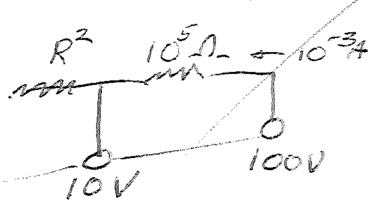
0

R_1 = ~~X~~ 0
R_2 = ~~X~~ 9000
R_3 = ~~$10^5\Omega$~~ 90000



$R_3 = \frac{V_3}{i_3} = \frac{100}{10^{-3}} = 10^5\Omega$

$V = 10^{-3}10^5 = 100$



$\frac{1V}{10^{-3}}$

000

(16) BOB MARKS

5

[illegible]

... $\vec{E} = 6.1 \times 10^{20}$ ... [illegible] ...

b) On the figure, assuming the electron ... out of the plane of the drawing ... [illegible]

ORIENTATIONS OF E AND B

GIVEN USED:

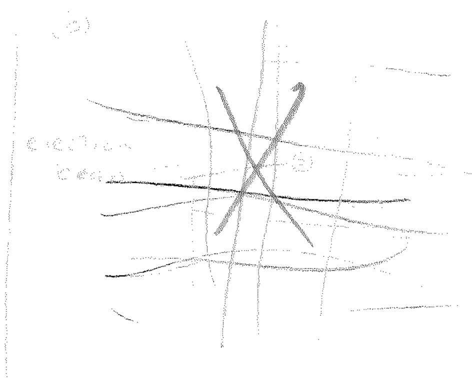
$e = 1.6 \times 10^{-19}$ coul

$m_e = 9.1 \times 10^{-31}$ kg

10^4 gauss = 1 weber/m²

1 eV = 1.6×10^{-19} joule

(b)



$F = qE + q_0 V \times B$

$\bar{F} = q_0 E + q_0 V \times B$

if $\bar{F} = 0$, $-E = V \times B$ ✓

~~$-6.1 \times 10^{20} = 1.422 \times 10^6 (B) \sin 90°$~~

~~$B = 4.3 \times 10^{14}$ COUL.~~

~~$B = 4.3 \times$~~

~~$B = 2.86 \times 10^{33}$~~

$6.1 \times 10^{20} = (1.42 \times 10^6)$~~$(1.6 \times 10^{-19})$~~$B \sin 90°$

$B = 2.86 \times 10^{33}$

(10)

$B = \mu_0 i n$

... THE DIRECTION OF THE CURRENT ... WHAT WILL BE THE DIRECTION OF $\vec{B}$ INSIDE THE ...

To the right X O

2. ... THE FIELD AT EVERY POINT INSIDE THE SOLENOID HAS THE SAME MAGNITUDE B_0 AND SAME DIRECTION (the direction being that deduced in 1). Assume the field outside the solenoid is zero. What is the value of the following two line integrals? In both cases the path is the straight (dotted) line connecting the two points

$\int_A^B \vec{B}\cdot\vec{dl} = 0$ ✓ 4

$\int_A^C \vec{B}\cdot\vec{dl} =$ ~~B_A~~ $B \sin\theta \int_A^C dl$

$= B \sin\theta \, (C-A)$ 5

What does this mean

3) AMPERE'S LAW STATES THAT $\oint \vec{B}\cdot\vec{dl} = \mu_0 i_{ENT}$. WHAT IS THE ... OF i_{ENT} FOR THE CLOSED DOTTED PATH ACDGA?

$\oint B\cdot dl =$

$\int_A^C B\cdot dl + \int_C^D B\cdot dl + \int_D^G B\cdot dl + \int_G^A B\cdot dl$

(0 above the second and third terms)

~~$\int_A^C B\cdot dl = \int_A^C B\cos(\tfrac{\pi}{2}-\theta)\,dl$~~

$\int_G^D B\cdot dl = -BA$

~~$= B\sin\theta(C-A) + B(D-G)$~~

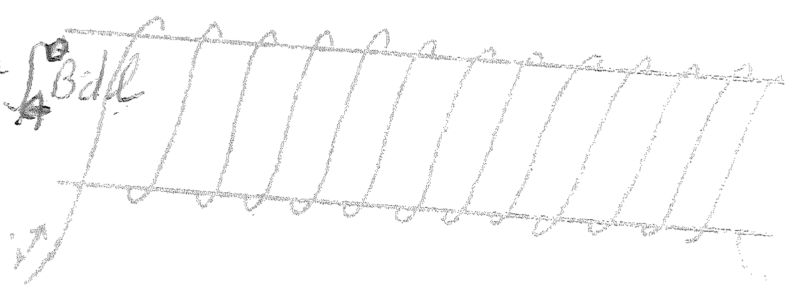
~~$= B[(\sin\theta)(C-A) + (D-G)]$~~

~~$B[(\sin\theta)(C-A) + D - G]$~~

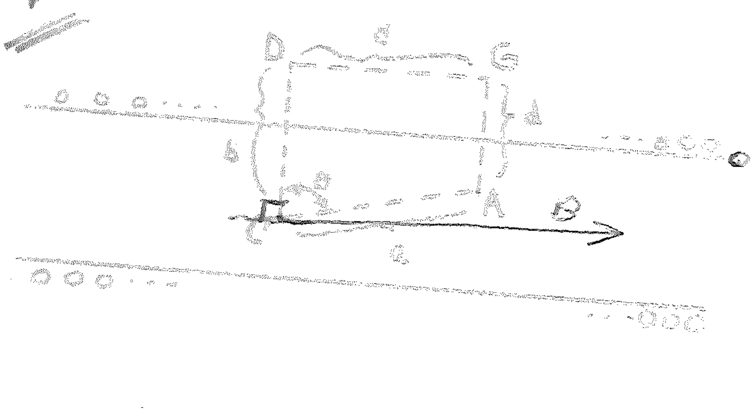
~~$\mu_0 n$~~

$B\sin\theta(C-A) - BA = \mu_0 i$

$i = \dfrac{B[(C-A)\sin\theta - A]}{\mu_0}$



1

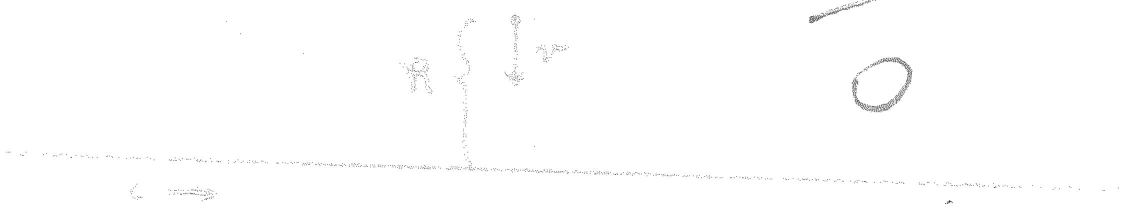


... WIRE ...

... DIRECTLY TOWARD THE ...

INDICATED. WHAT IS THE MAGNITUDE AND DIRECTION

OF THE FORCE ON THE ELECTRON AT THE INSTANT



$V = \frac{a}{2}t^2 + V_0$

$a = \frac{2(V - V_0)}{t}$

$B = \frac{\mu_0 \mu}{4\pi\varepsilon_0 x^3}$

$B = \frac{\mu_0 i}{2\pi R}$

$F = i l B$

$r = \frac{mv}{q_e B}$

A SINGLE LOOP OF WIRE OF RADIUS R [illegible] UNIFORM MAGNETIC FIELD OF MAGNITUDE B_0. THE [illegible] OF THE FIELD IS PARALLEL TO THE PLANE OF THE LOOP AS INDICATED. WHAT IS THE MAGNETIC [illegible] THROUGH THE AREA OF THE LOOP.

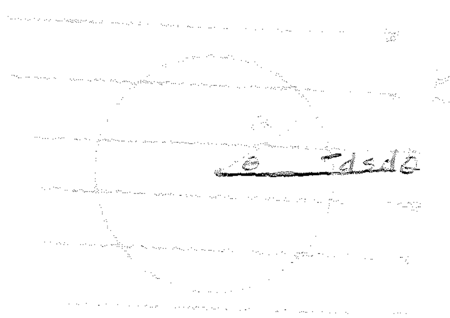
$\Phi_B = \int B \cdot ds$

$= R\int_0^{2\pi} B_0 \, d\theta$ $\quad s = r\theta$

$ds = R\,d\theta$

$= B_0 R 2\pi$

0



$\oint B \cdot dl = \mu_0 i$ $\quad dl_1 = b\,d\theta$

$\oint_0^{2\pi} B \cdot a\,d\psi + \oint B\,b\,d\theta$ $\quad dl_2 = a\,d\psi$

$2\pi B(b-a) = \mu_0 i$

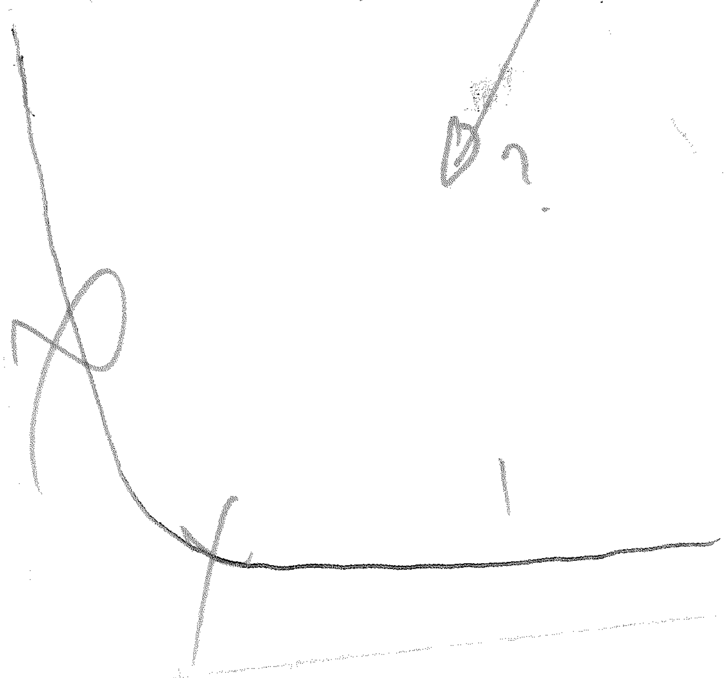
$B = \dfrac{\mu_0 i}{2\pi(b-a)}$ r dependence?

B = const. ?

contour?

the graph below sketch B vs r

$B = \dfrac{\mu_0 i}{2\pi R}$



B ?

(1)

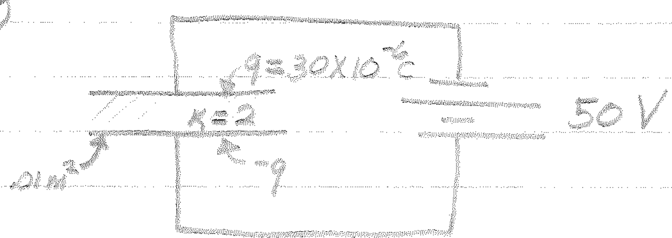
ANS. TEST II

I) A) $2 + 3 = 5$

$$\frac{1}{C} = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

$$C = 5/2 = 2.5\ \mu f$$

~~B)~~



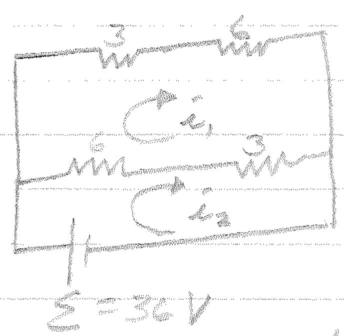
$$V_d = V_0/K$$

$$C = K\epsilon_0 A/d$$

$$\frac{V_0}{V} = K$$

$$C = \frac{K\epsilon_0 A}{d}$$

III)



$$9i_1 + 9(i_1 - i_2) = 0$$

$$18i_1 = 9i_2$$

$$2i_1 = i_2$$

$$9(i_2 - i_1) + 36 = 0$$

$$9i_2 - 9i_1 = -36$$

$$18i_1 - 9i_1 = -36$$

$i_1 = 4$ AMPS

$i_2 = 8$ AMPS

$V_a = 12V \qquad V_B = 24V$

ETC.

I) THE MAGNETIC FIELD

A) GENERAL

1) $\vec{B}$ = MAGNETIC FIELD
2) $\Phi_B = \int \vec{B} \cdot d\vec{S}$ = MAGNETIC FLUX
3) $|\vec{B}| = \frac{F_\perp}{q_0 V}$
   a) $F_\perp$ = MAXIMUM FORCE
   b) q_0 = CHARGE OF PARTICLE
   c) V = VELOCITY
4) $\vec{F} = q_0 \vec{E}$
5) $\vec{B} = \frac{WEBER}{M^2} = \frac{NT\ SEC}{COUL\ M} = \frac{NT}{AMP\ M}$ } WEBER MEASURER OF Φ_B
6) 1 WEBER/M^2 = 10^4 GAUS
7) $\vec{F} = q_0 \vec{E} + q_0 \vec{V} \times \vec{B}$ IN MAGNETIC & ELECTRIC FIELD

B) MAGNETIC FORCE ON A CURRENT

1) IN A WIRE
   a) $F = i l B$
   b) $\vec{F} = i \vec{l} \times \vec{B}$
   c) $d\vec{F} = i\, d\vec{l} \times \vec{B}$

C) TORQUE ON A CURRENT LOOP

1) $\tau = N\tau' = NiAB \sin\theta$
   a) N = # OF TURNS
2) $\tau = p \times E$
   a) p = ELECTRIC DIPOLE MOMENT
   b) $p = 2qa$
3) μ = MAGNETIC DIPOLE UNIT
   a) $\mu = NiA$
   b) $\tau = \mu \times B$
   c) $U = -\mu \cdot B = -p \cdot E$

D) CIRCULATING CHARGES

1) $r = \frac{mv}{qB}$
2) $\omega = V/R = \frac{q\vec{B}}{m}$
3) ν = FREQ = $\frac{\omega}{2\pi} = \frac{qB}{2\pi M}$
4) $K = \frac{1}{2} m v^2 = \frac{q^2 B^2 R^2}{2m}$

II) AMPERE'S LAW ($\oint \vec{B} \cdot d\vec{l} = \mu_0 i$)

A) $\mu_0 = 4\pi \times 10^{-7} \frac{WEBER}{AMP\ MTR}$

B) 2 // CONDUCTORS

$$F_b = i_b l B_a = \frac{\mu_0 l i_b i_a}{2\pi d}$$

C) B FOR A SOLENOID

$B = \mu_0 i_0 n \Rightarrow n$ # OF TURNS PER UNIT LENGTH

D) FOR A TOROID

$$B = \frac{\mu_0 i_0 N}{2\pi r}$$

N = # OF TURNS

E) B FOR A WIRE

$$B = \frac{\mu_0 i}{2\pi R}$$

PROPERTY	DIPOLE TYPE	EQUATION
τ IN EXTERNAL FIELD	ELEC	$\tau = p \times E$
	MAG	$\tau = \mu \times B$
E IN EXT. FIELD	E	$U = -p \cdot E$
	M	$U = -\mu \cdot E$
FIELD AT DISTANT POINTS ALONG AXIS	E	$E = p / 2\pi\varepsilon_0 x^3$
	M	$B = \mu_0 \mu / 2\pi x^3$
FIELD AT DISTANT POINTS ALONG ⊥ BISECTOR	E	$E = p / 4\pi\varepsilon_0 x^3$
	M	$B = \mu_0 \mu / 4\pi\varepsilon x^3$

Physics III Lab. Fall 1969

Experiment 3

Forced Oscillations and Resonance

Purpose

To study forced oscillation and resonance of a mechanical system.

Reference

Study in H. & R. the linear harmonic oscillator (Sec. 15-2 and 15-3) and its rotational analog, the torsional pendulum (15-5).

Also study sec. 15-9 and 15-10. See H. & R., Chap. 35, section 3 and questions 2, 4, for discussion of damping mechanism used in this experiment.

The oscillating system to be observed consists of a round flat disc which rotates about its axis of (cylindrical) symmetry. The "axle" of the disc moves in low-friction bearings. A spring provides an elastic restoring torque τ (proportional to angular displacement θ from equilibrium). An electromagnet provides a damping torque proportional to the angular velocity $d\theta/dt$, providing in effect a variable amount of "friction". (Too much damping would "brake" the motion sufficiently to prevent oscillation.) You have seen references (above, text) to linear and rotational analogs. The equation of motion (15-6) of the linear oscillator, $d^2 x / dt^2 + (k/m)\, x = 0$, its periodic solution (15-8) and the period (15-10), have their analog, equations (15-22), (15-23), and (15-10). There is a one-to-one correspondence between the sets of physical quantities:

linear		angular	
x	$\longrightarrow$	θ	displacement
t	$\longrightarrow$	t	time
k	$\longrightarrow$	K	"elastic constant"
m	$\longrightarrow$	I	"inertial constant"

In our experimental study we need to carry the analogy further (Sec. 15-9 and 15-10). Thus b, damping const. $\longrightarrow$ B in the (electromagnetic) damping torque, $(-B\, d\theta/dt)$, and F_m, max. driving force $\longrightarrow$ τ_m, maximum driving torque, and ω'' driving freq. $\rightarrow \omega_d$, freq. of driving torque. The equation of motion (15-40) of the damped, driven linear oscillator becomes, for the damped, driven angular oscillator (our disc)

$$-K\theta - B\, d\theta/dt + \tau_m \cos\omega_d t = I\, d^2\theta/dt^2 . \qquad (1)$$

Eq'n. (1) asserts that the sum of the spring's restoring torque, $-K\theta$, the damping torque, $-B\, d\theta/dt$, and the periodic driving torque, $\tau_m \cos\omega_d t$, produces an angular acceleration, $d^2\theta/dt^2 = \alpha$, proportional to it:

$$\tau\,(\text{restor.}) + \tau(\text{damp.}) + \tau(\text{driving}) = I \cdot \alpha$$

I = constant

difference between the two periods (if any) in the direction predicted by theory? Is this difference too great to be explained by experimental error?.

II. A. With no current in the electromagnet start the disc from rest with a large initial amplitude x_0 and record peak positive displacements for the first six successive cycles. Repeat and average the corresponding peak displacements (amplitudes) A.

B. Repeat part A with the terminals of the damping electromagnet attached to the 32 volt supply.

C. On a single set of axes, plot graphs of ln A versus t from the data of the two preceding parts and determine the slopes at t = 0. Use these slopes to find out the approximate relaxation times t_{r1} and t_{r2}

Hint: $A = x_0 e^{-t/t_r}$

$$\ln A = (-1/t_r)t + \ln x_0$$

and y = ln A versus x = t should yield a straight line of slope $(-1/t_r)$ <u>if the damping torque is always proportional to the angular velocity</u>

III. A. Plug in the motor controller and allow it to warm up. With the damping terminals connected to the 24 volt supply and the driving rod connected to the driving lever arm record the driving frequency and amplitude of oscillation for various motor speeds. Note: The apparatus must be positioned so that the pointer swings just as far in one direction as in the other. The driving rod should be connected to the lever arm in such a way as to give a large (on scale) amplitude at resonance. Observe phase relations (ques. 4 below). The frequency may be determined by timing several cycles. Be sure to cover the range of amplitudes near resonance carefully; the amplitude should be measured at several frequencies close to the resonant (maximum amplitude) frequency on each side. Before leaving this part make sure that you have enough points to plot a reasonable curve without having to guess at what happens to the curve between points. How does the resonant frequency compare with the natural frequency determined in part I?

B. Repeat Part A with 48 volts on the damping electromagnet terminals. On one set of axes plot graphs of the amplitude versus the driving frequency for this and the preceding part. (See fig. 15-20, H. + R.)

Physics III Lab. Fall 1969

Experiment 1 - Pendulum

Purpose

Determine quantitative relations between parameters of oscillating physical pendulums: e.g., size, mass, amplitude and frequency.

References

Text; and others on mechanics
Mechanics, Berkeley Physics Course, Vol. 1, by Kittel etal
(p. 197; p.225, topic 1) for anharmonic pendulum.

With a set of pendulums formed from metal rod in the shape of isoceles triangles, use simple devices (clock, meter stick) to study their behavior.

How can you improve the precision of your measurement of period of oscillation? Partners should share in measuring techniques.

Record observations in a systematic way. Organize your work with the above purposes in mind.

Compare empirical relations - those found from your measurements- with theoretical relations - those deduced from physical laws. A table or a graph might be used. Show whether any discrepancies between theoryand experiment might be due to measurement uncertainties.

For example, suppose a calculated quantity were $I = m\ x^2/3$, the moment of inertia of a rod about a certain axis perpendicular to it. If m is measured with an experimental uncertainty $\pm\Delta m$, and x with uncertainty $\pm\Delta x$, then the resulting uncertainty in I is found as follows:

$$d\ I = d\ (m\ x^2/3) = (m/3)\ d\ (x^2) + (x^2/3)\ dm$$

$$= (2/3)\ m\ x\ dx + x^2\ dm\ /3,$$

or $$d\ I\ /\ I = d\ I\ /\ (M\ x^2/3) = 2\ dx\ /x + dm\ /m.$$

This relates the fractional uncertainties $\Delta I\ /I$, etc. approximately, if they are small, or $\Delta x\ /\ x \ll 1$.

In terms of percent uncertainty ("% error"), dividing through by 100 gives (% error in I) = 2 (% error in x) + (% error in m).

Questions to consider in report

Could g (acceleration of "free fall") be calculated from your data. If so, indicate now. (Do it if you have time.)

Is frequency dependent on amplitude? This question is discussed in your text for the analogous case of the simple pendulum; see also Berkeley text cited above. (The ambitious investigator might try to answer this question quantitatively.)

OVER

Physics Laboratory - General Instructions

I. Purpose of Laboratory

Laboratory work in physics has two important objectives - first, to give the student direct experience with some of the natural phenomena upon which physical principles are based, and second, to develop in the student some understanding of the experimental procedures. It is felt that some experience in the laboratory is necessary to give the student an insight into the methods of physics (or for that matter any experimental science). Without it he would be merely accepting principles as they were handed to him without an understanding of the experimental procedures on which they are based.

In the laboratory the student will work with real, rather than ideal, apparatus. This equipment (and the experimenter as well) will be subject to limitations which cause errors that must be taken into account before any conclusions can be drawn from the experimental results. Therefore error analysis is an essential part of all good laboratory work.

Although you will be assigned a certain group of experiments to do this quarter, and in many cases the procedure to be followed in performing the experiment is described in an instruction sheet, it is hoped that the student will use some of his own ingenuity in performing the experiments; it is intended that the instructions be used as an aid to understanding rather than something to be followed mechanically without thought. We also want to encourage students to think about possible experiments that they might do in place of one of the prescribed set. Within the limitations of equipment and time, substitution of an experiment which is more interesting to the individual student is permitted, provided it is a physics experiment and it is cleared with the instructor.

II. Preparation for an Experiment

In order to perform an experiment thoroughly and accurately in the time allotted, it is necessary to put in some time beforehand thinking about the experiment. If an instruction sheet has been provided it is to be studied carefully before the laboratory period. You should come to the laboratory with as thorough an understanding as possible of what you are going to do during the period and why. This may require that you spend some time in the library, looking up references etcetera.

III. Performance of the Experiment

An essential part of the method of solving an experimental problem is the preparation of a clear, concise record of the data taken during the performance of the experiment. This record should contain, in a clear and legible form, all the "raw" data and information with which to make corrections (don't try to make corrections "in your head" while taking data) and also enough explanation of what you are doing and why so that your pages of data can be analyzed later without confusion or ambiguity. Your instructor may require that this record be kept in a permanent notebook or he may ask you to keep this record on data sheets which are later included in a report on the experiment. In either case, all observations should be recorded directly into the notebook or on the data sheets (nothing on scratch paper and later copied) and an estimate of the accuracy of each set of measurements should be made and recorded also. Corrections can be made by crossing out errors with a single line (no erasures). Before leaving the laboratory, the student should do enough calculation and graphical work to ensure that the data collected "makes sense" and there are no gaps in it which need to be filled in before he can continue the analysis without having to make any "wild guesses or assumptions. Your data record must be approved by the instructor before you leave the laboratory.

IV. Laboratory Notebook (Data Record)

The following are specific suggestions concerning the form of the laboratory record of the experiments.

A. If the instructor has you keep a permanent laboratory notebook it should be one having cross-ruled pages (useful for graphs) and it must be labeled with the following information.

1. On the front cover in ink:

   Physics Laboratory

   Your Name

2. Inside the front cover at the top:

   Fall (or whatever) Quarter

   Lab. day and hours

   Group Number

B. For each experiment the student should record the title of the experiment and the date performed at the top of the data record. A very brief (not detailed) description of the procedure followed should precede the data record, which is preferably in tabular form. Label the data carefully with the proper column headings and units. Whenever possible, the type and identifying number of instruments being calibrated or used in measurement should be recorded for later reference.

C. As suggested above the next step is to do the calculations required by the analysis of the experiment and draw the graphs. Repeat any measurements which appear doubtful and make new measurements where needed to fill in gaps in the data.

D. If you are using a laboratory notebook rather than data sheets and if the instructor informs you that no report is required on a particular experiment, then the experiment should be completed in the notebook by writing a summary and conclusions. Final calculations should be summarized in tabular form and whatever additional graphs are required should be completed. State a conclusion in your own words and discuss the experiment briefly (for example a discussion of accuracy is always desirable). On graphs and in your final summary give the page number of the data or discussion referred to. The summary and conclusions may be left for the report when one is being written.

V. <u>Report</u>

When a report is required on an experiment it is due at the beginning of the period one week after the experiment was performed. The report is to be written independently by each student in ink (or typewritten) on white, unlined 8½ x 11 paper (graph paper for graphs). Each report must have:

A. A cover sheet containing the following information -- course, experiment title, your name, laboratory period day and hours, group number, date experiment was performed, and date of report.

B. A statement of the purpose of the experiment and a brief summary of how you went about performing it (not detailed), data and observations (if you used data sheets rather than a notebook these may be submitted as they are), sample calculations, tabulated results, graphs, conclusions, and a discussion of the experiment. The discussion section of a report should be more thorough and complete than the corresponding section in the notebook. It may include a discussion of what was learned in doing the experiment, as well as the results and the accuracy of the results. It should also contain a discussion of any points which the instructor may have brought to your attention through questions written on the instruction sheets, and of any other points of interest that may occur to you.

It is customary to use the passive voice in scientific writing (e.g. "The time required for the pendulum to swing through twenty complete cycles was measured...etc.") thus not calling attention to the observer. The following styles are not to be used in a report: I" (we) swung the pendulum and..." or "Swing the pendulum and measure the time for twenty complete cycles...". If you quote or paraphrase any outside sources in writing your report (including your own text book) give credit to the original author in a footnote.

References:

1. Baird, "Experimentation", chapter 7

2. Olson, "Experiments in Modern Physics", section 1.4

Measurement, Probability, and Experimental Errors

I. Types of Error

Whenever a measurement is made of any physical quantity there is a certain amount of uncertainty in the result. Determination of the amount of uncertainty in a measurement is not usually easy but an attempt should always be made to do so, even if it is no more than an educated guess. Without some estimate of the uncertainties associated with experimental measurements one has no indication of the accuracy of the results and it is difficult to come to any conclusion about what the experiment has shown (or not shown). In all of the experiments which follow in the physics laboratory sequence the student will be expected to make some estimate of the accuracy of his quantitative experimental results.

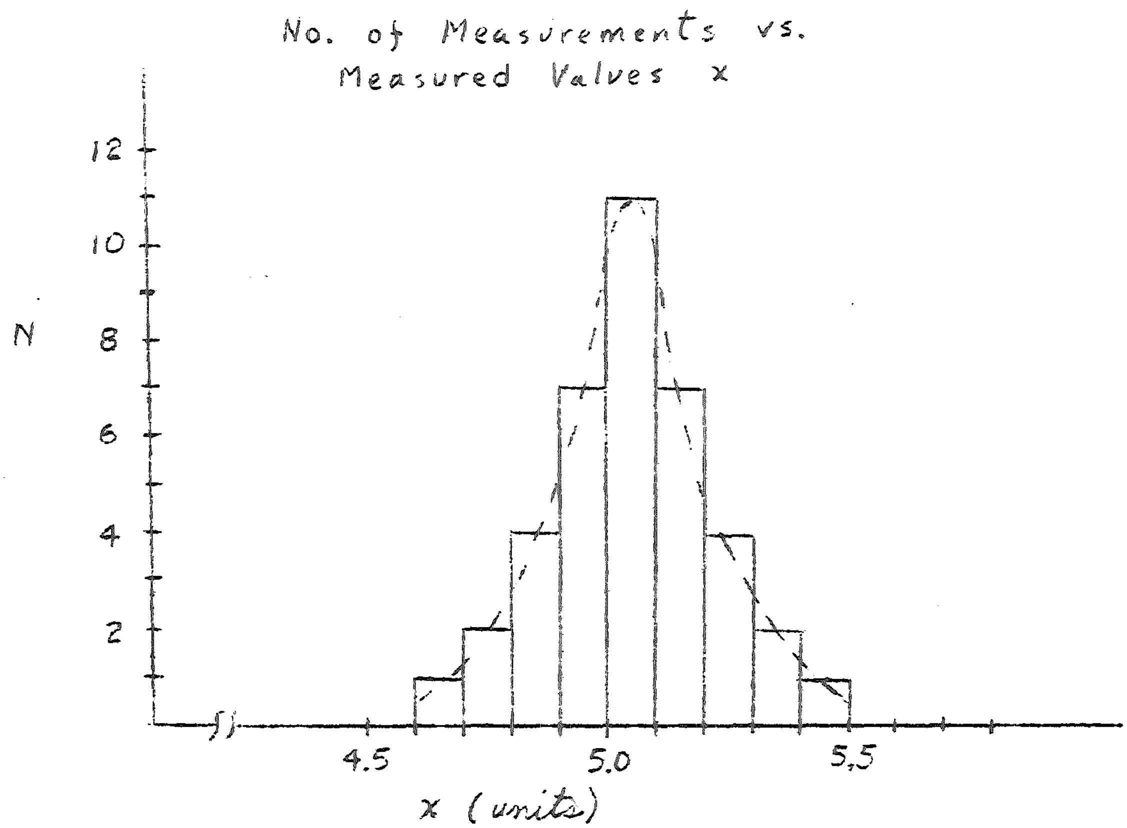
There are two types of errors which may occur in the measurement process, systematic errors and random errors. Systematic errors tend to make all the observations of one item too small or too large. For example if voltage measurements were taken in an electric circuit using a voltmeter which consistantly read 0.1 volt too high, a systematic error would be present. Other common examples of causes of systematic error are worn weights, clocks which gain or lose time, friction, and personal bias of the observer which causes him to make readings which are consistently high or low. When systematic errors are recognized in an experiment it is often possible to find out how large their effect is and to correct for it. The error in the voltmeter which reads 0.1 volt too high, for example, can be discovered by calibrating the instrument against some sort of standard (accurately known voltage), and a correction of -0.1 volt made to all the readings. Error due to an observer's bias may be minimized by having another observer make the same measurement independently (bias is best eliminated if each observer knows nothing of the other's result until after both measurements are completed).

Random errors result from chance variations in the quantity being measured, in the measuring devices, or in the observer, and are just as likely to produce too large a value as too small. For example, if one measures the diameter of a metal rod several times with a micrometer the readings will probably fluctuate slightly in a non-systematic fashion due to actual differences in the rod's diameter at different positions, variations in pressure when the micrometers jaws are closed, and changes in the observer's estimate of the scale reading. Random errors are present in all measurements, although they may be too small to be noticeable, and they cannot be corrected for because of their random nature.

II. Determination of Precision

Suppose that several measurements of the same quantity x were made and all systematic error in the measurements eliminated or corrected (assuming this were possible). As discussed above there would still be a certain amount of rnadom fluctuation apparent in the measurements if they are "fine" enough to make it noticeable. If a histogram was plotted showing the number of measurements N falling within different intervals of size Δx it might look like that shown in Fig. 1.

Fig. 1



The meaning of the histogram is that one measurement of x fell between 4.6 and 4.7 units, two between 4.7 and 4.8 units, four between 4.8 and 4.9 units, and so forth. The completely symetrical distribution shown usually results only if a large number of measurements are made and if the fluctuations are entirely random. In such cases the envelope of the distribution often has a particular form called a "normal" or "Gaussian" distribution which is represented by the mathematical equation

$$y = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\bar{x})^2/2\sigma^2} \qquad (1)$$

where σ is a constant which determines the "sharpness" of the peak (high, narrow peaks are characterized by small values of σ). The quantity $\bar{x}$ is the average of the individual measurements

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots\ldots\ldots}{n} = \frac{\sum x_i}{n}$$

where n is the total number of measurements, and because of the symmetry of the Gaussian function $\bar{x}$ corresponds to the most probable value of x obtained from a measurement of x (peak of curve). Thus $\bar{x}$ is the best estimate that one may make of the true value of x from these measurements.

The individual measurements of x differ from the average or most probable value $\bar{x}$ by an amount d called the deviation of that measurement

$$d_1 = x_1 - \bar{x}\ , \quad d_2 = x_2 - \bar{x}\ , \ \ldots\ldots\ldots$$

The standard deviation

$$\sigma = \left[\frac{d_1^2 + d_2^2 + \ldots\ldots d_n^2}{n - 1}\right]^{1/2} = \left[\frac{\sum (d_i)^2}{n - 1}\right]^{1/2}$$

is an indication of the precision of a set of measurements since narrow Gaussian distributions indicate precise measurements with small deviations from the average and a small standard deviation σ. If a large number of measurements is made, 68% of them will be in the range $\bar{x} \pm \sigma$, 95% in the range $\bar{x} \pm 2\sigma$, and 99% in the range $x \pm 3\sigma$, a fact which can be verified by determining the area under a Gaussian curve between the various limits. If after having determined x and σ from a large number of measurements one makes a single measurement x, he then will have about a two thirds chance of getting a value between $\bar{x} + \sigma$ and $\bar{x} - \sigma$, etcetera.

Although increasing the number of measurements of quantity x would have little effect on the standard deviation σ (the scatter of the data) except to give a more accurate picture of what it really is, increasing the number of measurements should improve the reliability of the average value $\bar{x}$. It can be shown from statistics that the standard deviation in the mean $\bar{x}$ is given by the equation

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

which means that there is a 68% chance that the true value of x will be in range $\bar{x} \pm \sigma_m$ assuming the distribution is normal and there are no systematic errors present. Thus the precision of the mean $\bar{x}$ can be increased (σ_m reduced) by taking more observations, but the improvement is slow because of the $\sqrt{n}$ factor (90 readings only 3 times as good as 10 readings). The final result of a set of measurements may be stated

$$x = \bar{x} \pm \sigma_m$$

It is quite often useful to represent the standard deviation σ_m as a percentage of the value $\bar{x}$. The calculation required is:

$$\text{per cent std. dev.} = (\sigma_m/\bar{x}) \cdot (100\%)$$

Although the normal or Gaussian distribution (equation 1) is very often a good representation of the kind of distribution found in repeated measurements of physical quantities, it should not be assumed that this distribution always gives an accurate description of the results of such measurements, even when a large number of measurements are made. There are a number of cases where the distribution is non-Gaussian and perhaps even non-symmetrical. For example, if one makes several determinations of the number of nuclei which decay by particle emission in a certain time, he obtains the Poisson distribution

$$y \propto \frac{\bar{x}^x}{x!} e^{-\bar{x}} \tag{2}$$

where $\bar{x}$ is the average number of counts and y is the probability of obtaining x counts in a given trial. This distribution is very unsymmetrical about the mean $\bar{x}$ when the number of counts $\bar{x}$ is small but closely resembles a Gaussian distribution with standard deviation $\sqrt{\bar{x}}$ when $\bar{x}$ is large.

III. Propagation of Errors If one uses experimental observations, with their associated random errors, to calculate a result, the precision of the result will be determined by the precision of the quantities involved in the calculation. The standard deviation of the result may be determined from those of the separate quantities σ_{m1}, σ_{m2}, etc. by keeping in mind the following rules.

A. The standard deviation of the result of addition and/or substraction is the square root of the sum of the squares of the standard deviations of the separate terms.

Example:

$x_1 = 5.30 \pm 0.20$ units

$x_2 = 1.70 \pm 0.10$ units

$x_3 = 7.20 \pm 0.01$ units

$$x_1 - x_2 + x_3 = (5.30-1.70+7.20) \pm \left[(0.20)^2 + (0.10)^2 + (0.01)^2\right]^{1/2}$$
$$= 10.80 \pm 0.22 \text{ units}$$

Note that most of the standard deviation in the result comes from the largest standard deviation present in the separate terms (0.22 ≃ 0.20).

B. **The percentage standard deviation in the result of mulitplication and/or division is the square root of the sum of the squares of the percentage std. deviations of the factors.**

example: x_1, x_2, x_3 as above

$$(\%\text{ std. dev.})_1 = \frac{0.20}{5.30} \times 100\% = 3.8\%$$

$$(\%\text{ std. dev.})_2 = \frac{0.10}{1.70} \times 100\% = 5.9\%$$

$$(\%\text{ std. dev.})_3 = \frac{0.01}{7.20} \times 100\% = 0.1\%$$

$$y = \frac{(x_1)\ (x_2)}{x_3} = 1.25 \pm \text{std. dev.}$$

$$(\%\text{ std. dev.})_y = \left[(3.8)^2 + (5.9)^2 + (0.1)^2\right]^{1/2} = 7.0\%$$

$$(\text{std. dev.})_y = (.07)\ (1.25) = 0.09$$

$$y = 1.25 \pm 0.09 \text{ units}$$

Note that in this case the largest contribution to the standard deviation in the result comes from that quantity with the largest percentage standard deviation.

C. **In case a quantity is raised to the n^{th} power its percentage standard deviation is multiplied by n.**

The process of carrying standard deviations through calculations is useful not only indetermining the precision of the result but also in determining which quantity contributes most to random error in the result. It may be possible to reduce the deviations in this quantity by using more care or different techniques.

IV. Accuracy of Experimental Results

Determination of the standard deviation in an experimental result will tell you how much uncertainty is present due to random errors, but this is an indication of the accuracy of the result only in the case where systematic errors are negligible compared to random errors. For example, if in a particular experiment you obtained a percentage standard deviation of 1% but the instruments used to obtain the measurements were accurate only to within 5% (all readings may be too high or low by 5%), then the 5% accuracy is a better indication of the reliability of the results than the 1%. Some attempt should be made by the student to determine the reliability of his results in each experiment, although in some cases this will involve making some educated guesses as to the accuracy with which a particular measurement may be made with a particular measuring device. In all cases try to eliminate as much systematic error from the measurement as possible within the time available. An experimental result does not agree with a prediction of a theory unless the theoretically predicted result lies within the range given by the experimental result plus and minus the probable error; an experiment does not disagree with a theory unless the predicted result lies outside this range.

V. Significant Figures

The term "significant figures" refers to the digits of a measurement made in the laboratory, including all the certain digits and one additional doubtful one based on the observer's estimate of a fraction of a scale division. The numbers which represent data or the results of calculations should always be given with neither more nor fewer significant figures than are justified by the precision of the observations and computations. The number of significant figures in a measurement (or a calculated quantity) may be determined using the following rules.

(a) The first significant figure is the first non-zero digit.

(b) Zeros which occur between significant digits are considered significant.

(c) Zeros which occur to the right of the last non-zero digit are considered significant when they are to the right of the decimal point (the significance of such zeros to the left of the decimal point is indeterminate).

(d) If numbers having a different number of significant figures are added, substracted, multiplied or divided, the answer is given so as to have the same number of significant figures as the term or factor which has the least.

Examples:
.0001906 has 4 significant figures
10,937 has 5
93,000 has an indeterminate number
9.3×10^4 has 2
9.30×10^4 has 3

VI. Comparison of Results

Sometimes an experimental result is arrived at by two different methods which should both theoretically give the correct result. If there is no reason to believe that one of the results is much more accurate than the other, it might be instructive to see how much difference there is between the two. This difference is usually given in terms of the "percentage difference" which is defined.

$$\%\ \text{diff.} = \frac{\text{diff. between values}}{\text{average value}} \times 100\%$$

References:

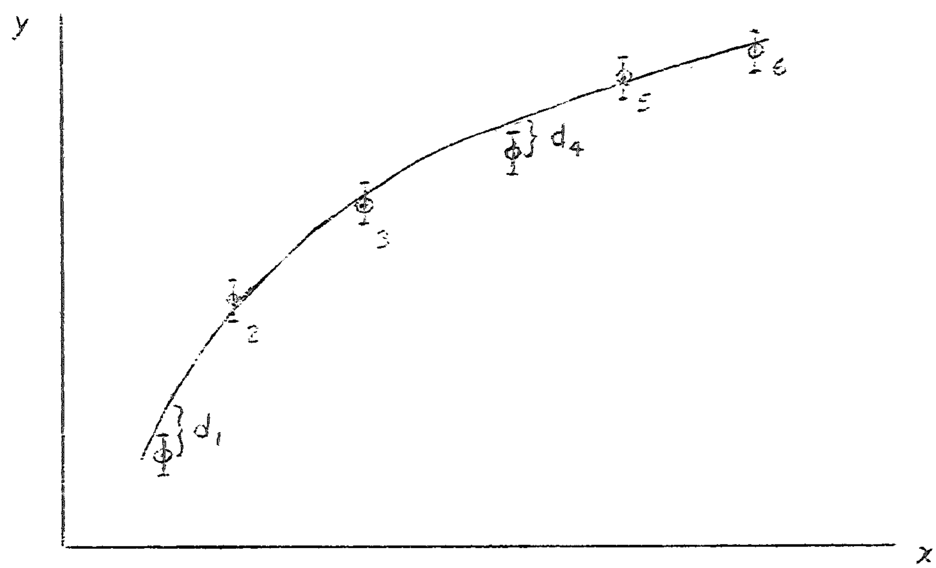
1. Young, "Statistical Treatment of Experimental Data"
2. Barford, "Experimental Measurements: Precision, Error and Truth"
3. Baird, "Experimentation: An Introduction to Measurement Theory and Experiment Design"
4. Braddick, "The Physics of Experimental Method"
5. Pugh and Winslow, "The Analysis of Physical Measurements"
6. Bevington, "Data Reduction and Error Analysis for the Physical Sciences"

METHOD OF LEAST SQUARES

One of the fundamental problems that comes up again and again in the laboratory is that of finding, from simultaneous measurements of quantities y and x, the dependence of quantity y on quantity x (the dependence of the period of a pendulum on its length for example). Often this dependence is revealed by making a graph of y versus x from the data. However, a certain amount of judgement is always involved in making a graph from experimental data since deviations in the measurements usually make it impossible to draw a smooth curve through all the data points. One usually tries to draw a smooth curve among the points in such a way that it appears that the deviations of the points from the line (positive and negative) add up to approximately zero. In other words, in the graph shown below

$$|d_1| + |d_3| + |d_4| + \ldots\ldots \simeq |d_2| + |d_5| + \ldots\ldots\ldots$$

where the deviations here and in the analysis to follow will be assumed to be deviations in y for precisely known values of x.



If a high degree of precision is required in the expression relating y to x, this method of balancing deviations "by eye" might not be sufficient. In this case a more scientific approach, based on statistics, is followed. It can be shown that the most probable disposition of the line representing the dependence of y on x is that for which the sum of the squares of the deviations of the points from the line is a minimum (hence the name "least squares").

$$\Sigma(d_i)^2 = d_1^2 + d_2^2 + d_3^2 + d_4^2 + \ldots\ldots = \text{a minimum}$$

This statement is called the "principle of least squares" and it is the basis of a method for finding the relationship between y and x which best fits the data points (for which the sum of the squares of the deviations is a minimum).

Actually the problem of determining the line which "best" fits a set of data points (x_i, y_i) is several different problems, depending on the type of curve which is to represent the relationship between x and y. If it has been predetermined from the data or from theory that y depends on x linearly so that $y = Ax + B$, the problem becomes one of picking out, from all possible straight lines, the one with values of slope A and intercept B such that the sum of the d_i^2 will be as small as possible. If (x_1, y_1) are the coordinates of the first data point, (x_2, y_2) the coordinates of the second and so forth, and if it is assumed that the deviations are only in the y measurement for precisely known x ' s, then

$$\Sigma(d_i)^2 = (Ax_1 + B - y_1)^2 + (Ax_2 + B - y_2)^2 + \ldots\ldots$$

If the "best" straight line is that which makes the sum of the squared deviations or a minimum.

$$\frac{d[\Sigma(di)^2]}{dA} = 0 = 2x_1(Ax_1 + B - y_1) + 2x_2(Ax_2 + B - y_2) + \ldots$$

$$\frac{d[\Sigma(di)^2]}{dB} = 0 = 2(Ax_1 + B - y_1) + 2(Ax_2 + B - y_2) + \ldots\ldots$$

are the conditions which should lend to the "best" values of A and B. These equations may be rewritten:

$$B\,\Sigma x_i + A\,\Sigma x_i^2 - \Sigma x_i y_i = 0 \qquad (1)$$

$$nB + A\,\Sigma x_i - \Sigma y_i = 0 \qquad (2)$$

where n is the number of points.

The method is illustrated below for a set of $n = 5$ points.

Point No.	1	2	3	4	5
x	1.00	1.90	2.60	3.20	4.00
y	0.90	3.00	4.00	5.50	6.90

A table is made as follows:

x	y	x^2	xy
1.00	0.90	1.00	0.90
1.90	3.00	3.61	5.70
2.60	4.00	6.76	10.40
3.20	5.50	10.24	17.60
4.00	6.90	16.00	27.60

$\Sigma x_i = 12.70$ $\Sigma y_i = 20.30$ $\Sigma x_i^2 = 37.61$ $\Sigma x_i y_i = 62.20$

Substituting in (1) and (2).

$$12.70\ B + 37.61\ A = 62.60$$

$$5\ B + 12.70\ A = 20.30$$

Solving simultaneously, $B = -0.989$ $A = 1.988$

The equation of the straight line which best fits the data points is

$$y = 1.988\ x - 0.989$$

In other words the sum of the squares of the deviations of the points from the straight line is a minimum for a line of slope 1.988 and y intercept -0.989.

It is generally shown in books on statistics that the standard deviations in these values obtained for the slope A and intercept B may be found using the equations (3 and 4):

$$\sigma_A = \left[\frac{\Sigma d_i^2}{n \Sigma x_i^2 - (\Sigma x_i)^2}\right]^{1/2} = \left[\frac{\Sigma (Ax_i + B - d_i)^2}{n \Sigma x_i^2 - (\Sigma x_i)^2}\right]^{1/2}$$

$$\sigma_B = \left\{\frac{(\Sigma d_i^2)\ (\Sigma x_i^2)}{n^2 \Sigma x_i^2 - n\ (\Sigma x_i)^2}\right\}^{1/2} = \left\{\frac{\left[\Sigma (Ax_i + B - y_i)^2\right]\left[\Sigma x_i^2\right]}{n^2\ \Sigma x_i^2 - n\ (\Sigma x_i)^2}\right\}^{1/2}$$

A table is made as follows:

x	y	x^2	xy
1.00	0.90	1.00	0.90
1.90	3.00	3.61	5.70
2.60	4.00	6.76	10.40
3.20	5.50	10.24	17.60
4.00	6.90	16.00	27.60
$\Sigma x_i = 12.70$	$\Sigma y_i = 20.30$	$\Sigma x_i^2 = 37.61$	$\Sigma x_i y_i = 62.20$

Substituting in (1) and (2).

$$12.70\ B + 37.61\ A = 62.60$$

$$5\ B + 12.70\ A = 20.30$$

Solving simultaneously, $B = -0.989$ $A = 1.988$

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$$\sigma_A = \left[\frac{\Sigma d_i^2}{n \Sigma x_i^2 - (\Sigma x_i)^2}\right]^{1/2} = \left[\frac{\Sigma(Ax_i + B - d_i)^2}{n \Sigma x_i^2 - (\Sigma x_i)^2}\right]^{1/2}$$

$$\sigma_B = \left\{\frac{(\Sigma d_i^2)\ (\Sigma x_i^2)}{n^2 \Sigma x_i^2 - n\ (\Sigma x_i)^2}\right\}^{1/2} = \left\{\frac{\left[\Sigma(Ax_i + B - y_i)^2\right]\left[\Sigma x_i^2\right]}{n^2\ \Sigma x_i^2 - n\ (\Sigma x_i)^2}\right\}^{1/2}$$

In cases where a nonlinear curve is to be fit to a set of data points in such a way as to make $\Sigma(di)^2$ a minimum, equations (1), (2), (3), and (4) no longer apply. Often one can get around this difficulty, however. For example, suppose some data points are to be fit with a parabola of the type $y = Ax^2 + B$. If the quantity $X = x^2$ is calculated for each of the points, the method may then be applied to quantities y and X, since y versus X will be a straight line ($y = AX + B$) even though y versus x is not.

The least squares method is not confined to finding the constants of a straight line, however; it can be applied to any kind of curve. For example, if one has a set of data points and wants to determine the constants of the "best fit" parabola $y = AX^2 + BX + C$, he can apply the conditions that minimize $\Sigma(di)^2$ with respect to variables A, B, and C and will obtain the equations:

$$\Sigma x_i^2 y_i = C\,\Sigma x_i^2 + B\,\Sigma x_i^3 + A\,\Sigma x_i^4$$

$$\Sigma x_i y_i = C\,\Sigma x_i + B\,\Sigma x_i^2 + A\,\Sigma x_i^3$$

$$\Sigma y_i = nC + B\,\Sigma x_i + A\,\Sigma x_i^2$$

which may be solved simultaneously for constants A, B, and C.

References:

1. Young, "Statistical Treatment of Experimental Data", section 14.

2. Baird, "Experimentation", Appendix 2

3. Barford, "Experimental Measurements", Chapter 3

4. Pugh and Winslow, "The Analysis of Physical Measurements", Chapter 10.

5. Bevington, "Data Reduction", Chapters 6 and 11

6. Gerhold, "Least-Squares Adjustment of Weighted Data to a General Linear Equation", American Journal of Physics, Vol. 37, p. 156.

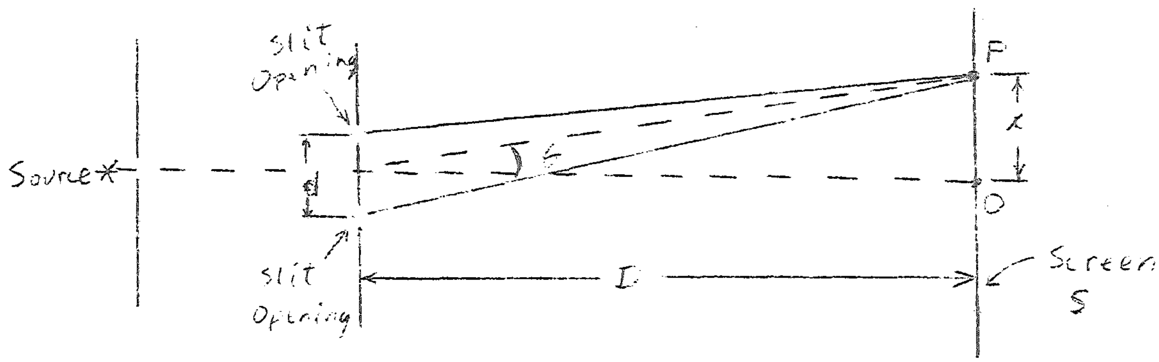
BOB MARKS
PHYSICS V

9/10 D.R.

Interference and Diffraction

According to Huygen's principle, each point along a wavefront may be regarded as a new source of waves. Whenever something obstructs part of the wavefronts, interference between "wavelets" emanating from different parts of the unobstructed wavefronts produce a diffraction pattern which is characteristic of the geometry of the obstruction (or opening in object which blocks the light) and of the wavelength of the light. It is shown in nearly all introductory physics textbooks, for example (see Resnick and Halliday, section 43-1), that when light waves pass through a double slit arrangement like that shown below they interfere constructively and destructively at different positions to form fringes on the screen S such that intensity maximum appear at positions

$$x = n\left(\frac{D\lambda}{d}\right) \qquad n = 0,1,2,3\ldots\ldots.$$



In somewhat the same way wavelets passing through different parts of a single slit interfere to produce a single slit diffraction pattern with destructive interference causing diffraction minima at angles θ such that (see Resnick and Halliday, section 44-2)

$$a \sin \theta = m\lambda \qquad m = 1,2,3,\ldots\ldots.$$

with maxima approximately half way between (the exact intensity expressions are given in section 44-3), where a is the slit width. A circular aperture of diameter d results in fringes having circular symmetry with the first minimum appearing at a distance from the center such that (see Resnick and Halliday, section 44-5)

$$\sin \theta = 1.22\,\lambda/d$$

Experiment:

Your light source will be a monochromatic beam from a helium-neon laser having wavelength $\lambda = 6328$ Å. This experiment is somewhat open ended in that you are not told exactly what to do or how to do it. The object is to investigate the nature of interference and diffraction effects. The details are left to you. You might for example put a single or double slit in the beam and determine the spacing or width of the slits, perhaps checking your results with a direct measurement using the optical comparator. You might try to do an experiment which would confirm the constant 1.22 in the expression for the first fringe minimum from a circular aperture or compare the fringe pattern of aperture or slits of different sizes. An experimental study of diffraction by rectangular openings or by a repeated pattern of openings (such as found in a seive) might be of interest.

I) SINGLE SLIT

$D = 54$ cm

$X_1 = .9$ cm

$X_2 = 1.65$ cm

II) DOUBLE SLIT

A) $D = 51$ cm

$X_1 = .13$ cm

$X_2 = .29$ cm

B) $D = 245.5$ cm

$X_1 = .65$ cm

$X_2 = 1.35$ cm

$X_3 = 2.0$ cm

III) PIN HOLE

$D = 1.04$ m

$X = .23$ cm

I) COMPUTATION OF d FROM DOUBLE SLIT DATA

$$d = \frac{nD\lambda}{x} \quad \text{(INTENSITY AT MAXIMUM)}$$

A) $d_1 = \frac{(1)(2.45)(6.33\times10^{-7})}{.65\times10^{-2}}$

$= .239$ mm

B) $d_2 = \frac{2(2.45)(6.33\times10^{-7})}{1.35\times10^{-2}}$

$= .230$ mm

C) $d_3 = \frac{3(2.45)(6.33\times10^{-7})}{2.0\times10^{-2}}$

$= .232$ mm

D) $d_4 = \frac{(1)(.51)(6.33\times10^{-7})}{.13\times10^{-2}}$

$= .249$ mm

E) $d_5 = \frac{(2)(.51)(6.33\times10^{-7})}{.29\times10^{-2}}$

$= .223$ mm

$$\bar{d} = \frac{d_1+d_2+d_3+d_4+d_5}{5} = .235 \text{ mm}$$ ↔ WIDTH BETWEEN SLITS

II) COMPUTATION OF a, FROM SINGLE SLIT DATA

$$a \sin\theta = \left(m+\tfrac{1}{2}\right)\lambda \quad \text{(INTENSITY AT A MAXIMUM}$$

$(\sin\theta \approx \tan\theta \approx \frac{x}{D}$

A) $a_1 = \frac{(6.33\times10^{-7})(54)}{2\,(.9)}$

$= .190$ mm

B) $a_2 = \frac{(6.33\times10^{-7})(54)}{(1.65)}$

$= .207$

$\bar{a} = .198$ ↔ WIDTH OF SINGLE SLIT

III) CIRCULAR APERTURE

$$\sin\theta = \frac{1.22\lambda}{d} \approx \tan\theta \approx \frac{x}{D}$$

$\therefore\ d = \frac{1.22\lambda D}{x}$

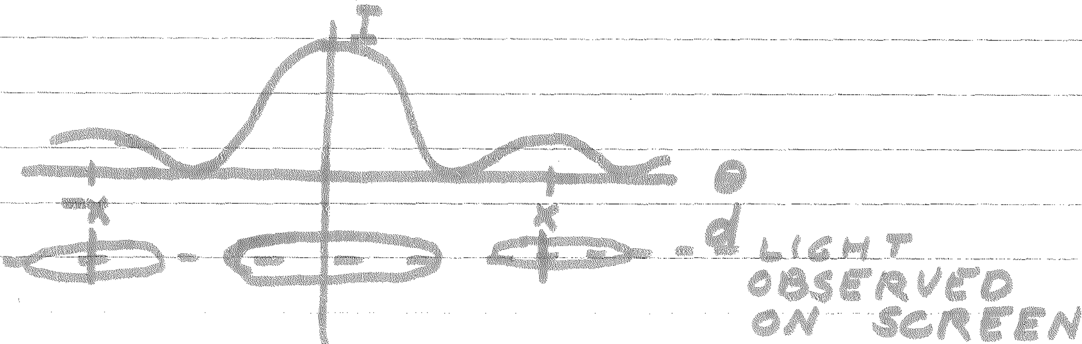
$= \frac{(1.22)(6.33\times10^{-7})(1.04)}{.23\times10^{-2}}$

$= .349$ mm ← DIAMETER OF HOLE

CONCLUSIONS AND OBSERVATIONS:

1) THE INTENSITY OF THE LIGHT ON THE SCREEN DISIPATES RAPIDLY. THIS FOLLOWS, FROM THE DERIVED FORMULA FOR INTENSITY

$$I_\theta = I_{MAX}\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \left(\alpha = \frac{\pi a}{\lambda}\sin\theta\right)$$



$$\alpha = \frac{\pi a}{\lambda}\sin\theta$$

$$\cong \frac{\pi a}{\lambda}\,\frac{\bar{x}}{D} \xrightarrow{n=1} = \frac{\pi(.198\times10^{-3})(1.72\times10^{-3})}{(6.33\times10^{-7})(54\times10^{-3})}$$

$$\cong 3.14 = \pi$$

WHICH CONFIRMS THAT FOR

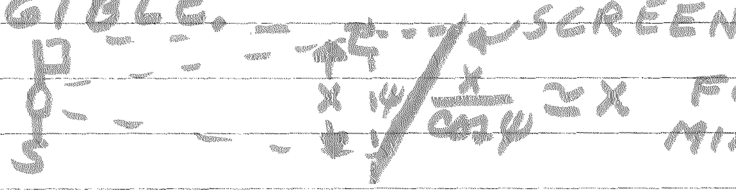
$$I_{\theta=\pi} = 0$$

2) THE FACT THAT LIGHT DISPLAYS WAVE PROPERTIES HAS CLEARLY BEEN SHOWN, THE ~~HOL~~ MATHEMATICAL CONCEPTS OF SUPERPOSITION OF WAVES HAS BEEN SHOWN EXPERIMENTALLY, BY USE OF REFRACTION RESULTING IN WAVE INTERFERENCE.

ERROR ANALYSIS:

FROM THEORY, MEASUREMENT ERROR FROM THIS EXPERIMENT SHOULD NOT HAVE BEEN OF ANY GREAT MAGNITUDE, FOR THE RELATIONSHIPS ARE LINEAR, AS OPPOSED TO, SAY, THE ELECTRICAL OSCILLATION LABS, WHERE RELATIONSHIPS WERE EXPODENTIAL, AND A SMALL ERROR IN MEASUREMENT YIELDS A RATHER LARGE EXPERIMENTAL ERROR. EVEN THE ASSUMPTION $\sin\theta \simeq \tan\theta = \frac{x}{L}$ IN THIS EXPERIMENT WOULD NOT YIELD SUBSTANTIAL ERROR. THIS PROVED TO BE THE CASE EXPERIMENTALLY. IN COMPUTATION OF d, THE DISTANCE BETWEEN THE SLITS, DATA ONLY DEVIATED FROM THE MEAN BY 6%. IN COMPUTATION OF a, THE WIDTH BETWEEN THE SLITS, BOTH ANSWERS WERE REASONABLY CLOSE.

THERE COULD BE LITTLE ERROR IN THE SET-UP, THAT IS, SETTING THE SLIDE $\parallel$ TO THE SCREEN. IF A MAXIMUM ANGLE OF ERROR OF 5° WERE SET, THE ERROR WOULD BE NEGLIGIBLE.



$\frac{x}{\cos\psi} \simeq x$ FOR MINUTE ψ

4. The method of least squares determines the most probable line of a certain type (e.g. straight line) through a set of data points.

(a) The line is placed so as to minimize____________________

____

____.

(b) Use the method of least squares to determine the slope and y intercept of the straight line graph I versus V^2 in question 3. How do your results compare with the values taken directly from the graph in question 3 ?

GRAPHICAL ANALYSIS

Often one of the aims of an experimental investigation is the determination, from measurements made in the laboratory, of how one of two interdependent quantities, y, depends on the other, x. Graphical methods provide us with a very useful tool in this type of analysis.

I. Plotting Graphs

Suppose one is interested, for example, in finding in a particular experiment a methemical relationship which expresses the velocity of a moving object v as a function of the time t. In this case velocity is the "dependent variable" whose dependency on the "independent variable" time is to be established from the following data.

Time (sec)	Velocity (magnitude) (cm/sec)
1.00	1.9
2.00	1.9
3.00	3.0
4.00	3.9
5.00	6.5
6.00	11.0

Suppose that in this experiment the time measurements are very precise and their errors can be ignored while the velocity measurements are estimated to have a standard deviation (see instruction sheet on "Measurement, Probability, and Experimental Errors) of about ± 0.30 cm/sec. The steps to be followed in constructing a graph which illustrates the dependence of velocity v on time t (or any quantity y on another quantity x) are summarized below.

(a) The dependent variable (quantity whose dependency on the other is to be determined) is plotted vertically (velocity versus time rather than vice versa).

(b) Scales should be chosen which are easy to plot and easy to read and which make the graph large enough to be read easily and accurately (occupying a full page if possible).

(c) Scales usually start at zero but sometimes this would cause the data to be crowded into one part of the graph. In such a case it is a good idea to suppress the zero (start the scale at some value other than zero or show a break in the scale). However, it should be made obvious to someone looking at the graph that the zero has been suppressed.

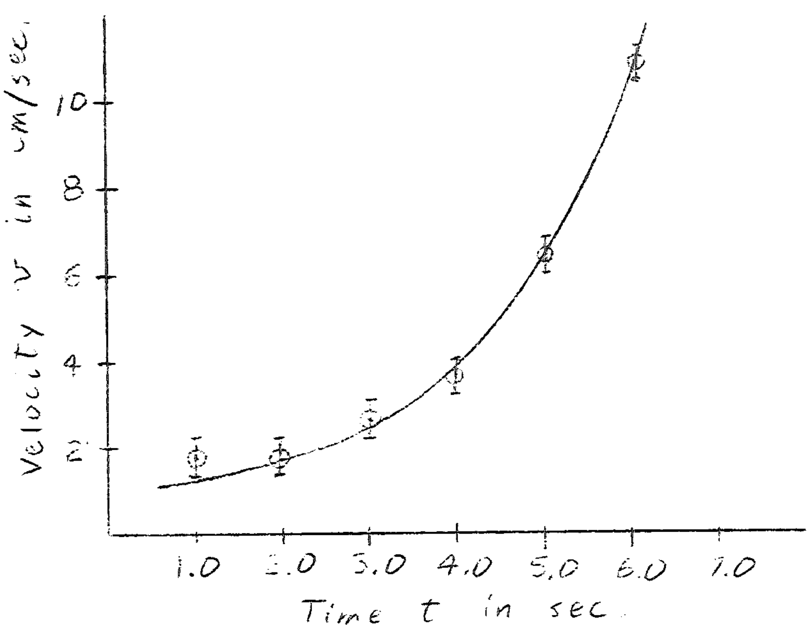
(d) The graph should have a title and each of the axes should show the quantity plotted along that axis and the numerical scale and units for that quantity.

(e) The experimental points are marked clearly on the graph by drawing a small circle around each of them and drawing an "error line" (in the above example extending 0.30 cm/sec above and below the data point) at each point.

(f) Draw the simplest possible smooth line or curve (i.e. the simplest curve is a straight line, the next is a curve whose curvature is always in the same direction and doesn't change magnitude suddenly, etc) among the points, with no more details of shape and curvature than is justified by the size of the estimated errors. If the magnitude of the standard deviations are estimated correctly and the line is drawn correctly the curve should cut about two thirds of the error lines (very roughly).

When these steps are applied to the example of the moving object given above, a graph results such as that shown in the following figure.

Velocity v versus Time t



(This sample graph is actually too small for accuracy

II. Determination of a Mathematical Relationship

If a graph of dependent variable y versus independent variable x turns out to be a straight line, the dependence of y on x is expressed by the equation

$$y = ax + b \tag{1}$$

The slope a and y intercept b of the line can be taken directly from the graph (see part III) thus establishing the relationship between quantity y and quantity x in this experiment.

If the graph of y versus x is curved, however, as it is in the case of the velocity of an object versus the time in part I, the quantities must be related by some other equation. For example, one might guess that y is related to x according to an equation of the type

$$y = ax^n + b \tag{2}$$

where n might be an integer $-1, \pm 2, \pm 3, \pm 4, \ldots\ldots\ldots\ldots$ or a fraction $\pm 1/2, \pm 1/3, \pm 1/4, \ldots\ldots\ldots\ldots\ldots$ To decide which values of n are truly possibilities one should study the graph of y versus x and equation (2). In the case of the velocity versus time graph of part I, for example, negative values of n should be immediately discounted since equation (2) would predict a decrease in y for increasing x. Fractional values of n are just as unlikely since as x increases, the graph shows y increasing faster and faster (perhaps indicating n = + 2 or + 3, etc.).

To see if the velocity - time (y = v, x = t) data for the moving object example of part I fits equation (2) with n = + 2 one could graph Y = v versus $X = t^2$ from the experimental values of v and the corresponding values of t^2. If the graph of Y versus X from the data is a straight line, the experimental results fit a relationship

$$Y = a X + b$$

or $$v = a t^2 + b \qquad \text{(equation 2 with n = + 2)}$$

where a and b are the slope and intercept of the line. If such a graph was not straight, but was straighter than a graph of v versus t, then one might try a graph of Y = v versus $X = t^3$ and so on until a straight line was found. The same general procedure could be followed in cases where n is thought to be a fraction or have a negative value. If the data are to be represented by the equation

$$y = ax^{-1/3} + b \tag{3}$$

then a graph of y versus $x^{-1/3}$ should yield a straight line.

Another type of relationship between quantities which appears often is

$$y = Ae^{ax} \tag{4}$$

where A and a are positive or negative constants. If equation (4) accurately represents the data, then

$$\ln y = ax + \ln A$$

$$\text{or} \qquad Y = ax + b$$

making the substitutions $Y = \ln y$ and $b = \ln A$. Therefore if $Y = \ln y$ is plotted vertically against x horizontally, a straight line of slope a and intercept $b = \ln A$ should result. The values of a and A can be determined from this line.

III. Determination of Slope and Intercept

The slope and intercept of a straight line are found as follows: First the x and y coordinates of two widely separated points on the line are determined (note that the points must be widely separated for accuracy and the points are points on the line, not data points). The slope of the line is defined

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

and should have the same value (for a straight line) regardless of what two points are chosen. The y intercept is obtained by extending the line back to $x = 0$ and noting the value of y at this point on the line (this is the intercept b).

A more reliable determination of slope, a, and y intercept, b, results when one computes the slope and intercept of the straight line which minimizes the sum of the squares of the deviations of the data points from the line (see instruction sheet on "Method of Least Squares").

References:

1. Kruglak and Moore, "Basic Mathematics for the Physical Sciences", chapter 7.

2. G. Wootan, Inc., "Graphs"

3. Ford, "Basic Physics", section 7.6

Physics III Lab. Fall 1969

Experiment 2 'Mapping' an Electric Field in Two Dimensions

Purpose

To provide a look at equipotential lines in a two dimensional system. Hopefully an electric field may become more real (less abstract) to you.

The 2-D system is a thin layer of slightly-conducting graphite on the surface of a board. The equipotenital lines (each, the locus of points having a given constant potential) correspond to equipotential surfaces in a three dimensional system. You know where to look these up - refresh your memory!

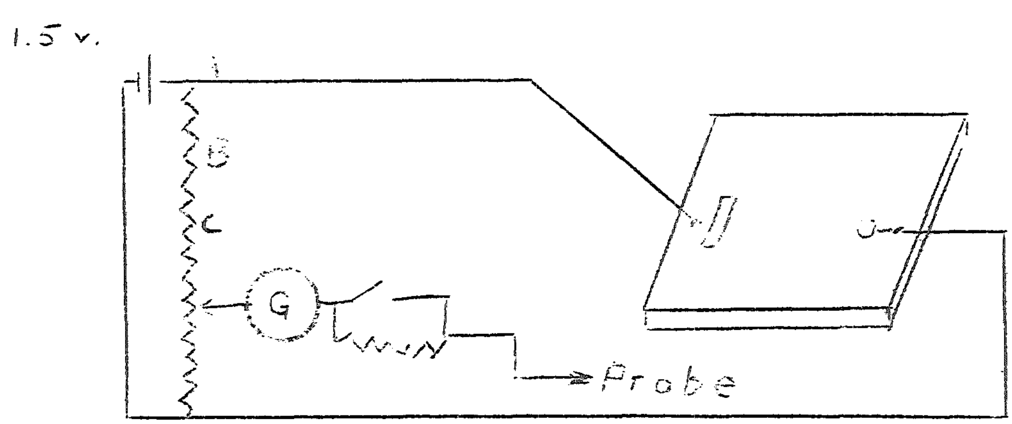
When two points in a conducting body or circuit are at different potentials a current flows between the points. You will use a galvanometer G to determine a condition of zero-current flow; i.e., the condition of zero-potential difference between the points to which the galvanometer is connected. If you have not learned the principles of this instrument, you need only assume that zero-deflection means zero-current in the galvanometer.

Map at least two field-electrode configurations (at least one for the report of each student). Tentatively sketch some field lines ("lines of force").

Take care: some galvanometers have two button switches to connect them: Use R first until the deflection is very small, then use the more sensitive O-button.

Discuss in report the relation between field lines and equipotentials, and the theory behind this relation. Indicate the direction of the E-vector on your plot.

Explain the analogy between your equipotential plot and another kind of map. (Incidentally, can the energy concept be included in your explanation?)



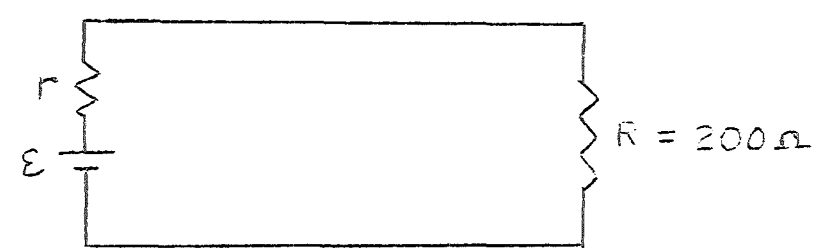
POTENTIOMETER MEASUREMENT OF POTENTIAL DIFFERENCE

OBJECT: To calibrate a potentiometer and to become familiar with its use in measuring potential differences.

THEORY: One of the most obvious advantages of using ammeters, voltmeters, etc. in making circuit measurements is their convenience; they can be easily moved about from one part of the circuit to another. For this reason and also because their accuracy is sufficient for the purpose at hand in many instances, they are widely used in measuring the currents, voltages, etc. in a variety of circuit applications. However, it should be remembered that there are different types of voltmeters, for example, having different characteristics and for use under different conditions. One should not assume that all voltmeters, if operating correctly, will automatically read correctly the voltage he wishes to measure in any circuit.

One cause of incorrect meter readings is of course that the instrument is not properly calibrated. This can be quickly remedied by recalibrating it against a standard and making a calibrating graph which shows the correct value for any meter reading.

Often, however, the difficulty is not in the calibration but in the use that is made of a meter. Suppose in the circuit below for example, that you wish to measure the voltage across the 200 ohm resistor with a voltmeter having only 200 ohms resistance.



An ideal voltmeter would be one with infinite resistance; if it were placed in parallel with R it would draw no current and the total resistance of the parallel combination would remain 200 ohms (there would be no change in the operation of the circuit due to the introduction of the voltmeter). In the case of a 200 ohm voltmeter, however, the introduction of the meter causes the resistance of the parallel combination to be cut in half and the total current to increase, half going through R and the other half through the voltmeter. The meter will correctly read the voltage across its terminals, but that voltage is no longer the same as it was when the meter was not present. This difficulty can usually be avoided by taking care that the resistance of the voltmeter be large compared to the total resistance of the circuit between the points to which the meter is connected.

The potentiometer is in effect an "ideal" voltmeter. It draws no current from the circuit at the instant of measurement and thus doesn't change conditions in the circuit from what they were before its introduction.

The potentiometer circuit is shown on the next page. The power source supplies direct current through a variable resistance to a uniform resistance slide wire CD. The variable resistance may be increased or decreased in order to control the amount of current I flowing in the wire. Since the slide wire is uniform along its length, its resistance per unit length is the same everywhere so that

$$\frac{V_{TD}}{V_{CD}} = \frac{I\,R_{TD}}{I\,R_{CD}} = \frac{L_{TD}}{L_{CD}} \qquad (1)$$

Thus if a certain fixed voltage or potential difference V_{CD} is impressed across points CD, the potential difference V_{TD} may vary from zero to V_{CD} depending on the position of the sliding tap T. The laboratory potentiometers are equipped with dials which tell where along the wire point T is for a particular setting of the dial --- a reading of .400 for example means l_{TD} = .400 l_{CD} (decimal point omitted on dial).

OPERATION:

The process of measuring an unknown potential difference with a potentiometer is one of comparing the unknown to an accurately known voltage. For example in the circuit shown on the next page suppose an unknown voltage is placed across terminals X and switch S_1 is to the right. Suppose in addition that it is known that the voltage drop across CD is exactly V_{CD} = 2.000 volts. Now suppose that tap T is moved along the wire until a point is found for which there is no noticeable deflection of the galvanometer when either the R or Q switch is closed. This would mean that there was no current flow from T to E or vice versa, a condition which could only arise if T and E were at the same potential. Therefore at this balance point $V_X = V_{ED} = V_{TD}$.

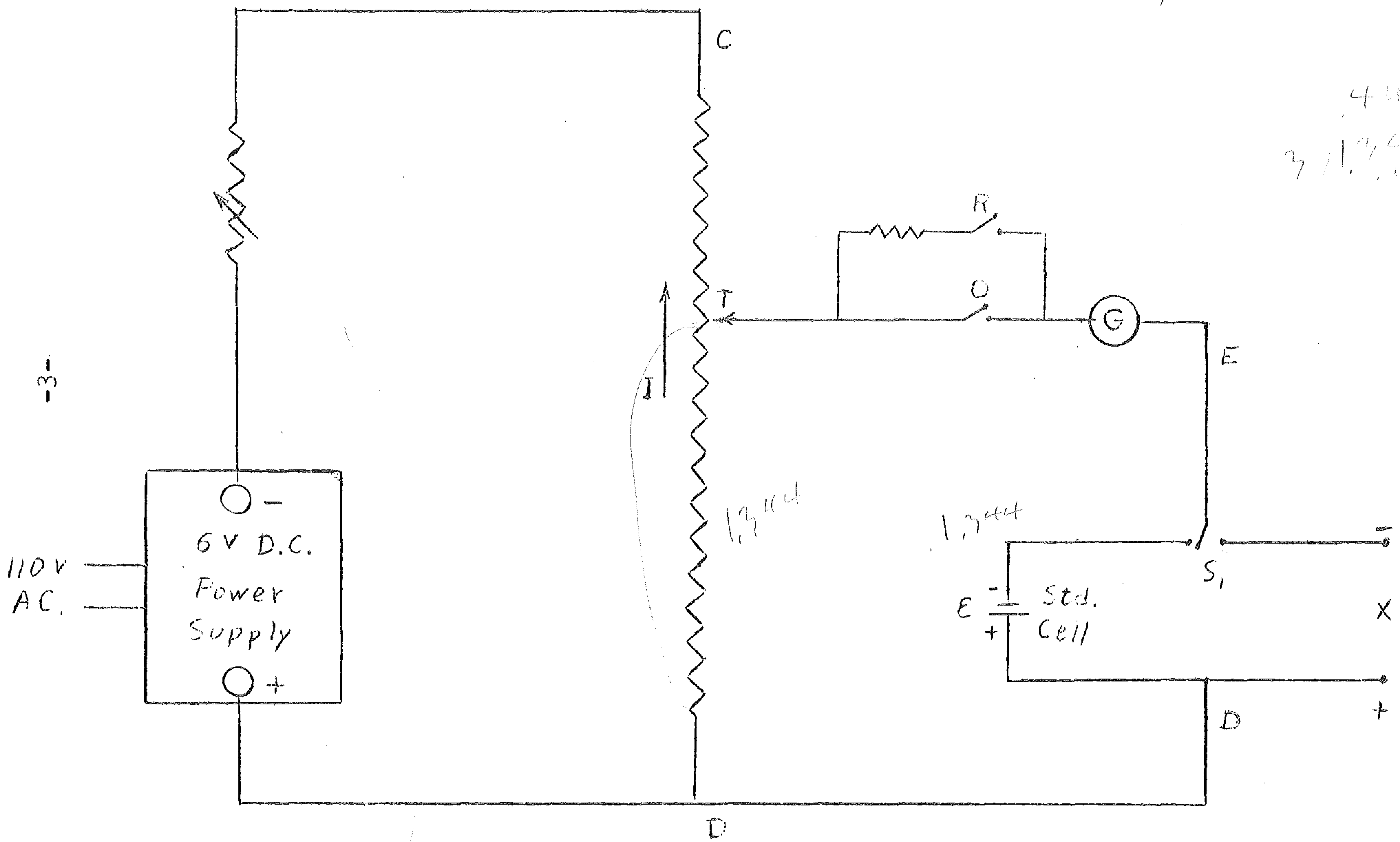
But equation (1) tells us that if T is halfway between C and D, V_{TD} = 1.000 volts, or if it is 0.600 of the way from D to C, V_{TD} = (0.600) (2.000) = 1.200 volts, etc. Thus V_X is determined by noting what fraction of the wire (fraction of the 2.000 volts across the wire) it will balance against. However, the process of getting an accurately known voltage (2.000 volts here) across CD must be accomplished frist -- a process called calibration.

The potential difference V_{CD} is controlled by the variable resistances in series with resistance R_{CD}. If the variable resistance is increased, the current I is decreased and thus the voltage $V_{CD} = IR_{CD}$ decreased; decreasing the variable

$\frac{1000}{3} = \frac{x}{1.344}$

.448
3)1.344

248



resistance increases V_{CD}. Suppose now that switch S_1 makes contact with the standard cell (left) which is an accurately known, constant emf. having the value (for example) $\mathcal{E}$ = 1.500 volts. As before, if at any position of the tap T there were no current through the galvanometer when the R or O switch is closed, the voltage drop across TD would have to equal $\mathcal{E}$. If one puts the tap T three-fourths of the way from D to C, then adjusts the variable resistance until there is no galvanometer current, he is assured of having placed 1.500 volts across TD (3/4 of the wire) and by proportion [equation (1)] of having placed 2.000 volts across CD (the whole wire). The process of calibration becomes one of placing the standard voltage across a certain fraction of the wire in order to have the desired voltage V_{CD} across the whole.

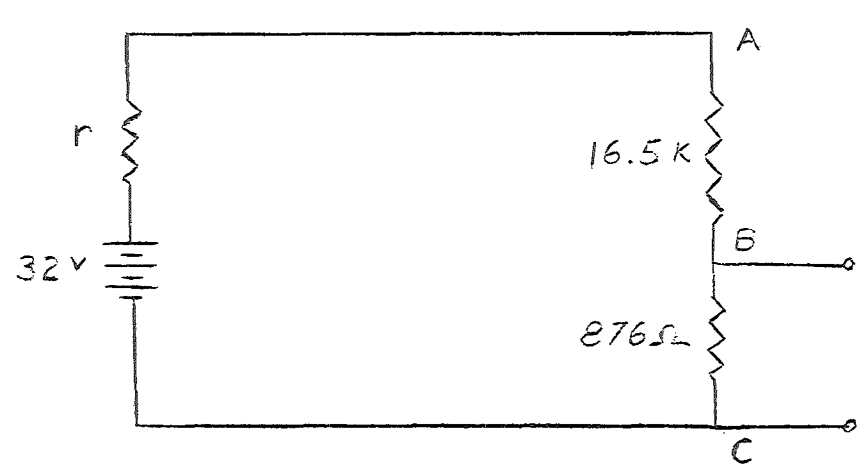
IMPORTANT NOTE: In balancing the potentiometer always tap switch R first until the galvanometer shows no deflection and only then start using switch O. Do not hold either switch down; do not touch switch O until balance is made with R -- very small currents drawn from the standard cell for a short time will ruin it.

INSTRUCTIONS:

I. A. Calibrate the potentiometer to read a maximum potential V_{CD} = 3.00 volts using the standard cell mounted on the potentiometer board.

B. Connect a dry cell to terminals X (note correct polarities) and measure its emf. Next, check the calibration again to see if V_{CD} has changed. If it has, repeat the calibration and measurement.

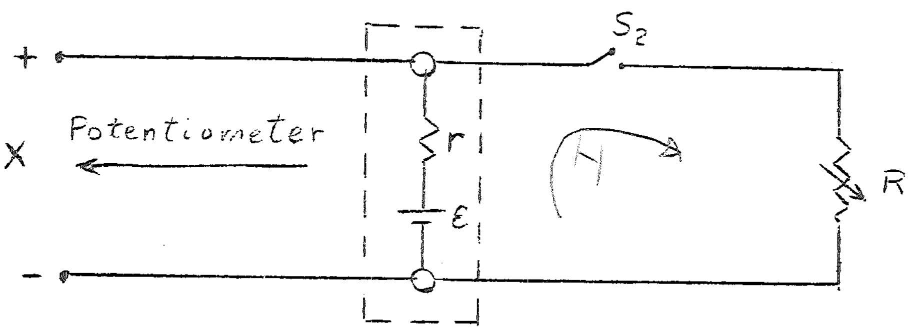
C. Set up a circuit as shown below and measure the voltage V_{BC} using a voltammeter and again using the potentiometer.



Assuming the internal resistance r of the 32 volt source to be negligible compared to the other resistances, what should voltage V_{BC} be according to theory (roughly -- the source voltage is only approximately 32 volts)? How do the voltages measured using the voltmeter and potentiometer compare with this value? Explain discrepancies and calculate the internal resistance of the voltmeter from these measurements assuming that the meter is calibrated correctly. From your data what is the best value that you can obtain for the actual emf of the "32 volt" source?

II. A. Connect the unknown ("black box") to terminals X (this contains a source of emf. and an internal resistance) and measure the emf.

Now set the dials of a resistance box on 500 ohms first and then connect it across the terminals of the unknown as shown below.



Measure the terminal voltage V_T of the "black box" with a 500 ohm resistor drawing current from it. Repeat with R = 200, 100, 70, 50, 40, 30, 20, and 10 ohms checking the calibration occasionally (caution: be very careful to have switch S_2 open while turning dials on the resistance box -- an accidental resistance of less than 5 ohms across the terminals will draw enough current to burn out the box).

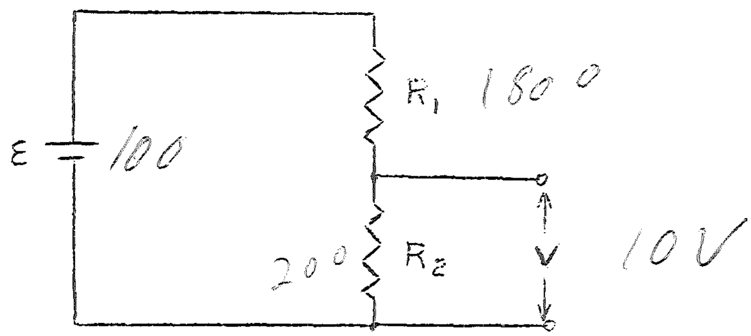
B. Plot a graph of the terminal voltage V_T of the "black box" versus the current I it supplies to the series loop shown above. Since

$$V = \mathcal{E} - I r = IR$$

this graph should be a straight line with slope (-r) and intercept $\mathcal{E}$ (if it is not, perhaps $\mathcal{E}$ is changing as current is drawn from the box). Determine the value of the internal resistance of the unknown from the slope of the line or, if it is curved, from the slope of the tangent at I = 0. Record the values of $\mathcal{E}$ and r for the unknown.

EXERCISES:

Given a circuit such as that shows below.



1. Show that potential difference $V = \left(\frac{R_2}{R_1 + R_2}\right) \mathcal{E}$

   If V is to be equal to (1/10) $\mathcal{E}$, what fraction must R_2 be of R_1? This circuit arrangement is called a voltage divider.

2. If R_2 = 200 ohms, R_1 = 1800 ohm, and $\mathcal{E}$ = 100 volts, what is the smallest resistance that a (100% accurate) voltmeter can have if it is to measure the voltage V with an error of less than 5%?

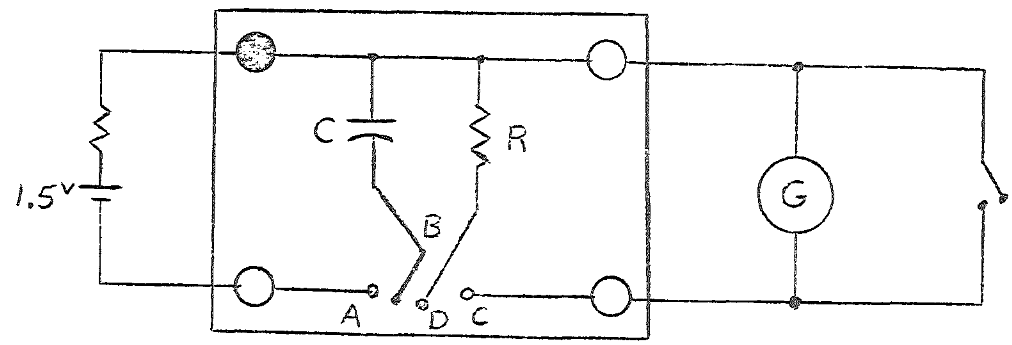
$$R_x = \frac{200\, R_{int}}{200 + R_{int}}$$

$$\frac{100}{1800 + R_x} = \frac{9.5v}{R_x}$$

HIGH RESISTANCE MEASUREMENT

OBJECT: To measure a high resistance using the ballistic galvanometer.

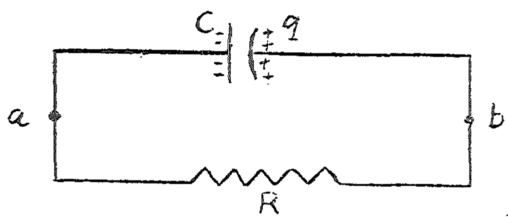
CIRCUIT:



THEORY:

A. Capacitor Discharge

If a capacitor is charged so that charge q_o is on each plate and then allowed to discharge through a resistor as shown below, the potential difference across the capacitor must equal that across the resistor (they are across the same two points in the circuit ab) at all times during the discharge.



q = charge on plates at any time t

i = current at the same time

$$V_{ab} = \frac{q}{C} = i\,R \qquad i = -\frac{dq}{dt}$$

$$\frac{1}{C}\,q = -R\,\frac{dq}{dt} \tag{1}$$

from which one can derive the expression for the charge still on the plates at any time t after discharging starts

$$q = q_o e^{-t/RC} \tag{2}$$

or

$$\ln(q/q_o) = -\frac{1}{RC}\,t \tag{3}$$

B. The Ballistic Galvanometer

The moving element of this type of galvanometer consists of a rectangular coil which is suspended between the poles of a magnet by a fine wire. When a charge q passes through the coil, the forces exerted by the magnetic field on the moving charges produce a turning moment or torque on the coil. The torque gives the coil an angular momentum, but the coil has a relatively large moment of inertia so that very little actual motion occurs in the time that it takes for charge q to pass through the coil. The coil continues to rotate however, twisting the suspension wire. The twisted suspension wire now exerts a restoring torque which decreases the angular momentum of the rotation, brings the rotation to a stop at some angle θ, and increases the angular momentum in the opposite direction. The result would be oscillatory motion of angular amplitude θ as long as no mechanical energy was lost from the system (it is almost frictionless). It is not difficult to show that the angle θ is proportional to the charge q which passed through the galvanometer coil.

If one wishes to damp the oscillations of the galvanometer it is useful to recall that a coil rotating in a magnetic field generates an induced emf. Short-circuiting the galvanometer terminals completes an external circuit so that this emf can cause a current flow in the low resistance short-circuit. Thus the mechanical energy of the rotating coil is converted to electrical energy as in a generator, and this electrical energy is in turn dissipated as heat by the circuit resistance. The loss of mechanical energy by the system results in a quick damping of the oscillation.

INSTRUCTIONS:

I. A. Charge the capacitor to the battery voltage by shorting A to B. Then short B to C and allow the capacitor to discharge through the galvanometer, noting the deflection D_o.

B. Recharge the capacitor as before, but this time connect B to D for a measured time t (5 seconds, 10 seconds, or whatever proves suitable) and allow the capacitor to discharge through resistance R for that time interval before discharging the remaining charge through the galvanometer. Again record the galvanometer deflection D.

C. Repeat step B for longer and longer time intervals t. Plot a graph of galvanometer deflection D versus discharge time t. How does this graph relate to a graph of the charge q on a discharging capacitor as a function of time?

II. A. Prove that $\ln (D/D_o) = -\frac{1}{RC} t$

B. Plot a graph of some function of D versus t which may be reasonably expected to yield a straight line. Determine the value of the resistance R from the slope of this line.*

III. To see if leakage of charge off the capacitor's plates is a factor which might cause error, charge the capacitor once again and let it sit for 5 minutes before discharging through the galvanometer. Compare the deflection due to the charge which remained on the plates with that due to the original charge q_o.

* The capacitors have the following values for the different circuit boards:

no. 1	C	=	0.88 μf
no. 2	C	=	0.90 μf
no. 3	C	=	0.88 μf
no. 4	C	=	0.88 μf
no. 5	C	=	0.91 μf

Electrical Conduction in Semiconductors

References: Halliday and Resnick, Physics, part II, Chap. 31, sec. 1-4; example 2; chap. 32, prob. 15 on Wheatstone bridge; Holden, A., "The Nature of Solids", chap. 13,14; Feynman, "Lectures on Physics", Vol. III, chap 14, sec. 1-5; and Purcell, "Electricity and Magnetism" (Berkeley Physics Course) Chap. 4.

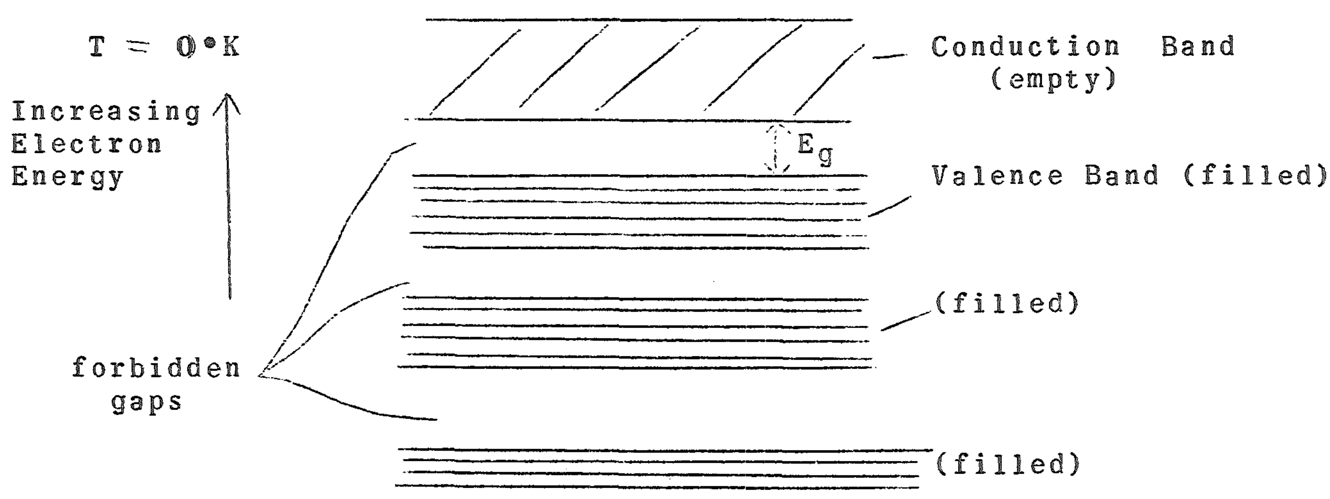
Object

1 - to investigate the temperature-dependence of resistance of a semiconductor

2 - to use a d.c. bridge circuit as a null-reading technique.

Theory

According to quantum mechanics an electron which is moving along in a crystalline solid may only have certain energies, these allowed energies being grouped in bands with gaps of forbidden electron energies between. There is a limit to the number of electrons which can take up energies within a given band; when this limit is reached the band is said to be filled. The distribution of electrons among the allowed energies is different in different solids but it is found in all pure semiconductors at T equals 0°K, that the lower bands are completely filled and the upper ones completely empty. The highest energy filled band is called the "valance" band and the next higher energy band the "conduction" band. The energy gap E_g between these two bands is small ($\sim$1 electron volt) in these materials (in comparison with insulators).



It can be shown that electrons of a completely filled band do not contribute to the conduction process; they can not be given, as a group, a net drift in one direction by applying an electric field. Thus a pure semiconductor at absolute zero would be a perfect insulator (conductivity $\sigma = 0$), since the application of an electric field produces no current. However, if one puts some energy into such a system, raising its temperature, some of this energy will be taken on by electrons in the valence band which then (in the energy absorption process) jump the forbidden gap and appear in the conduction band. Thus at finite temperatures there will be some electrons in the conduction band and an equal number of "holes" in the valence band. When an electric field is applied to the sample the electrons in the c. b. are given a drift velocity in the opposite direction to that of the field; the val. b. holes act like positive particles and drift in the same direction as the field. The conductivity of a given sample is determined by the numbers of conduction electrons and holes per unit volume and by the mobilities (average drift velocity per unit applied field E of these charge carriers.

$$\sigma = n e \mu_e + p e \mu_h \qquad (1)$$

where n = concentration of electrons in c.b.
p = conc. of holes in v.b.
e = electronic charge magnitue, hole charge magnitude
μ_h, μ_e = mobility of holes, electrons

Note that increasing the temperature of one of these materials causes more electrons to absorb energy and jump from the valence to the conduction band, increasing the number of conduction electrons n and holes p. So raising the temperature of a pure semiconductor increases its conductivity instead of decreasing it as in the case of metals where the number of carriers is not a function of temperature and the controlling factor is the decrease of mobility with increasing temperature. In pure ("intrinsic") semiconductors the relationship between temperature and the number of each type of carrier per unit volume is

$$n = p = c\, T^{3/2}\, e^{-E_g/2kT} \qquad (2)$$

where k is the Boltzmann constant, and c is a constant. Note that increasing T increases the number of both types of carriers and that decreasing the energy gap E_g has the same effect since this makes it easier for valence band electrons to gain enough energy to jump to the conduction band.

If one substitutes equation (2) into equation (1) and takes the natural logarithm of both sides he obtains

$$\ln \sigma = - E_g/2kT + \ln \left[ce(\mu_e + \mu_h)T^{3/2} \right]$$

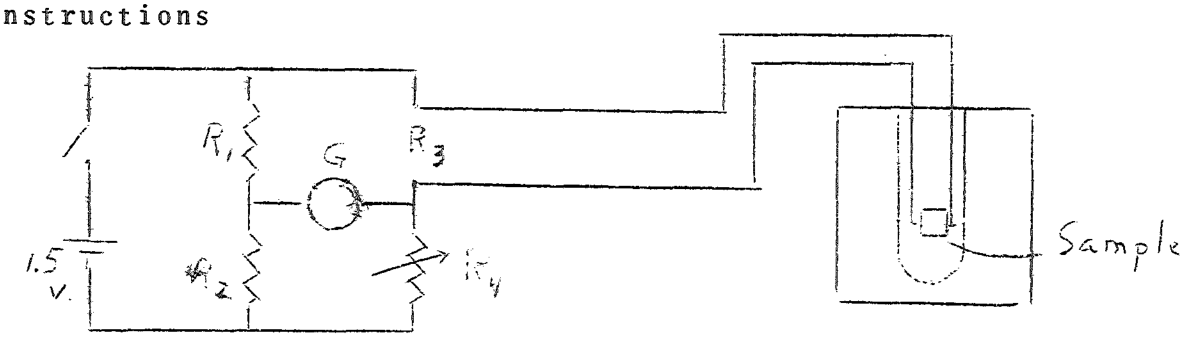
or in terms of the resistivity ρ of the sample,

$$\ln \rho = E_g/2kT - \ln \left[ce(\mu_e + u_h)T^{3/2} \right] .$$

The first term on the right varies much more strongly with temperature than the second; over a limited temperature range one can consider the second term nearly constant. Therefore if a graph of $\ln \rho$ versus

1/T is plotted, the result should be a straight line of slope ($E_g/2k$), to a good approximation. In this way the energy gap may be found.

Instructions



The resistance of the sample is measured using a Wheatstone bridge. The sample is placed in a calorimeter containing water, which may be heated .

I. Before coming to lab review the bridge theory and operation. Derive the fact that if no current flows throught the galvanometer in the above circuit when the switch is closed, $R_3/R_4 = R_1/R_2$ (from which one of the R's may be determined if the other three are known). In the set up you will use, there are two switches. Verify for yourself that when one of these is used $R_1 = R_2$, and that use of the other produces a 10 to 1 ratio in these two resistances.

II. Connect the circuit as shown and measure the unknown resistance R_3 by adjusting R_4 until there is no deflection of the galvanometer when you tap the O button on its case. (Start by using the less sensitive R button until you are close to the balance point.) Record R_3 and the temperature. Turn on the heater and raise the temperature of the sample about five digrees C. Disconnect heater, wait til T stabilizes, and again measure R and T. Measure four more points or so. <u>Do not allow the temperature to exceed 90°C or R to go below 15 ohms</u>. If time permits, measure at water-ice temperature.

III. Determine the energy gap graphically. Discuss errors carefully. Note that the meter you used need not be calibrated, as only zero ("null") deflections were used at bridge balance. This fact removes one source of systematic error, namely, an improperly calibrated scale.

Compare your results with those you would expect were the sampee a metal, such as copper. Discuss reasons for any differences.

Linear and Nonlinear Circuit Elements

Object: This experiment actually has two objects, (1) to demonstrate the use of the method of differences in the determination of the mathematical relationship between two variables and (2) to find out how the current through various circuit elements depends on the potential difference applied to them.

Theory: The resistance of a circuit element is defined as the potential difference across the terminals of the element divided by the current flowing in the element. (If the potential difference is in volts and the current in amperes, then the resistance is in ohms.) In a number of circuit elements, for example the common resistor, the current through the element is directly proportional to the voltage across it, provided that the current is not large enough to cause a significant amount of heating or the voltage large enough to cause electrical breakdown. When the current is proportional to the voltage the resistance R in the equation $I = V/R$ is constant and the element is referred to as being linear since a graph of I versus V would yield a straight line through the origin.

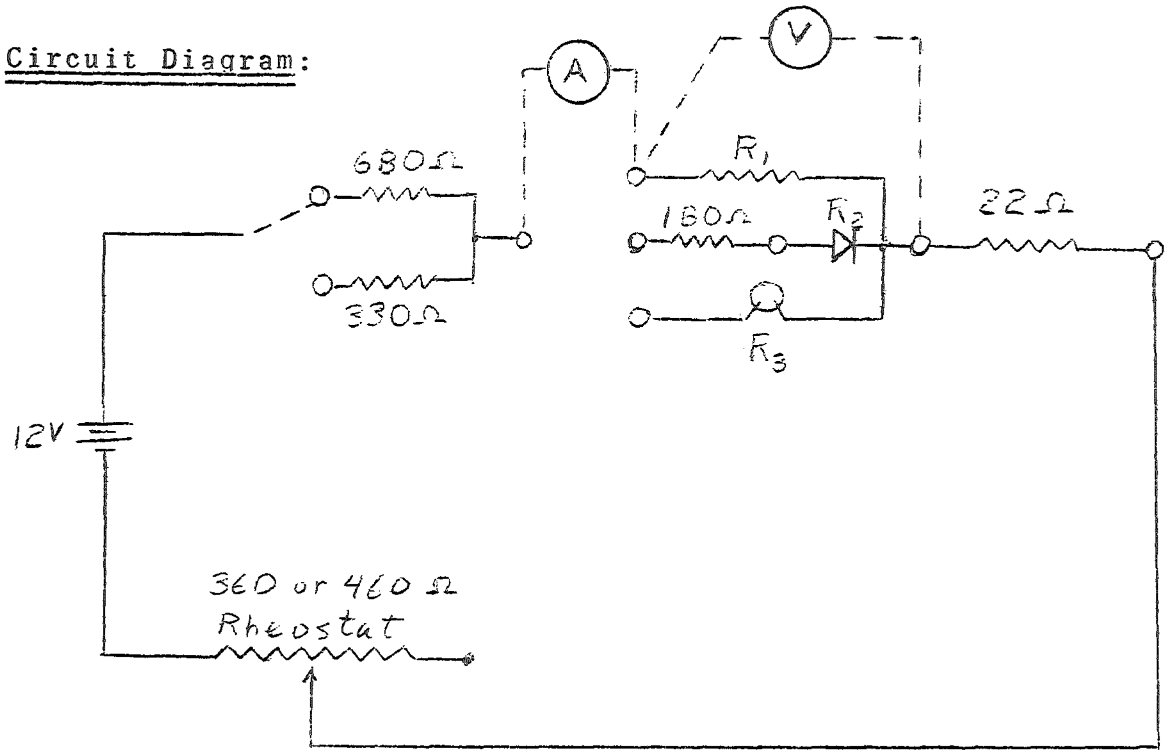
Nonlinear elements are those in which, for one reason or another, the current is not directly proportional to the potential difference across them. One can still define $R = V/I$ for these elements but it is apparent that R is no longer a constant in this case, but is a function of the current through the element. One example of a nonlinear element is an ordinary light bulb which, as the current is increased, heats up more and more, resulting in an increase in the filament resistance. Some other circuit elements are intrinsically nonlinear (without any temperature change or other changes in experimental conditions), such as the vacuum tube diode or solid state diode. These particular examples are not only nonlinear but also non-symmetric, having a much higher resistance to current flow in one direction ("backward") than to flow in the other ("forward"). Some diodes are nearly linear in the forward direction and are useful in circuits where it is desirable to have I proportional to V when the current flow is in the forward direction, and negligible current in the backward direction when the voltage tends to cause a flow in that direction (rectification). There are other elements which are symmetric but nonlinear.

The circuit elements investigated in this experiment are (1) a wire wound resistor, (2) a globar resistor which is actually a semiconductor (whose resistance changes with temperature), and (3) a solid state diode. Further information about semiconductors and solid state diodes is contained in the following references:

Shortley and Williams, "Elements of Physics", 4th edition, pp. 712-14

Feynman, "Lectures on Physics", Vol. III, Chapter 14.

Circuit Diagram:



Instructions:

1. Connect the circuit as shown in the preceding diagram and have the instructor check your connections. Record the voltage across, and the current through, the resistor R_1 as the voltage is varied in 0.1 volt steps from 0 to 1.5 volts. The voltage is varied by changing the resistance in series with resistor R_1 using the variable resistance rheostat in series with the 680 or 330 ohm resistor, or by itself. The 22 ohm resistor is never to be taken out of the circuit or electrically by-passed during the entire experiment. When you have obtained your highest voltage and current readings reverse the polarity of the power source and the voltmeter and ammeter and record the voltage and current. Does the direction of the current have any effect on the resistance? Plot graphs of I versus V, making one direction of current flow and one potential difference polarity positive on the graph and the other negative. Also plot the resistance R of the resistor for positive (one direction) and negative (the other direction) current.

1 reading

2. Repeat part 1 for the diode R_2 (1N100) and the globar resistor R_3 (Workman Electronic Products, model FR9). The 180 ohm resistor should always be left in series with the diode and the voltage across the diode may be varied in 0.05 volt steps from about 0.6 volts to 1.0 volts. Increase V across the globar resistor in 0.2 volt steps between 2 and 5 volts. In the case of the globar resistor you will need to keep readjusting the voltage after you set it since the resistance of the element will continue to change until the temperature levels off at the new value. Record voltage and current readings only after equilibrium is obtained at the required voltage.

3. Using the method of differences (see separate instruction sheet) determine power series $I = A + BV + CV^2 + \ldots$ which approximate the behaviour of these three circuit elements (for one direction of current flow only....the direction in which you have complete data). In each case make a table such as that shown below and compute first, second, etcetera differences. Stop taking differences when the last column shows no trend but only random deviations.

TABLE I

Potential Difference V (volts)	Current I (ma)	First Difference ΔI (ma)	Second Difference $\Delta^2 I$ (ma)
0	0		
		11	
0.1	11		16
		27	
0.2	38		14
		41	
0.3	79		18
		59	
0.4	138		
	etc.		

Record a decision as to the number of terms that must be kept in each case (resistor, diode, globar resistor) in order to closely approximate the correct relationship between I and V for each of these elements.

Finally, take the three expressions which approximately relate I to V for the three circuit elements (e.g. $I = A + BV + CV^2$), and substitute equally spaced values of voltage 0, Δv, $2\Delta v$, $3\Delta v$, $4\Delta v$....into the expressions. Make another difference table for each of the three expressions such as that shown below. By comparing each table with the corresponding table made from the data determine the constants A,B,C....for that circuit element.

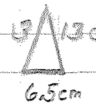
TABLE II

V	I	ΔI	$\Delta^2 I$
0	A	$B\Delta v + C\Delta v^2$	$2\ C\Delta v^2$
Δv	$A + B\Delta v + C\Delta v^2$	$B\Delta v + 3\ C\Delta v^2$	$2\ C\Delta v^2$
$2\Delta v$	$A + 2B\Delta v + 4\ C\Delta v^2$	$B\Delta v + 5\ C\Delta v^2$	$2\ C\Delta v^2$
$3\Delta v$	$A + 3\ B\Delta v + 9\ C\Delta v^2$	$B\Delta v + 7\ C\Delta v^2$	
$4\ \Delta v$	$A + 4\ B\Delta v + 16\ C\Delta v^2$		

PENDULUMS

10/10/69 ①

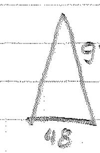
ALL PEND. RELEASED AT 40°



PERIOD (OSS/SEC) Ⓐ ±.02	
3.4/5.0	= .680
3.6/5.0	= .720
3.4/5.0	= .680
3.6/5.0	= .720
7.2/10.0	= .720
7.0/10.0	= .700
6.8/10.0	= .680
10.5/15.0	= .700
10.6/15.0	= .707
13.8/20.0	= .690
14.0/20.0	= .700
14.3/20.0	= .701
17.4/25.0	= .697
17.2/25.0	= .688
16.9/25.0	= .677
17.0/25.0	= .680
20.8/30.0	= .694
21.0/30.0	= .700
21.4/30.0	= .713

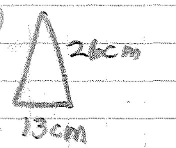
PERIOD (OSS/SEC) Ⓒ ±.02	
7/5	= 1.40
7/5	= 1.40
14.2/10	= 1.42
13.8/10	= 1.38
13.6/10	= 1.36
20.4/15	= 1.36
20.8/15	= 1.39
21.2/15	= 1.41
20.6/15	= 1.37
27.7/20	= 1.39
28/20	= 1.40
27.5/20	= 1.37
34.2/25	= 1.37
34.7/25	= 1.39
35.3/25	= 1.41

PERIOD (OSS/SEC) Ⓑ ±.02	
5/5	= 1.00
5/5	= 1.00
10/10	= 1.00
15/14.9	= 1.01
15/15	= 1.00
19.8/20	= .99
19.9/20	= 1.00
20.0/20	= 1.00
24.8/25	= .99
24.9/25	= 1.00
29.9/30	= 1.00
29.9/30	= 1.00



PERIOD (OSS/SEC) Ⓓ ±.02	
9.6/5	= 1.92
9.8/5	= 1.95
18.8/10	= 1.88
27.2/15	= 1.81
36.3/20	= 1.81
19/10	= 1.90
27/15	= 1.81
36.6/20	= 1.83
180.6/100	= 1.81
91.0/50	= 1.82

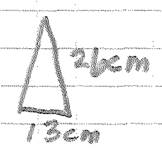
Weshah
10X

(E) 

(more mass than (B))

PERIOD		(OSS/SEC) ±.02
5.3/5	=	1.06
4.9/5	=	.98
10/10	=	1.00
10.2/10	=	1.02
15/15	=	1.00
20.3/20	=	1.01

$M_B - M_A = 205g$

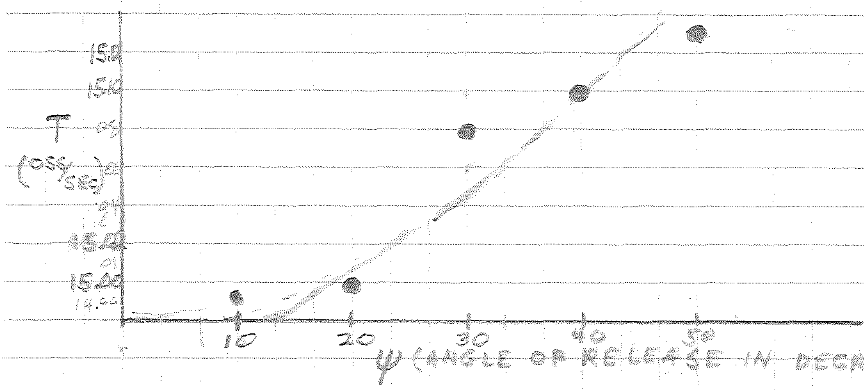
(F) 

PEND. RELEASED AT DIFFERENT ANGLES

°RT OF ⊥	PERIOD (OSS/SEC)	$\overline{T}$
10°	15.0/15	14.99
10°	15.0/15 ✓ small	
10°	14.9/15	
20°	14.9/15	15.00
20°	15.0/15	
20°	15.1/15	
30°	15.1/15	15.08
30°	15.1/15	
30°	15.0/15	
40°	15.2/15	15.10
40°	15.0/15	
40°	15.1/15	
50°	15.1/15	15.13
50°	15.2/15 ✓ large	
50°	15.1/15	

CONCLUSIONS CONCERNING ANGLE OF RELEASE

EXAMINATION OF DATA IN PT. F OF PENDULUM EXPERIMENT SHOWS CLEARLY THAT THE PERIOD SHOWS CLEARLY THAT THE PERIOD INCREASES VERY SLOWLY AS THE ANGLE OF RELEASE INCREASES (<.97% in 40°)



IT MAY BE STATED THAT THE PERIOD OF THE PENDULUM IN NEARLY INDEPENDENT OF THE ANGLE OF RELEASE.

POSSIBLY THE FRICTION CAUSED FROM THE BALL BEARINGS AT THE PENDULUM'S ROTATIONAL AXIS INTRODUCED SOME SYSTEMATIC ERROR, IN THAT BY INCREASING THE ANGLE OF RELEASE, ONE INCREASES THE ACCELERATION WHICH INCREASES THE FORCE ACTING OVERALL ON THE PENDULUM WHICH INCREASES THE AMOUNT OF FRICTION.

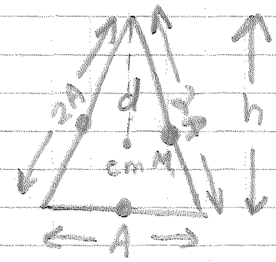
CONCLUSIONS CONCERNING PENDULUM'S MASS

THE PERIOD OF A PENDULUM IS ALSO INDEPENDENT OF THE MASS AS LONG AS THE PENDULUM'S DIMENSIONS ARE IDENTICAL. THIS CAN BE DEDUCED FROM PARTS B AND E WHERE PENDULUMS OF SUBSTANTUALLY DIFFERENT MASSES HAVING IDENTICAL DIMENTIONS PRODUCED IDENTICAL PERIODS (1.00 ± .02 OSC/SEC)

COMPUTATION OF g FROM THE PERIOD

$$g = \frac{4\pi^2 I}{m d T^2}$$

I) CALCULATION OF d



$$c.m. = \frac{\frac{h}{2}M_1 + \frac{h}{2}M_1 + h\frac{M_1}{2}}{M_1 + M_1 + \frac{M_1}{2}}$$

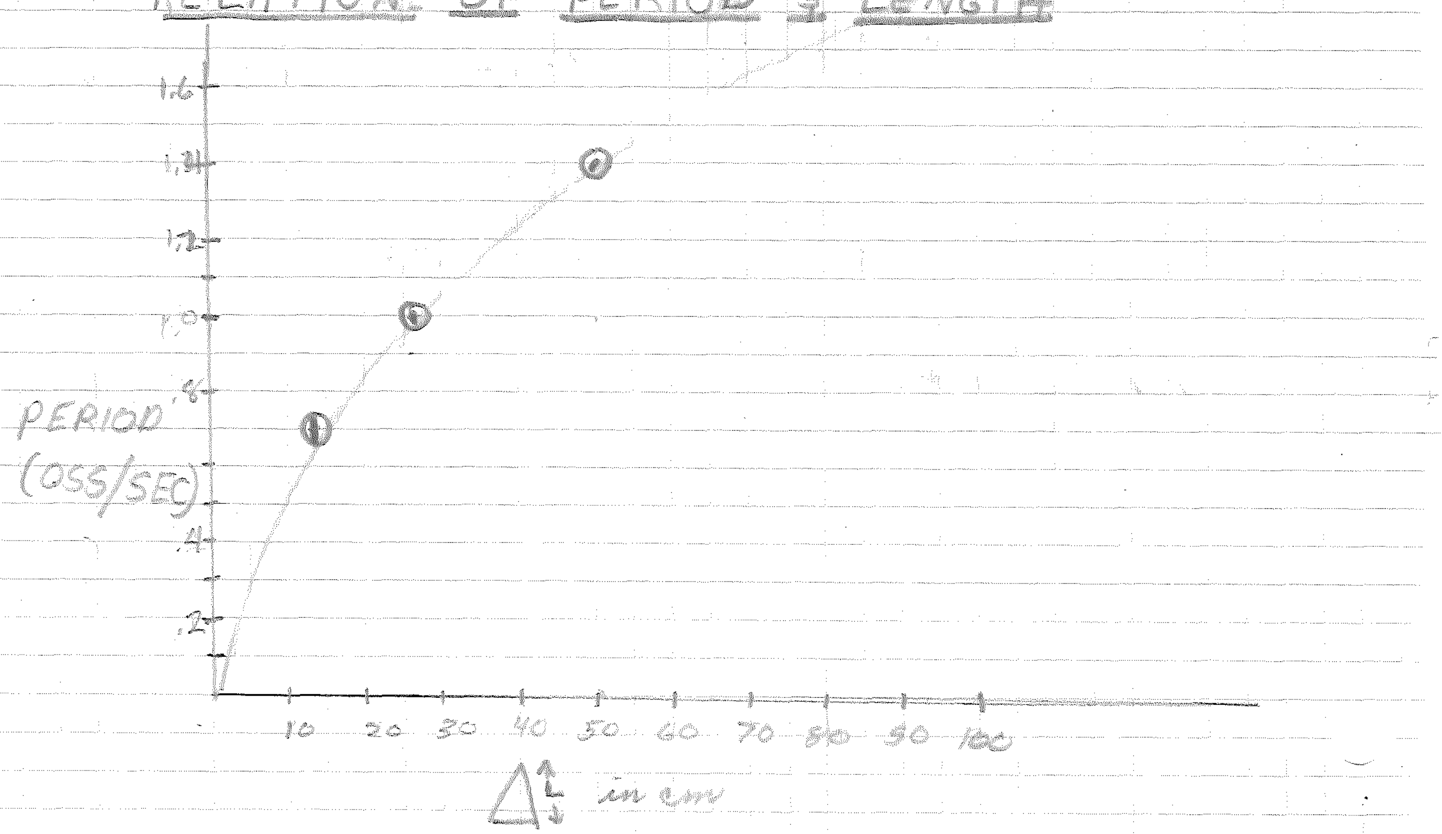
$$= \frac{3}{5}h = d$$

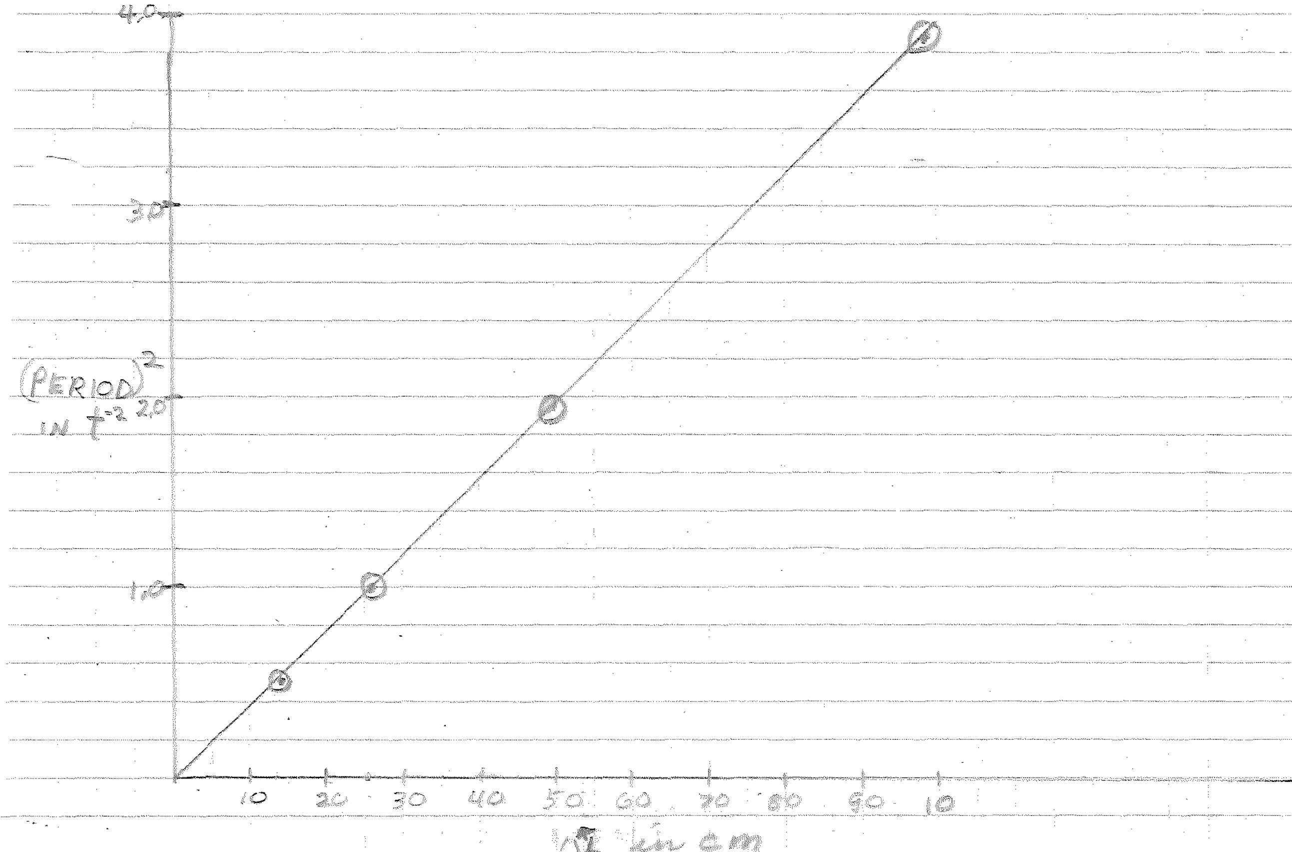
(CONT. ON Pg. 6)

RELATIONSHIP OF LENGTH AND PERIOD

THERE IS A QUADRATIC RELATIONSHIP BETWEEN THE SIMILAR SHAPES OF THE PENDULUMS AND THEIR PERIODS. ($L = f(T^2)$) WHERE IS ANY LINEAR LENGTH OF THE PENDULUM. IN THIS CASE, THE PERIOD IS COMPARED WITH ONE OF THE ISOCOLES PENDULUM'S LEGS

RELATION OF PERIOD & LENGTH

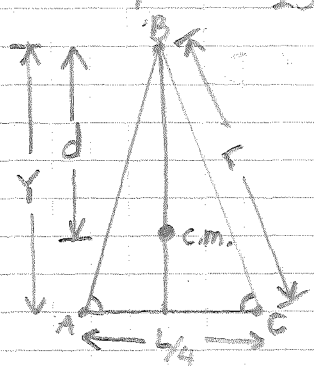




THE LENGTH OF SIMILAR PENDULUMS IS DIRECTLY PROPORTIONAL TO THE SQUARE OF THEIR PERIODS

$$L \propto T^2$$

$$T^2 = \frac{L}{25} \frac{\text{sec}^2}{\text{cm}}$$ FOR ALL ΔABC WITH AXIS AT B



SIMILARLY:

$$Y = \sqrt{15}\, L/4 \Rightarrow T^2 = \frac{4Y}{25\sqrt{15}}$$

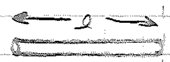
AND

$$d = 3Y/5 \Rightarrow T^2 = \frac{4d}{15\sqrt{15}}$$

COMPUTATION OF I

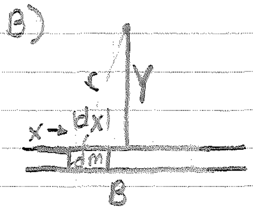
II) A) $I_1 = \frac{Ml^2}{3}$ $\quad l = 2B$

$M = \frac{2}{5} m$ where m = entire pendulum's mass



IBID Pg 272 AXIS I for Λ $= \frac{16 m B^2}{15}$

B)



$m_B = \frac{m}{5}$

$Y = \frac{\sqrt{15}\, B}{2} \Rightarrow Y^2 = \frac{15B^2}{4}$

$\sigma = \frac{M_B}{B} = \frac{m}{5B} = \frac{\Delta m}{5\Delta X}$

$dm = 5\sigma\, dx \qquad r^2 = Y^2 + \left(\frac{B}{2} - X\right)^2$

$= Y^2 + \frac{B^2}{4} - BX + X^2$

$= 4B^2 - BX + X^2$

$I = 5\sigma \int_0^B (4B^2 - BX + X^2)\, dx$

$= 5\sigma \left[4B^3 - \frac{B^3}{2} + \frac{B^3}{3}\right] = \frac{23 B^2 m}{6}$

$\Sigma I = (4.9)\, m B^2$

III) COMPUTATION OF g

$g = \frac{4\pi^2 I}{m d T^2}$ from DATA B & ABOVE

$B = .13\, m \qquad T = 1.00/\text{SEC}$

$I = 4.9 m B^2 \quad d = (3/5) Y$

$Y = \sqrt{15}\, B/2$

$g = \frac{4\pi^2 (4.9) m (.13)^2 (5)(2)(m)}{m (3) \sqrt{15}\, (.13)\, (1.00)^2\, (\text{SEC})^2} = 21.6 \frac{m}{\text{SEC}^2}$ X.

ERROR IN CALCULATION OF g OCCURED IN ALL PROBABILITY IN COMPUTATION OF I

9/10

CONCLUSIONS:

1) THE PERIOD OF A PENDULUM IS INDEPENDENT OF ITS MASS THUS DEPENDING ON ITS ACCELERATION, AND NOT THE FORCE ACTING UPON IT.
2) THE PERIOD OF A REAL PENDULUM WITH (FRICTION ≅ 0 IS NEARLY INDEPENDENT OF THE ARC SWEPT BY IT
3) ANY SIMILAR LINEAR MEASUREMENTS OF LINEARLY SIMILAR PENDULUMS ARE PROPORTIONAL TO THE SQUARE OF THEIR PERIODS
4) g CAN BE CALCULATED BY COMPUTING THE PENDULUM'S MOMENT OF INERTIA AND CENTER OF MASS & MEASURING ITS PERIOD
5) COMPUTATION OF THE MOMENT OF INERTIA FOR AN ISOCOLES PENDULUM IS A HAIRY PROCESS

FORCED OSCILLATIONS & RESONANCE

PURPOSE - TO STUDY FORCED OSCILLATION AND RESONANCE

II (PEAK AMPLITUDE) PERIOD IS CONSTANT

	TRIAL I	7.5	6.9	6.2	6.5	5.0	4.9	$T = 2.09 \frac{1}{sec}$
	" II	7.5	6.8	6.2	5.6	4.8	4.2	
DAMPED	TRIAL I	5.6	3.8	2.4	1.2	.4	.1	$T = 2.10 \frac{1}{sec}$
	II	5.6	3.6	2.3	1.2	.4	.1	

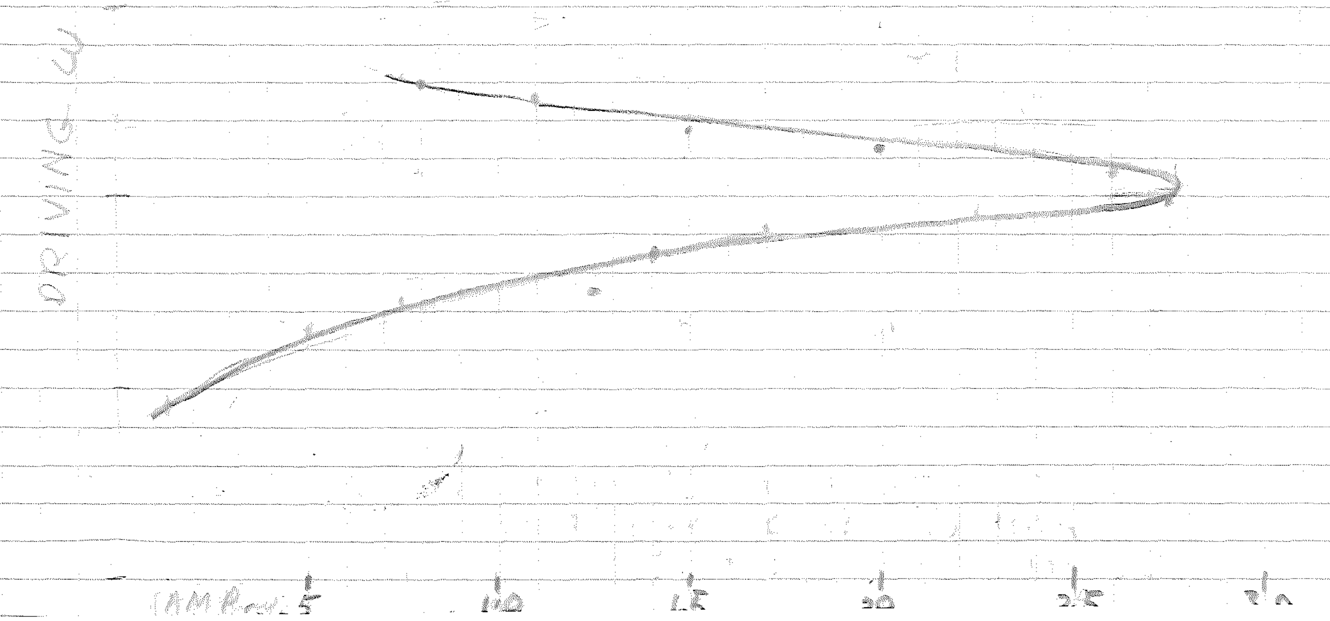
III)

T	$\frac{22.5}{5}$	$\frac{13.6}{5}$	$\frac{12.7}{5}$	$\frac{11.8}{5}$	$\frac{11.3}{5}$	$\frac{10.8}{5}$	$\frac{9.9}{5}$	$\frac{9.6}{5}$	$\frac{9.1}{5}$	$\frac{8.4}{5}$	$\frac{8.1}{5}$
A	.4	.8	1.1	1.5	2.0	2.6	2.75	2.25	1.7	1.4	1.25

T	$\frac{7.2}{5}$	$\frac{6.5}{5}$	$\frac{4.8}{5}$
A	.75	.5	1.25

Nephah
24th

5T



CALCULATIONS:

1) $Y = mX + B \longrightarrow A = mt + B$

$m = \frac{\Delta Y}{\Delta X}$ SLOPE AT $t=0$: FOR 0 VLTS $= \frac{-.25}{11.15} = -.022$

FOR 32 VLTS $= -.87/8.92 = -.098$

2) $-.022 = \frac{-1}{t_{r_1}}$ $t_{r_1} = 44.6$ SEC

$-.098 = -1/t_{r_2}$ $t_{r_2} = 10.2$ SEC

3) $\omega' = 2\pi\nu' = \sqrt{\frac{K}{m} - \left(\frac{b}{2m}\right)^2}$

CONCLUSIONS

1) DAMPING DOES NOT AFFECT THE THE PERIOD, BUT DOES AFFECT THE AMPLITUDE

2) IF DAMPING FORCE $\propto$ THE ANGULAR VELOCITY t_{r_1} & t_{r_2} CAN BE FOUND BY TA'KING THE NEGATIVE RECIPROCAL OF THE SLOPE. IF THEY ARE NOT PROPORTIONAL BUT A KNOWN RELATIONSHIP EXISTS, ON MAY FIND t_{r_1} AND t_{r_2} BY TAKING THE EQUATIONS DERIVITIVE

D.D

D.D

12/9

FALLING BODIES

NOV. 7, 1969

PURPOSE: TO USE INGENUITY IN THE QUANTITATIVE STUDY OF FALLING OBJECTS

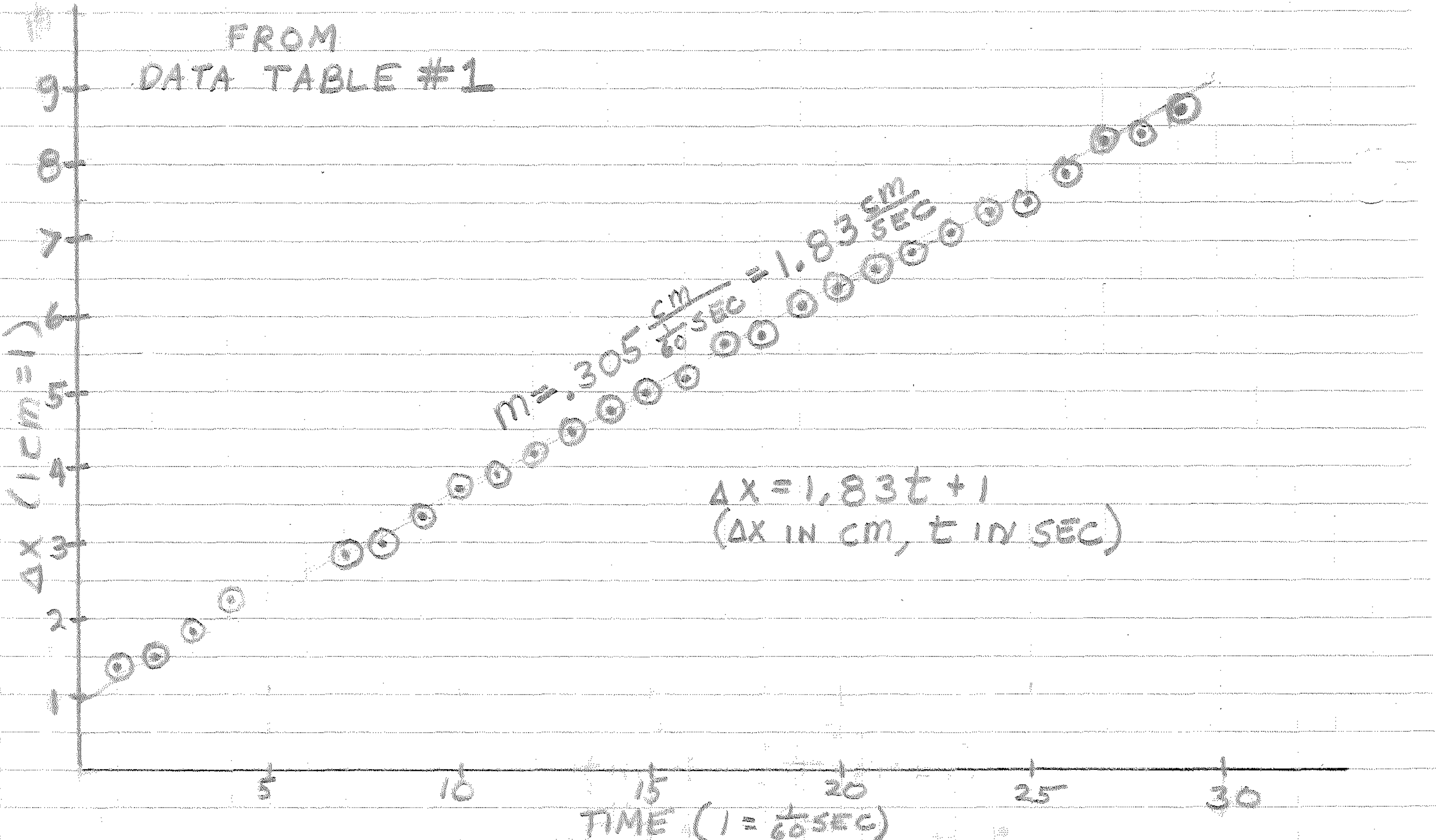
TABLE 1

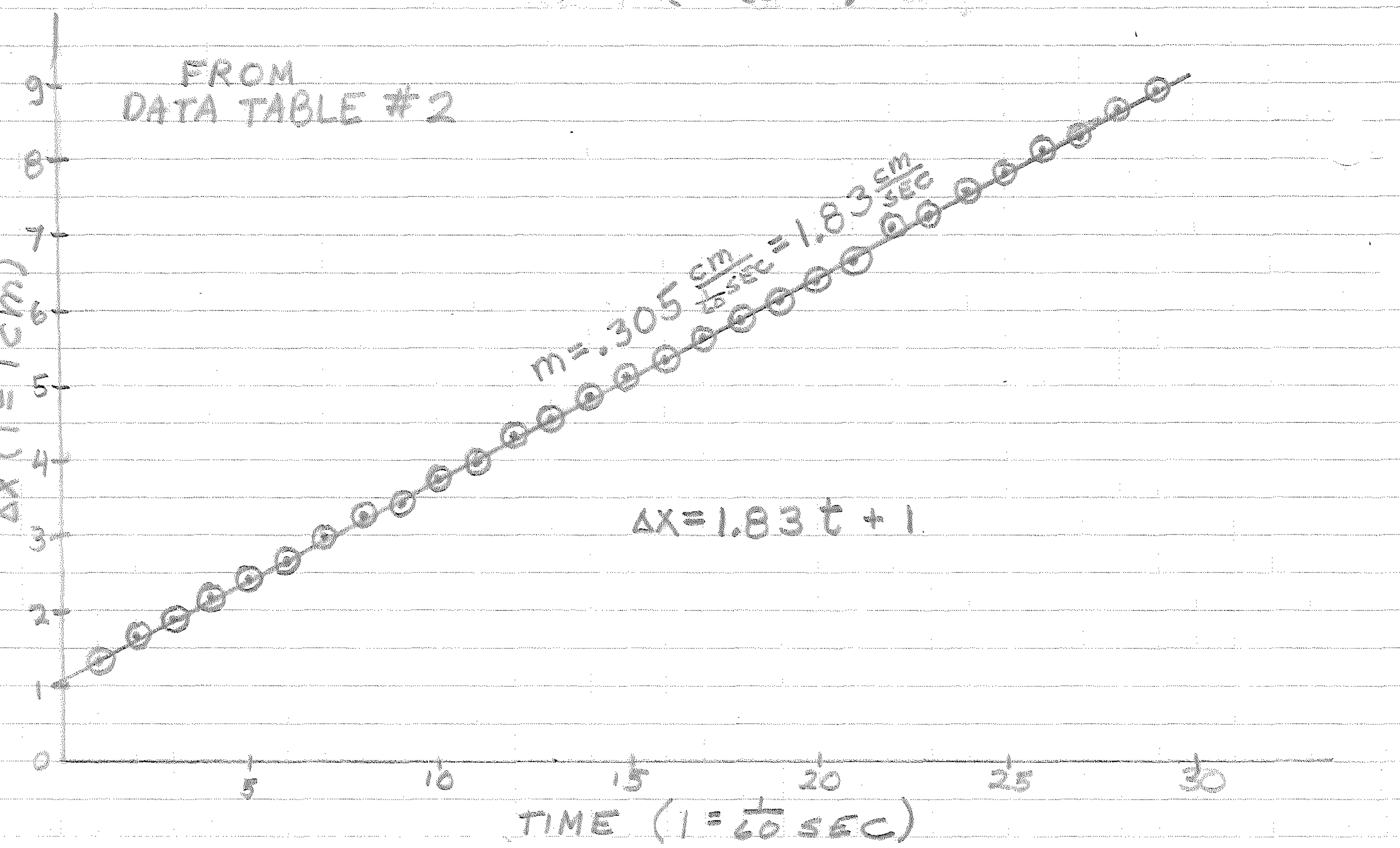
Δx (cm) ±.03	t ($\frac{1}{60}$ sec)
1.40	1
1.50	2
1.80	3
2.25	4
5.90	5
	6
2.80	7
3.00	8
3.35	9
3.70	10
3.95	11
4.20	12
4.50	13
4.75	14
5.00	15
5.20	16
5.65	17
5.75	18
6.15	19
6.40	20
6.60	21
6.85	22
7.15	23
7.40	24
7.50	25
7.90	26
8.30	27
8.40	28
8.75	29

TABLE 2

Δx (cm) ±.03 cm	t ($\frac{1}{60}$ sec)
1.35	1
1.65	2
1.85	3
2.20	4
2.45	5
2.65	6
3.00	7
3.25	8
3.45	9
3.75	10
4.06	11
4.35	12
4.55	13
4.85	14
5.15	15
5.35	16
5.70	17
5.85	18
6.20	19
6.45	20
6.70	21
7.10	22
7.25	23
7.55	24
7.80	25
8.10	26
8.30	27
8.65	28
8.90	29

D. D.





INTUITIVLY, AT $t=0$, $\Delta X=0$ LEADING TO THE CONCLUSION THAT THE INITIAL ΔX TAKEN MEASURMENT WAS NOT TAKEN AT $t=0$, HENCE $\Delta X = 1.83t$

DATA ANALYSIS OF FALLING BODY EXPERIMENT
FROM TABLE #2

QUADRATIC..... Y = A(1) + A(2)*X + A(3)*X*X

ERROR	PERCENT ERROR
-0.24601980	15.41458372
-0.16561540	5.23169699
-0.15379606	3.07358780
-0.06056179	0.85171608
0.01408742	0.14850889
0.02015156	0.16613202
0.10763064	0.71551658
0.17652463	0.96866616
0.17683359	0.81591031
0.20855748	0.82136919
0.22169633	0.75462604
0.31625008	0.94027602
0.34221878	0.89685190
0.39960241	0.93038117
0.48840099	1.01725625
-0.49138557	0.92120887
-0.38975712	0.66128051
-0.40671378	0.62757970
-0.34225547	0.48244233
-0.29638224	0.38318825
-0.26909393	0.32027711
-0.16039085	0.17633043
-0.12027293	0.12251461
-0.04873991	0.04613392
0.00420802	0.00371092
0.08857089	0.07295103
0.10434865	0.08045656
0.20154142	0.14578204
0.28014898	0.19048702

DATA ANALYSIS OF FALLING BODY EXPERIMENT
DATA — FROM TABLE #2

1.00000000	1.35000000
2.00000000	3.00000000
3.00000000	4.85000000
4.00000000	7.05000000
5.00000000	9.50000000
6.00000000	12.15000000
7.00000000	15.15000000
8.00000000	18.40000000
9.00000000	21.85000001
10.00000000	25.60000001
11.00000000	29.60000001
12.00000000	33.95000003
13.00000000	38.50000002
14.00000000	43.35000002
15.00000000	48.50000002
16.00000001	52.85000002
17.00000001	58.55000001
18.00000001	64.40000006
19.00000001	70.60000002
20.00000001	77.05000004
21.00000001	83.75000005
22.00000001	90.80000004
23.00000001	98.05000004
24.00000001	105.60000002
25.00000001	113.40000006
26.00000001	121.50000005
27.00000001	129.80000013
28.00000001	138.45000004
29.00000001	147.35000008

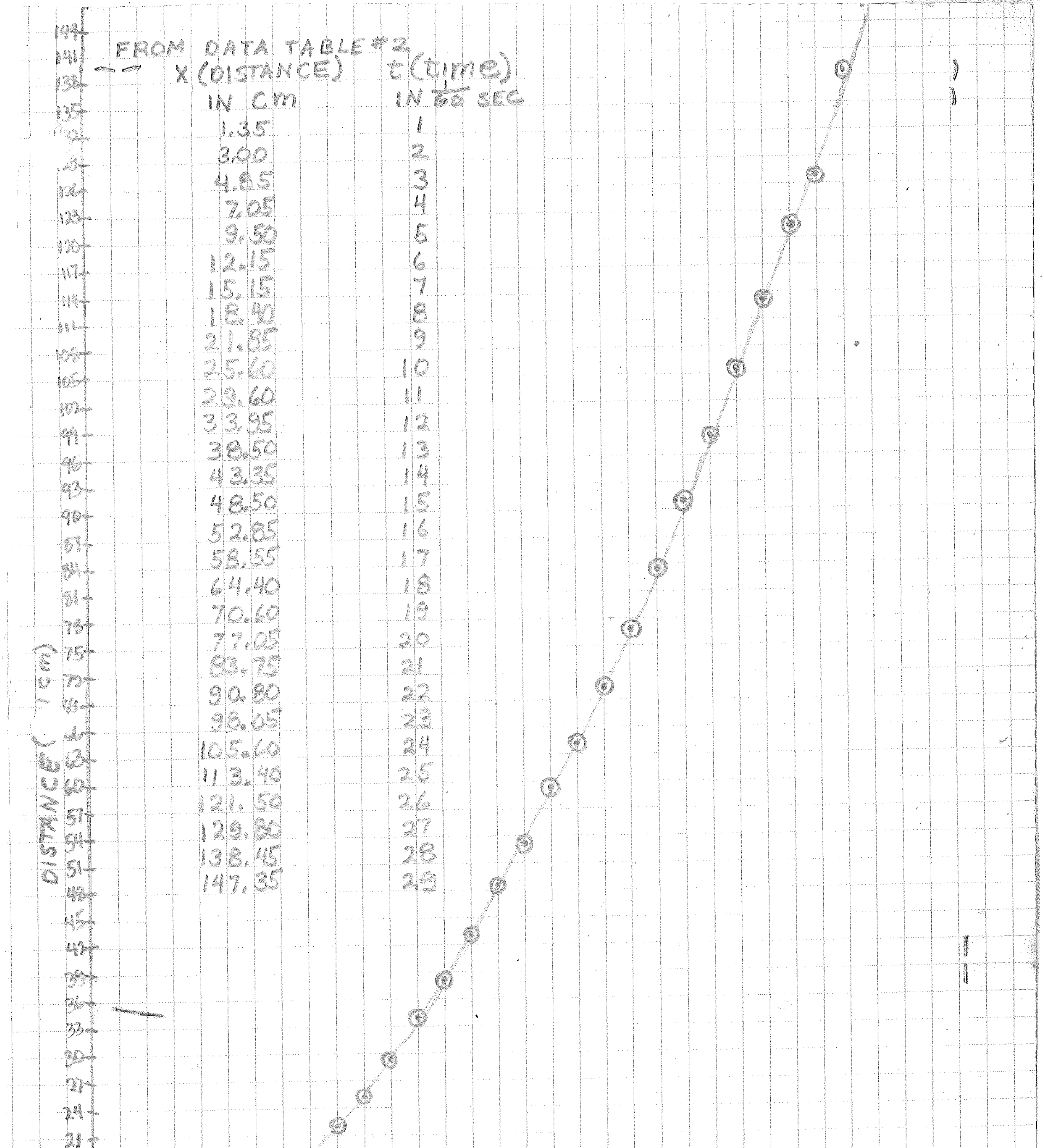
	A(3)	A(2)	A(1)	STD. DEV.
QUADRATIC......	0.1342925325	1.1667180005	0.2950092772	0.2614851326

FROM ABOVE THE EQUATION OF THE PRECEEDING GRAPH IS:

$x(t\ 2/) = (.134)t^2 + (1.17)t + .30$

FROM DATA TABLE #2

X (DISTANCE) IN cm	t (time) IN 1/30 SEC
1.35	1
3.00	2
4.85	3
7.05	4
9.50	5
12.15	6
15.15	7
18.40	8
21.85	9
25.60	10
29.60	11
33.95	12
38.50	13
43.35	14
48.50	15
52.85	16
58.55	17
64.40	18
70.60	19
77.05	20
83.75	21
90.80	22
98.05	23
105.60	24
113.40	25
121.50	26
129.80	27
138.45	28
147.35	29



FROM THIS;

$$\frac{g}{2} = \frac{d^2x}{dt^2} = \frac{.134\ cm}{(\frac{1}{60}\ SEC)^2} = 4.8\frac{m}{SEC^2}$$

$$\therefore g = 10.6\frac{m}{SEC}$$

ACCEPTED $g = 9.8\frac{m}{SEC^2} \Rightarrow 8.2\%$ ERROR

THE DISTANCE-TIME EQUATION DID NOT TAKE THE FORM $x = g/2\, t^2$ BECAUSE MEASUREMENTS WERE APARENTLY NOT TAKEN FROM $t=0$.

MORE SPECIFICALLY THE TIME FOR AN OBJECT TO DROP .30 cm (OR X_0) IS 7.8×10^{-4} SEC AT WHICH TIME IT WOULD ACHIEVE A VELOCITY OF 1.22 cm/$\frac{1}{60}$SEC, WHICH IS IN ERROR OF ONLY 4.46 % COMPAIRED WITH THE INITIAL VELOCITY DERIVED IN THE EQUATION:

$$X = \frac{g}{2}t^2 + V_0t + X_0$$
$$X = .134t^2 + 1.17t + .30$$

ERROR:

POSSIBLE MEASUMENT ERROR COULD HAVE ARISEN FROM:

1) DIRECT MEASUREMENT FROM THE TAPE
2) THE POSSIBILITY THE FALLING BODY APARATUS DIDN'T HAVE A FREQUENCY OF 60/SEC
3) OTHER OUTSIDE FORCES SUCH AS AIR RESISTANCE ACTING ON THE FALLING BODY

A DD

END

POTENTIOMETER (FORMAL WRITEUP) NOV, 14, 1969

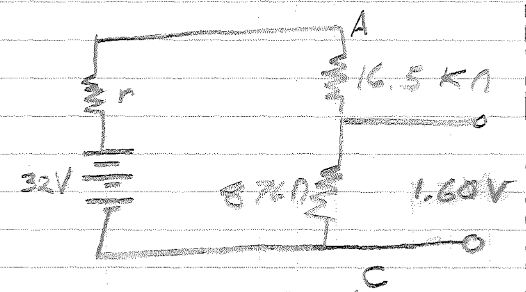
PURPOSE - TO CALIBRATE A POTENTIOMETER & BECOME FAMILIAR WITH ITS USE IN MEASURING POTENTIAL DIFFERENCES

VOLTAGE SOURCE 6 V

A) $\frac{1000}{3} = \frac{X}{1.344} \Rightarrow X = 449$ UNITS FOR GALV. = 0

B) $\frac{1000}{3} = \frac{521}{V_x} \Rightarrow V_x = 1.57V$

C) $\frac{1000}{3} = \frac{527}{V_x}$ $\quad V_x = 1.58V$



$V_C - V_A = 32V$

$$\frac{32V}{17.38K} = \frac{V_{CB}}{.876K}$$

$$V_{CB} = 1.60V$$

ASSUME Voltmeter internal resistance $= R_{INT}$

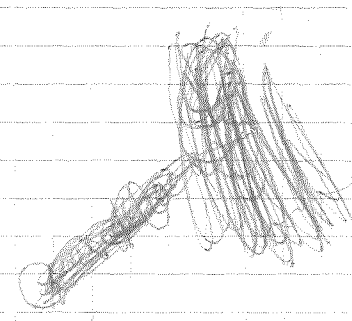
$$\frac{32V}{17.38} = \frac{(1.58V)(876 + R_{INT})}{876\ R_{INT}}$$

II) $\frac{1000}{3V} = \frac{498}{V_x}$ $\quad V_x = 1.50V$

CALIBRATION CHECK - OK

D.J.

R	X
500	497
200	470
100	443
70	423
50	398
40	378
30	349
20	303
10	216



HIGH RESISTANCE MEASUREMENT 11/21/68

URPOSE: TO MEASURE HIGH RESISTANCE USING THE BALLISTIC GALVANOMETER

PROCEDURE: A CHARGED CAPACITOR IS DISCHARGED ROTATIONALLY THROUGH A GALONOMETER AND A RESISTOR A NUMBER OF TIMES EACH TIME INCREASING THE LENGTH OF TIME OF DISCHARGE FROM THE RESISTOR, AND RECORD THE CORRESPONDING READING. FROM THE DATA AQUIRED, THE RELATIONSHIP BETWEEN THE GALVONOMETER DEFLECTION, THE RESISTANCE, THE CAPACITANCE AND THE TIME MAY BE COMPUTE'D.

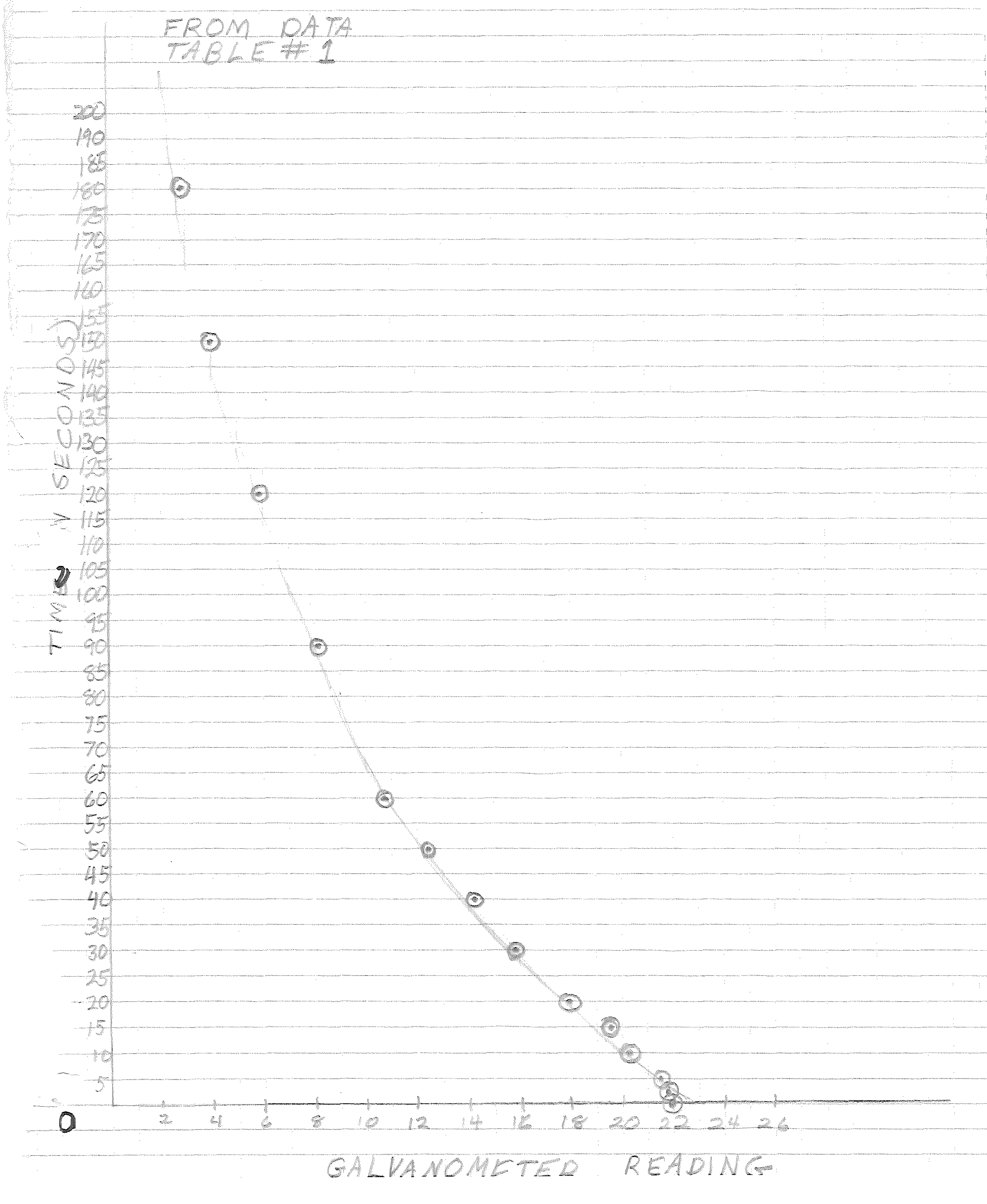
DATA:

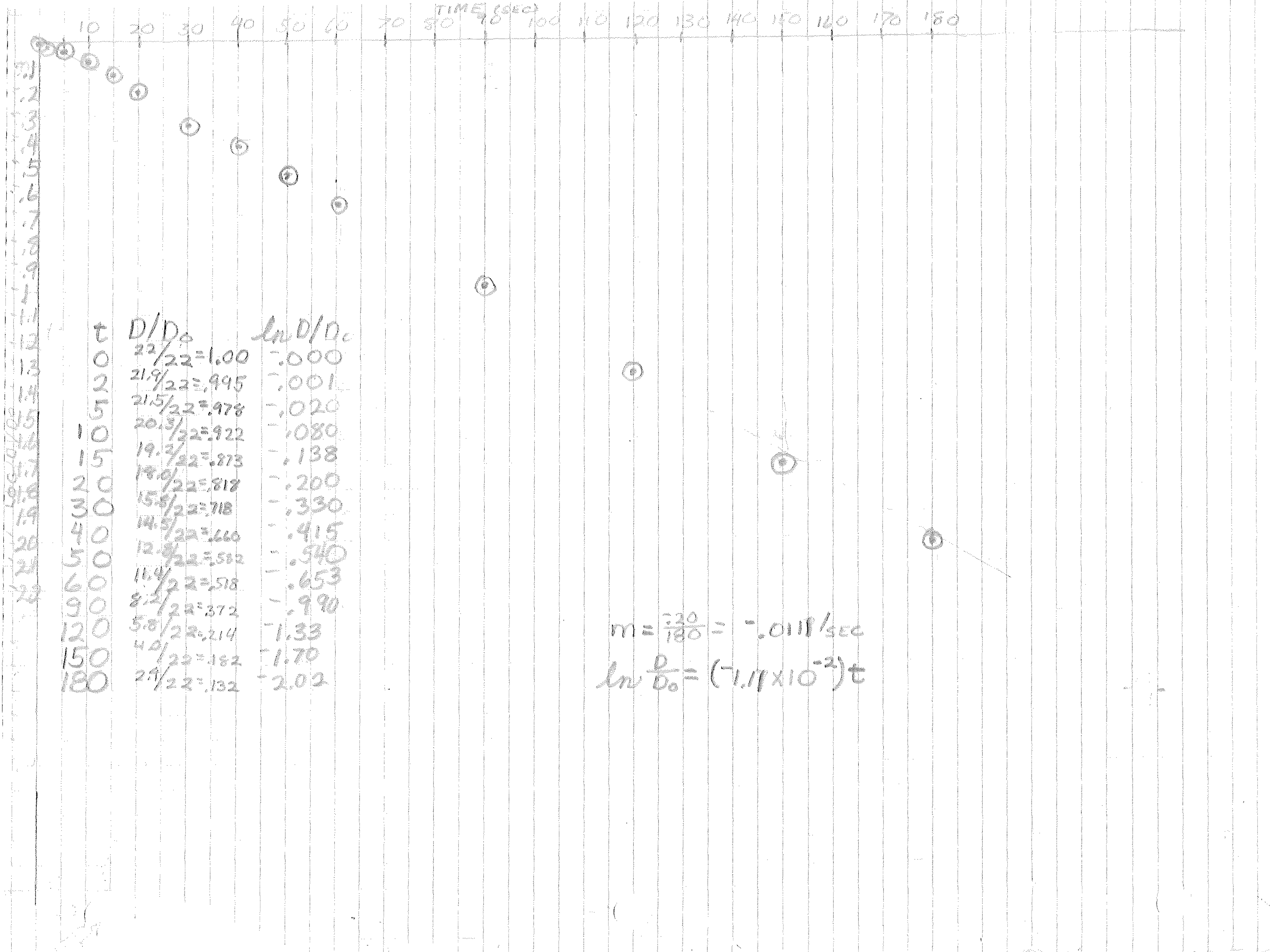
DATA TABLE #1

TIME OF DISCHARGE (IN SECONDS)	GALVANOMETER DEFLECTION (IN UNITS)
0	22.0
2	21.9
5	21.5
10	20.3
15	19.2
20	18.0
30	15.8
40	14.5
50	12.8
60	11.4
90	8.2
120	5.8
150	4.0
180	2.9

$C = .88\ \mu f$

$D_0 = 22$


FROM DATA
TABLE # 1
TIME IN SECONDS
GALVANOMETED READING



t	D/D_0	$\ln D/D_0$
0	22/22 = 1.00	-.000
2	21.9/22 = .995	-.001
5	21.5/22 = .978	-.020
10	20.3/22 = .922	-.080
15	19.2/22 = .873	-.138
20	18.0/22 = .818	-.200
30	15.8/22 = .718	-.330
40	14.5/22 = .660	-.415
50	12.8/22 = .582	-.540
60	11.4/22 = .518	-.653
90	8.2/22 = .372	-.990
120	5.8/22 = .264	-1.33
150	4.0/22 = .182	-1.70
180	2.9/22 = .132	-2.02

$m = \frac{-2.0}{180} = -.0111 / sec$

$\ln \frac{D}{D_0} = (-1.11 \times 10^{-2})t$

CALCULATIONS

$q(t)$ = CHARGE ON CAPACITOR PLATES AT TIME t
$i(t)$ = CURRENT AT TIME t

$$V_{ab} = i(t)R = \frac{q(t)}{c} \qquad i = -\frac{dq}{dt}$$

$$\frac{q(t)}{c} = -R\frac{dq}{dt}$$

A DIFFERENTIAL EQUATION, WHICH YIELDS

$$\ln(q/q_0) = -\frac{1}{RC}t$$

SINCE THE GALANOMETER DEFLECTION IS DIRECTLY PROPORTIONAL TO q,

$$\frac{D_1}{q(t_1)} = \frac{D_2}{q(t_2)} = \frac{D_0}{q_0}$$

OR $$\frac{D(t)}{D_0} = \frac{q(t)}{q_0}$$

$$\therefore \ln q/q_0 = \ln D/D_0 = -\frac{1}{RC}t$$

OR $$\frac{q}{q_0} = \frac{D}{D_0} = e^{-\frac{t}{RC}}$$

THE RESISTANCE IS THE PRO. TO THE SLOPE OF THE $\ln q/q_0$ VS. t GRAPH, WHICH IS EQUAL TO THE D/D_0 VS t GRAPH

$$m = -\frac{1}{R\ 88\times10^{-6}} = -.0111$$

R = 98 MEGOHMS

ERROR OF 11.4% ($R = 88\,M\Omega$)

CONCLUSIONS: THIS EXPERIMENT PROVIDES A MEANS OF MEASURING HIGH RESISTANCES, BY MEASURING THE GALVANOMETER DEFLECTION AND REALIZING IT IS DIRECTLY PROPORTIONAL TO THE CHARGE ON THE CAPACITOR.

ERROR COULD HAVE RESULTED FROM INACURATE READINGS OR NON-IDEAL APARATUS (CAPACITOR LEAKAGE ETC.)

LINEAR & NON-LINEAR ELEMENTS

12/5/69

PROCEDURE:

V-i RELATIONSHIPS FOR THREE DEVICES ARE GRAPHED AND MATHEMATICAL RELATIONSHIPS FOR THESE ARE DETERMINED BY THE METHOD OF DIFFERENCES.

DATA:

R_1:

V	i
.02	.011
.1	.041
.2	.073
.3	.101
.4	.139
.5	.164
.6	.20
.7	.228
.8	.263
.9	.300
1.0	.328
1.1	.363
1.2	.401
1.3	.428

(WITH POLARITY REVERSED).

-V	-i
.5	.163
1.0	.325
1.3	.425

R_2:

V	i
.60	.0081
.65	.0122
.70	.0163
.75	.0212
.80	.0269
.85	.0340
.90	.0410
.95	.0480

R_3:

V	i
2.0	.0094
2.2	.0104
2.4	.0114
2.6	.0124
2.8	.0136
3.0	.0146
3.2	.0154
3.4	.0169
3.6	.0178
3.8	.0193
4.0	.0204
4.2	.0220
4.4	.0236
4.6	.0256
4.8	.0272
5.0	.0296

(WITH POLARITY REVERSED).

-V	-i
2.0	.0100
3.0	.0150
4.0	.0212
5.0	.0296

mk

CALCULATIONS:

DIFFERENCES:

	V	i	1st Diff	R
R_1	0.0	11	30	0
	.1	41	32	4.1
	.2	73	31	14.6
	.3	101	35	30.3
	.4	139	25	55.6
	.5	164	36	82.0
	.6	200	28	120
	.7	228	35	160.6
	.8	263	37	210
	.9	300	28	270
	1.0	328	35	328
	1.1	363	38	399
	1.2	401	27	481
	1.3	428		556

$$i = .32V + .011$$

DIFFERENCES

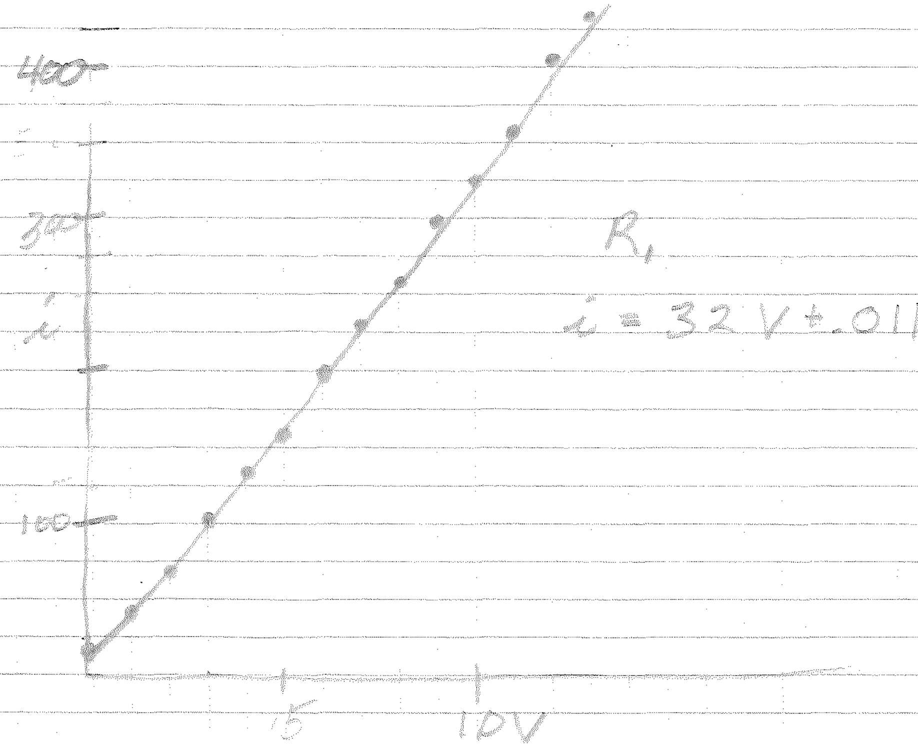
	V	i	1st Diff	2nd Diff	R
R_2	.6	8.1	4.1		4.8
	.67	12.2	4.1	0	8.2
	.70	16.3	4.9	.8	11.4
	.75	21.2	5.7	.8	14.8
	.80	26.9	7.1	1.4	21.5
	.85	34.0	7.0	-.1	28.9
	.90	41.0	7.0	0	36.9
	.95	48.0			45.6

$$i = .013 - .086V + .130V^2$$

DIFFERENCES

R_3

V	i	1st DIFF	R
2.0	9.4	1.0	18.8
2.2	10.4	1.0	22.9
2.4	11.4	1.0	27.4
2.6	12.4	1.2	32.2
2.8	13.6	1.0	38.1
3.0	14.6	.8	43.8
3.2	15.4	1.5	49.3
3.4	16.9	.9	57.5
3.6	17.8	1.5	64.1
3.8	19.3	1.1	73.3
4.0	20.4	1.6	81.6
4.2	22.0	1.6	92.4
4.4	23.6	2.0	103.8
4.6	25.6	1.6	117.8
4.8	27.2	2.4	136.6
5.0	29.6		145.0



CONCLUSIONS

A v-i RELATIONSHIP NEED NOT BE LINEAR. SOME DEVICES HAVE RADICALLY DIFFEREN RELATIONSHIPS WHEN POLARITY IS REVERSED. SUCH AS THE DIODE. SOME CAN BE LINEAR OVER A RANGE, SUCH AS THE GLOBAR RESISTOR.

ERROR COULD HAVE ARISEN FROM THE CURRENT DRAWN BY THE VOLTMETER & THE VOLTAGE DROP ACROSS THE AMMETER. THE GLOBAR RESISTOR TOOK TIME TO STABALIZE ITSELF AND THIS IS A POSSIBLE REASON FOR ERROR.

B

FORCED OSS. (CONT)

T (SEC)/5	AMP.	ln AMP
27.5/5 = 5.50	.4	−.916
13.6/5 = 2.72	.8	−.223
12.7/5 = 2.54	1.1	.093
11.6/5 = 2.32	1.5	.405
11.3/5 = 2.26	2.0	.693
10.8/5 = 2.16	2.6	.956
9.9/5 = 1.99	2.75	1.012
9.6/5 = 1.92	2.25	.811
9.1/5 = 1.82	1.7	.531
8.4/5 = 1.68	1.4	.336
8.1/5 = 1.62	1.25	.223
7.2/5 = 1.44	.75	−.287
6.5/5 = 1.30	.5	−.693
4.8/5 = .96	1.25	.223

ELECTRICAL CONDUCTION IN SEMICONDUCTORS

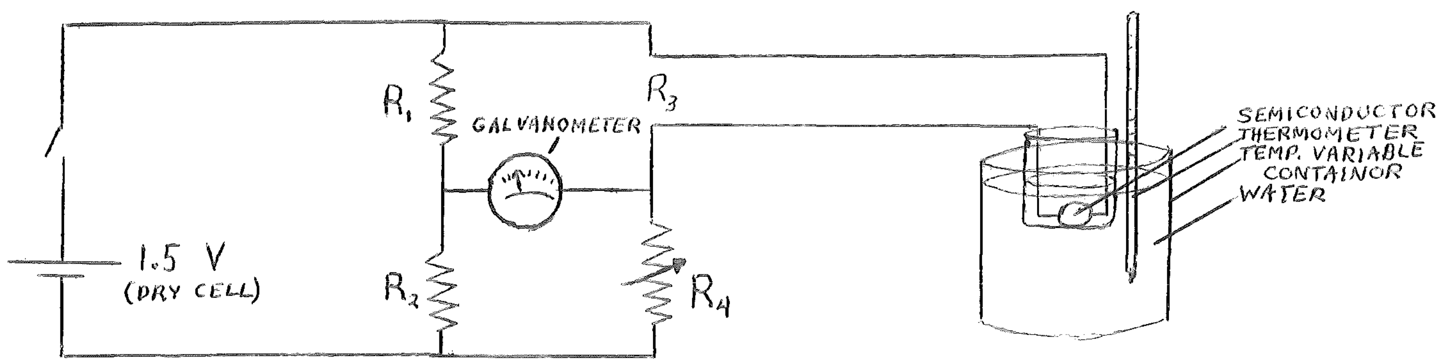
by Robert Jackson Marks II
Physics III
Group A
Friday; Periods 1-3
Performed: 11/29/69
Due: 12/5/69

A
D.D.

PURPOSE:

To investigate the temperature dependence of a resistance of a semiconductor by using a D.C. bridge circuit as a null-reading technique.

PROCEDURE:



The resistance is measured by the use of a "Wheatstone Bridge". The sample is placed in a calorimetercontaining water, which may be heated or cooled. Temperature is measured off of the thermometer, which is placed near the semiconductor. Resistance is measured by varying it so the galvonometer reads zero, keeping i=0 and making calibration of the galvanometer unnessisary.

Since:

$$\ln(r) = E_g/2kT - \ln(ce(u_e+u_h)T^{3/2})$$

where:

r = resistivity
T = temperature
E_g= energy gap
k = Boltzman's constant
c = constant (characteristic of semiconcuctor)
u_h= mobilite of holes
u_e= mobility of electrons

and $(-\ln(ce(u_e+u_h)T^{3/2}))$ changes minimally in comparison with $E_g/2kT$, a graph of ln(r) vs. 1/T would yield a good approximation of the energy gap (E_g), since its slope would be a good approximation of $E_g/2k$.

Resistance, however, is much easier to directly measure than is resistivity. One may derive one from the other from the formula:

r=RL

where :

R=resistance
L=number with dimention of length charicteristic of the semiconductor's geometry

The shape of the semiconductor changes minimally with the temperature, but this change is so small it can be disregarded. Therefore, L can be treated as a constant.

Therefore, if:

$$r = RL$$

then:

$$\ln(r) = \ln(RL)$$

since:

$$\ln(r) = E_g/2kT$$

$$\ln(RL) = \ln(R) + \ln(L) = E_g/2kT$$

or:

$$\ln(R) = E_g/2kT - \ln(L)$$

Ln(L), being a constant, would imply that the ln(r) vs. 1/T graph would have the sme slope as the ln(R) vs. 1/T graph, being $E_g/2k$.

DATA:

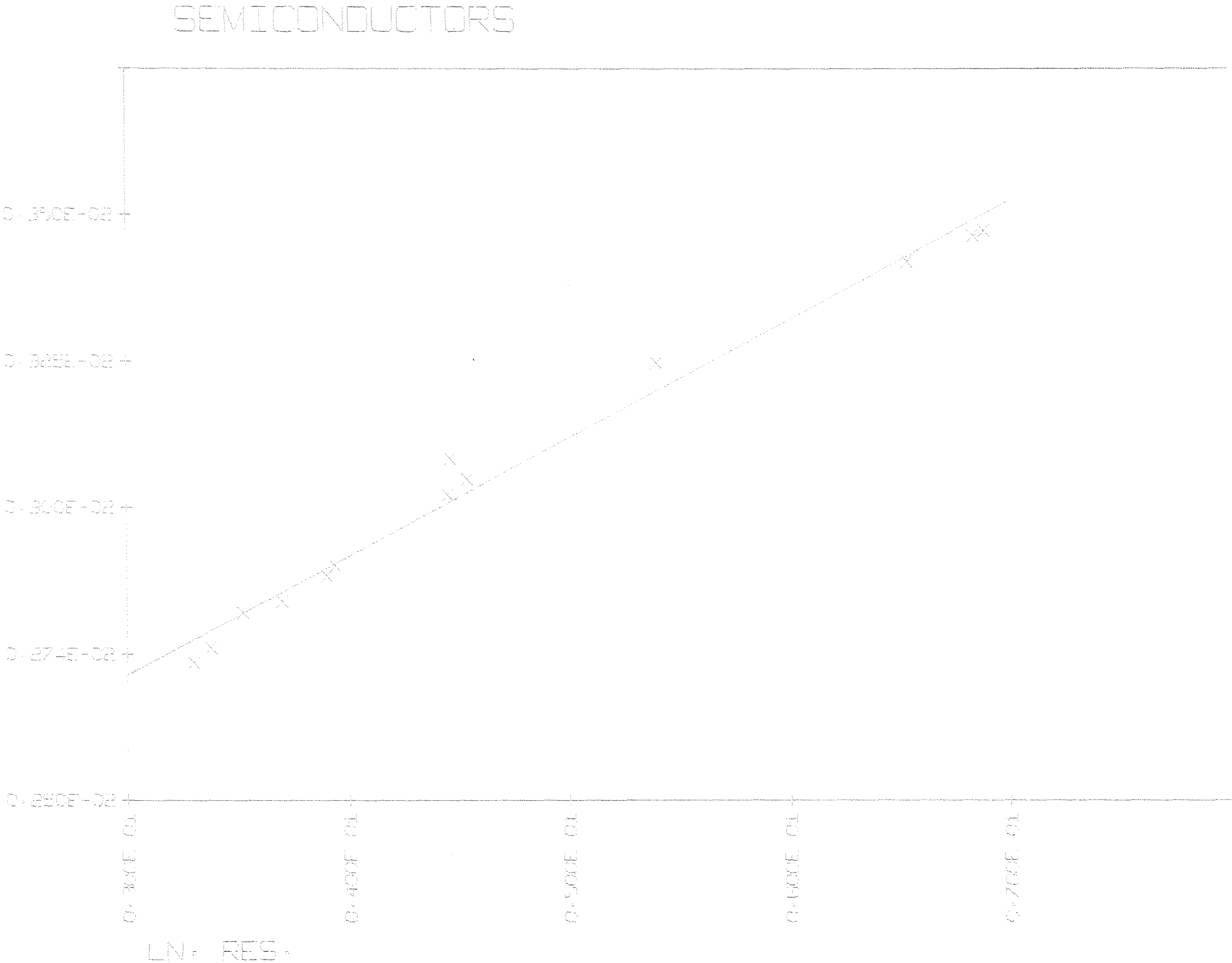
T °C ±.3°	T °K ±.5°	R-ohms ±.5 Ω	ln (R) ±.007	1/T ±1.5 X 10^{-5}
2.6	276	595	6.89	.00362
3.6	277	586	6.84	.00361
7.7	281	539	6.54	.00356
24.5	298	221	5.45	.00336
42.3	315	104	4.64	.00317
46.5	320	94.0	4.54	.00313
49.2	322	86.2	4.45	.00311
50.0	323	85.9	4.45	.00310
65.0	338	51.3	3.94	.00296
66.5	340	49.2	3.90	.00294
73.1	346	40.2	3.70	.00289
75.0	348	37.5	3.52	.00287
83.9	357	29.4	3.38	.00280

SEMICONDUCTORS

LINEAR......... Y = A(1) + A(2

ERROR	PER
-0.00003100	
-0.00002936	
-0.00000953	
0.00005583	
0.00008464	
0.00002602	
0.00001697	
-0.00000430	
-0.00001499	
-0.00001844	
0.00000345	
-0.00003395	
-0.00004533	

LINEAR.........



SEMICONDUCTORS
0.350E-02
0.325E-02
0.300E-02
0.275E-02
0.250E-02
1/T
0.300E 01
0.400E 01
0.500E 01
0.600E 01
0.700E 01
LN. RES.

CALCULATIONS:

From the preceeding data, E_g may be computed.

$$m \text{ } (\ln(R) \text{ vs. } 1/T \text{ graph}) = E_g/2k$$

$$E_g = (2.33 \times 10^{-4} \pm 3.8 \times 10^{-5})2(1.38 \times 10^{-23})$$

$$E_g = 6.43 \times 10^{-27} \text{ joules}$$

Also, the constant L may be computed, in that the "Y" intercept of the ln(R) vs. 1/T graph is equal to the ln 1/L.

$$\ln (1/L) = .00205$$

$$L = e^{-.00205}$$

$$L = 1.00 \text{ meters}$$

CONCLUSIONS:

The energy gap of an unknown semiconductor can be determined by finding the relationship of its resistance and change in temperature, keeping all other facets constent. A graph of ln(R) vs. 1/T yields a straight line, the slope of which is $E_g/2k$, and whose intercept is equal to the log of the linear constant by which the resistance is multiplied to yield the resistivity.

In this experiment, the energy gap was found to be equal to be $6.43 \times 10^{-27} \pm 8.35 \times 10^{-28}$ joules., and the linear constant equal to 1.00 meter.

The greatest amount of error in this experiment undoubtedly arose from the corrilation of a temperature and a resistance. The temperature read off of the thermometer was not that of the semiconductor, for though the temperature fell or rose slowly, the semiconductor's resistance could be visibly seen changing on the galvanometer. This temperature error could have varied as much as 5^{o}.

The deleting of the last part of the equation, the small change in the semiconductor's geometry, and the possible error in the calibration of the variable resistor are minimal compared to the temp-res reading error, and may be disregarded.

MFCBDLGDP

Physics Laboratory - General Instructions

I. Purpose of Laboratory

Laboratory work in physics has two important objectives - first, to give the student direct experience with some of the natural phenomena upon which physical principles are based, and second, to develop in the student some understanding of the experimental procedures. It is felt that some experience in the laboratory is necessary to give the student an insight into the methods of physics (or for that matter any experimental science). Without it he would be merely accepting principles as they were handed to him without an understanding of the experimental procedures on which they are based.

In the laboratory the student will work with real, rather than ideal, apparatus. This equipment (and the experimenter as well) will be subject to limitations which cause errors that must be taken into account before any conclusions can be drawn from the experimental results. Therefore error analysis is an essential part of all good laboratory work.

Although you will be assigned a certain group of experiments to do this quarter, and in many cases the procedure to be followed in performing the experiment is described in an instruction sheet, it is hoped that the student will use some of his own ingenuity in performing the experiments; it is intended that the instructions be used as an aid to understanding rather than something to be followed mechanically without thought. We also want to encourage students to think about possible experiments that they might do in place of one of the prescribed set. Within the limitations of equipment and time, substitution of an experiment which is more interesting to the individual student is permitted, provided it is a physics experiment and it is cleared with the instructor.

II. Preparation for an Experiment

In order to perform an experiment thoroughly and accurately in the time allotted, it is necessary to put in some time beforehand thinking about the experiment. If an instruction sheet has been provided it is to be studied carefully before the laboratory period. You should come to the laboratory with as thorough an understanding as possible of what you are going to do during the period and why. This may require that you spend some time in the library, looking up references etcetera.

III. Performance of the Experiment

An essential part of the method of solving an experimental problem is the preparation of a clear, concise record of the data taken during the performance of the experiment. This record should contain, in a clear and legible form, all the "raw" data and information with which to make corrections (don't try to make corrections "in your head" while taking data) and also enough explanation of what you are doing and why so that your pages of data can be analyzed later without confusion or ambiguity. Your instructor may require that this record be kept in a permanent notebook or he may ask you to keep this record on data sheets which are later included in a report on the experiment. In either case, all observations should be recorded directly into the notebook or on the data sheets (nothing on scratch paper and later copied) and an estimate of the accuracy of each set of measurements should be made and recorded also. Corrections can be made by crossing out errors with a single line (no erasures). Before leaving the laboratory, the student should do enough calculation and graphical work to ensure that the data collected "makes sense" and there are no gaps in it which need to be filled in before he can continue the analysis without having to make any "wild guesses" or assumptions. Your data record must be approved by the instructor before you leave the laboratory.

IV. Laboratory Notebook (Data Record)

The following are specific suggestions concerning the form of the laboratory record of the experiments.

A. If the instructor has you keep a permanent laboratory notebook it should be one having cross-ruled pages (useful for graphs) and it must be labeled with the following information.

1. On the front cover in ink:

   Physics Laboratory

   Your Name

2. Inside the front cover at the top:

   Fall (or whatever) Quarter

   Lab. day and hours

   Group Number

B. For each experiment the student should record the title of the experiment and the date performed at the top of the data record. A very brief (not detailed) description of the procedure followed should precede the data record, which is preferably in tabular form. Label the data carefully with the proper column headings and units. Whenever possible, the type and identifying number of instruments being calibrated or used in measurement should be recorded for later reference.

C. As suggested above the next step is to do the calculations required by the analysis of the experiment and draw the graphs. Repeat any measurements which appear doubtful and make new measurements where needed to fill in gaps in the data.

D. If you are using a laboratory notebook rather than data sheets and if the instructor informs you that no report is required on a particular experiment, then the experiment should be completed in the notebook by writing a summary and conclusions. Final calculations should be summarized in tabular form and whatever additional graphs are required should be completed. State a conclusion in your own words and discuss the experiment briefly (for example a discussion of accuracy is always desirable). On graphs and in your final summary give the page number of the data or discussion referred to. The summary and conclusions may be left for the report when one is being written.

V. Report

When a report is required on an experiment it is due at the beginning of the period one week after the experiment was performed. The report is to be written independently by each student in ink (or typewritten) on white, unlined 8½ x 11 paper (graph paper for graphs). Each report must have:

A. A cover sheet containing the following information -- course, experiment title, your name, laboratory period day and hours, group number, date experiment was performed, and date of report.

B. A statement of the purpose of the experiment and a brief summary of how you went about performing it (not detailed), data and observations (if you used data sheets rather than a notebook these may be submitted as they are), sample calculations, tabulated results, graphs, conclusions, and a discussion of the experiment. The discussion section of a report should be more thorough and complete than the corresponding section in the notebook. It may include a discussion of what was learned in doing the experiment, as well as the results and the accuracy of the results. It should also contain a discussion of any points which the instructor may have brought to your attention through questions written on the instruction sheets, and of any other points of interest that may occur to you.

It is customary to use the passive voice in scientific writing (e.g. "The time required for the pendulum to swing through twenty complete cycles was measured...etc.") thus not calling attention to the observer. The following styles are not to be used in a report: I" (we) swung the pendulum and..." or "Swing the pendulum and measure the time for twenty complete cycles...". If you quote or paraphrase any outside sources in writing your report (including your own text book) give credit to the original author in a footnote.

References:

1. Baird, "Experimentation", chapter 7

2. Olson, "Experiments in Modern Physics", section 1.4

Measurement, Probability, and Experimental Errors

I. Types of Error

Whenever a measurement is made of any physical quantity there is a certain amount of uncertainty in the result. Determination of the amount of uncertainty in a measurement is not usually easy but an attempt should always be made to do so, even if it is no more than an educated guess. Without some estimate of the uncertainties associated with experimental measurements one has no indication of the accuracy of the results and it is difficult to come to any conclusion about what the experiment has shown (or not shown). In all of the experiments which follow in the physics laboratory sequence the student will be expected to make some estimate of the accuracy of his quantitative experimental results.

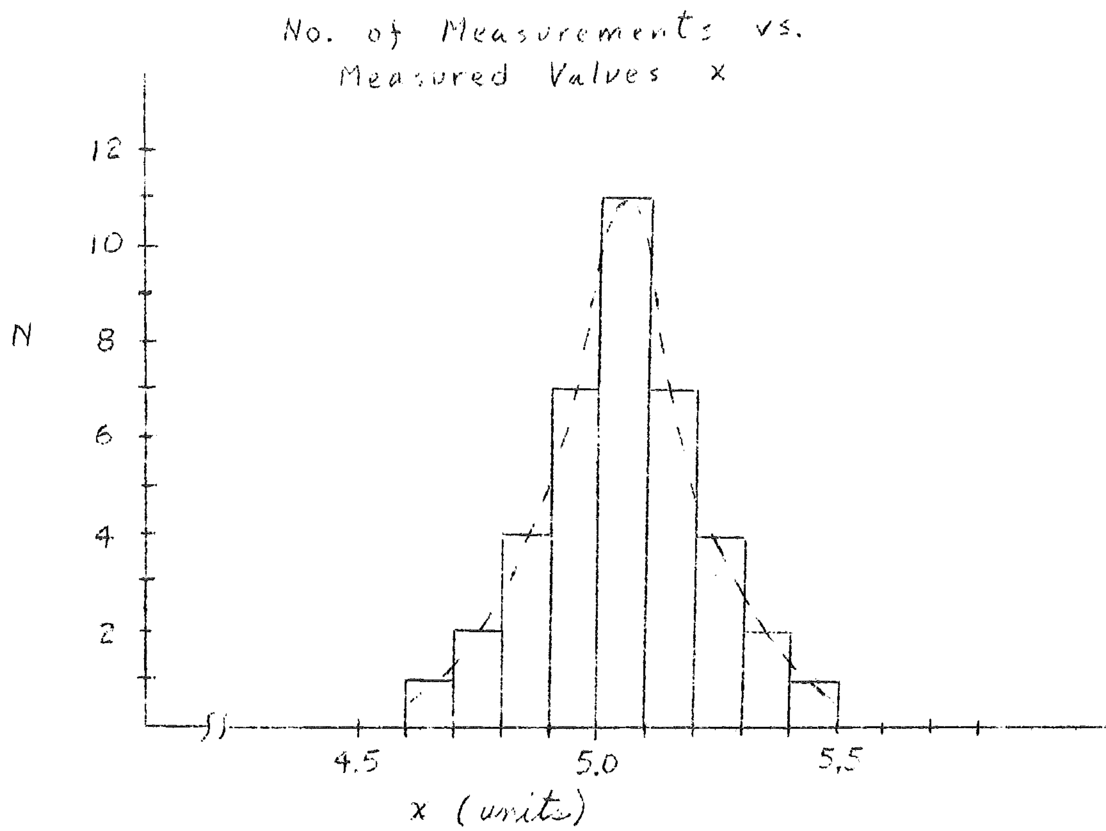
There are two types of errors which may occur in the measurement process, systematic errors and random errors. Systematic errors tend to make all the observations of one item too small or too large. For example if voltage measurements were taken in an electric circuit using a voltmeter which consistantly read 0.1 volt too high, a systematic error would be present. Other common examples of causes of systematic error are worn weights, clocks which gain or lose time, friction, and personal bias of the observer which causes him to make readings which are consistently high or low. When systematic errors are recognized in an experiment it is often possible to find out how large their effect is and to correct for it. The error in the voltmeter which reads 0.1 volt too high, for example, can be discovered by calibrating the instrument against some sort of standard (accurately known voltage), and a correction of -0.1 volt made to all the readings. Error due to an observer's bias may be minimized by having another observer make the same measurement independently (bias is best eliminated if each observer knows nothing of the other's result until after both measurements are completed).

Random errors result from chance variations in the quantity being measured, in the measuring devices, or in the observer, and are just as likely to produce too large a value as too small. For example, if one measures the diameter of a metal rod several times with a micrometer the readings will probably fluctuate slightly in a non-systematic fashion due to actual differences in the rod's diameter at different positions, variations in pressure when the micrometers jaws are closed, and changes in the observer's estimate of the scale reading. Random errors are present in all measurements, although they may be too small to be noticeable, and they cannot be corrected for because of their random nature.

II. Determination of Precision

Suppose that several measurements of the same quantity x were made and all systematic error in the measurements eliminated or corrected (assuming this were possible). As discussed above there would still be a certain amount of rnadom fluctuation apparent in the measurements if they are "fine" enough to make it noticeable. If a histogram was plotted showing the number of measurements N falling within different intervals of size Δx it might look like that shown in Fig. 1.

Fig. 1



The meaning of the histogram is that one measurement of x fell between 4.6 and 4.7 units, two between 4.7 and 4.8 units, four between 4.8 and 4.9 units, and so forth. The completely symetrical distribution shown usually results only if a large number of measurements are made and if the fluctuations are entirely random. In such cases the envelope of the distribution often has a particular form called a "normal" or "Gaussian" distribution which is represented by the mathematical equation

$$y = \frac{1}{\sqrt{2\pi}\ \sigma} e^{-(x-\bar{x})^2 / 2\sigma^2} \qquad (1)$$

where σ is a constant which determines the "sharpness" of the peak (high, narrow peaks are characterized by small values of σ). The quantity $\bar{x}$ is the average of the individual measurements

$$\bar{x} = \frac{x_1 + x_2 + \ldots\ldots\ldots\ldots}{n} = \frac{\sum x_i}{n}$$

where n is the total number of measurements, and because of the symmetry of the Gaussian function $\bar{x}$ corresponds to the most probable value of x obtained from a measurement of x (peak of curve). Thus $\bar{x}$ is the best estimate that one may make of the true value of x from these measurements.

The individual measurements of x differ from the average or most probable value $\bar{x}$ by an amount d called the deviation of that measurement

$$d_1 = x_1 - \bar{x}, \quad d_2 = x_2 - \bar{x}, \ldots\ldots\ldots$$

The standard deviation

$$\sigma = \left[\frac{d_1^2 + d_2^2 + \ldots\ldots d_n^2}{n - 1}\right]^{1/2} = \left[\frac{\sum (di)^2}{n - 1}\right]^{1/2}$$

is an indication of the precision of a set of measurements since narrow Gaussian distributions indicate precise measurements with small deviations from the average and a small standard deviation σ. If a large number of measurements is made, 68% of them will be in the range $\bar{x} \pm \sigma$, 95% in the range $\bar{x} \pm 2\sigma$, and 99% in the range $\bar{x} \pm 3\sigma$, a fact which can be verified by determining the area under a Gaussian curve between the various limits. If after having determined $\bar{x}$ and σ from a large number of measurements one makes a single measurement x, he then will have about a two thirds chance of getting a value between $\bar{x} + \sigma$ and $\bar{x} - \sigma$, etcetera.

Although increasing the number of measurements of quantity x would have little effect on the standard deviation σ (the scatter of the data) except to give a more accurate picture of what it really is, increasing the number of measurements should improve the reliability of the average value $\bar{x}$. It can be shown from statistics that the standard deviation <u>in the mean $\bar{x}$</u> is given by the equation

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

which means that there is a 68% chance that the true value of x will be in range $\bar{x} \pm \sigma_m$ assuming the distribution is normal and there are no systematic errors present. Thus the precision of the mean $\bar{x}$ can be increased (σ_m reduced) by taking more observations, but the improvement is slow because of the $\sqrt{n}$ factor (90 readings only 3 times as good as 10 readings). The final result of a set of measurements may be stated

$$x = \bar{x} \pm \sigma_m$$

It is quite often useful to represent the standard deviation σ_m as a percentage of the value $\bar{x}$. The calculation required is:

$$\text{per cent std. dev.} = (\sigma_m/\bar{x}) \cdot (100\%)$$

Although the normal or Gaussian distribution (equation 1) is very often a good representation of the kind of distribution found in repeated measurements of physical quantities, it should not be assumed that this distribution always gives an accurate description of the results of such measurements, even when a large number of measurements are made. There are a number of cases where the distribution is non-Gaussian and perhaps even non-symmetrical. For example, if one makes several determinations of the number of nuclei which decay by particle emission in a certain time, he obtains the Poisson distribution

$$y \propto \frac{\bar{x}^{x}}{x!} e^{-\bar{x}} \tag{2}$$

where $\bar{x}$ is the average number of counts and y is the probability of obtaining x counts in a given trial. This distribution is very unsymmetrical about the mean $\bar{x}$ when the number of counts $\bar{x}$ is small but closely resembles a Gaussian distribution with standard deviation $\sqrt{\bar{x}}$ when $\bar{x}$ is large.

III. Propagation of Errors If one uses experimental observations, with their associated random errors, to calculate a result, the precision of the result will be determined by the precision of the quantities involved in the calculation. The standard deviation of the result may be determined from those of the separate quantities σ_{m1}, σ_{m2}, etc. by keeping in mind the following rules.

A. The standard deviation of the result of addition and/or substraction is the square root of the sum of the squares of the standard deviations of the separate terms.

Example: $x_1 = 5.30 \pm 0.20$ units

$x_2 = 1.70 \pm 0.10$ units

$x_3 = 7.20 \pm 0.01$ units

$$x_1 - x_2 + x_3 = (5.30-1.70+7.20) \pm \left[(0.20)^2 + (0.10)^2 + (0.01)^2\right]^{1/2}$$

$$= 10.80 \pm 0.22 \text{ units}$$

Note that most of the standard deviation in the result comes from the largest standard deviation present in the separate terms ($0.22 \simeq 0.20$).

B. The percentage standard deviation in the result of mulitplication and/or division is the square root of the sum of the squares of the percentage std. deviations of the factors.

example: x_1, x_2, x_3 as above

$$(\% \text{ std. dev.})_1 = \frac{0.20}{5.30} \times 100\% = 3.8\%$$

$$(\% \text{ std. dev.})_2 = \frac{0.10}{1.70} \times 100\% = 5.9\%$$

$$(\% \text{ std. dev.})_3 = \frac{0.01}{7.20} \times 100\% = 0.1\%$$

$$y = \frac{(x_1)\ (x_2)}{x_3} = 1.25 \pm \text{std. dev.}$$

$$(\% \text{ std. dev.})_y = \left[(3.8)^2 + (5.9)^2 + (0.1)^2\right]^{1/2} = 7.0\%$$

$$(\text{std. dev.})_y = (.07)\ (1.25) = 0.09$$

$$y = 1.25 \pm 0.09 \text{ units}$$

Note that in this case the largest contribution to the standard deviation in the result comes from that quantity with the largest percentage standard deviation.

C. In case a quantity is raised to the n^{th} power its percentage standard deviation is multiplied by n.

The process of carrying standard deviations through calculations is useful not only indetermining the precision of the result but also in determining which quantity contributes most to random error in the result. It may be possible to reduce the deviations in this quantity by using more care or different techniques.

IV. Accuracy of Experimental Results

Determination of the standard deviation in an experimental result will tell you how much uncertainty is present due to random errors, but this is an indication of the accuracy of the result only in the case where systematic errors are negligible compared to random errors. For example, if in a particular experiment you obtained a percentage standard deviation of 1% but the instruments used to obtain the measurements were accurate only to within 5% (all readings may be too high or low by 5%), then the 5% accuracy is a better indication of the reliability of the results than the 1%. Some attempt should be made by the student to determine the reliability of his results in each experiment, although in some cases this will involve making some educated guesses as to the accuracy with which a particular measurement may be made with a particular measuring device. In all cases try to eliminate as much systematic error from the measurement as possible within the time available. An experimental result does not agree with a prediction of a theory unless the theoretically predicted result lies within the range given by the experimental result plus and minus the probable error; an experiment does not disagree with a theory unless the predicted result lies outside this range.

V. Significant Figures

The term "significant figures" refers to the digits of a measurement made in the laboratory, including all the certain digits and one additional doubtful one based on the observer's estimate of a fraction of a scale division. The numbers which represent data or the results of calculations should always be given with neither more nor fewer significant figures than are justified by the precision of the observations and computations. The number of significant figures in a measurement (or a calculated quantity) may be determined using the following rules.

(a) The first significant figure is the first non-zero digit.

(b) Zeros which occur between significant digits are considered significant.

(c) Zeros which occur to the right of the last non-zero digit are considered significant when they are to the right of the decimal point (the significance of such zeros to the left of the decimal point is indeterminate).

(d) If numbers having a different number of significant figures are added, substracted, multiplied or divided, the answer is given so as to have the same number of significant figures as the term or factor which has the least.

Examples:

.0001906 has 4 significant figures
10,937 has 5
93,000 has an indeterminate number
9.3×10^4 has 2
9.30×10^4 has 3

VI. Comparison of Results

Sometimes an experimental result is arrived at by two different methods which should both theoretically give the correct result. If there is no reason to believe that one of the results is much more accurate than the other, it might be instructive to see how much difference there is between the two. This difference is usually given in terms of the "percentage difference" which is defined.

$$\% \text{ diff.} = \frac{\text{diff. between values}}{\text{average value}} \times 100\%$$

References:

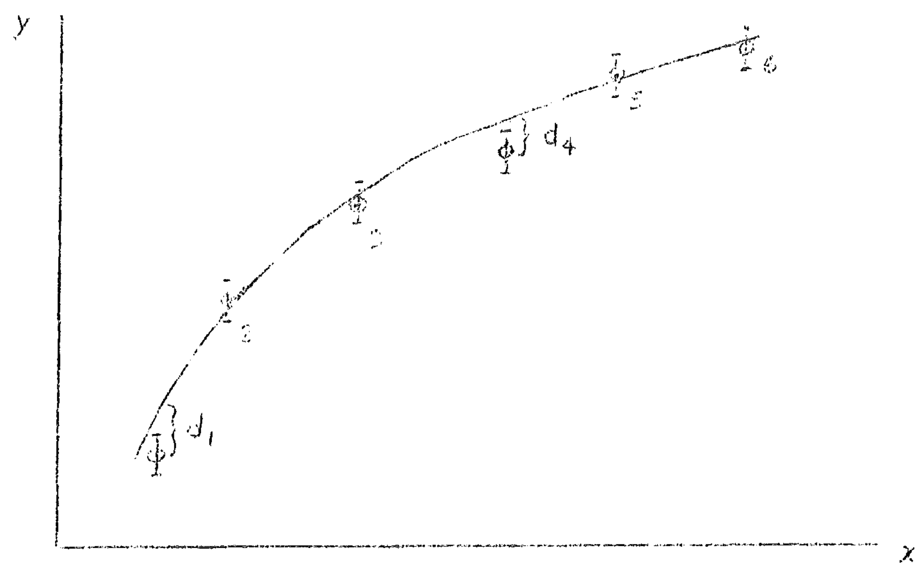
1. Young, "Statistical Treatment of Experimental Data"
2. Barford, "Experimental Measurements: Precision, Error and Truth"
3. Baird, "Experimentation: An Introduction to Measurement Theory and Experiment Design"
4. Braddick, "The Physics of Experimental Method"
5. Pugh and Winslow, "The Analysis of Physical Measurements"
6. Bevington, "Data Reduction and Error Analysis for the Physical Sciences"

METHOD OF LEAST SQUARES

One of the fundamental problems that comes up again and again in the laboratory is that of finding, from simultaneous measurements of quantities y and x, the dependence of quantity y on quantity x (the dependence of the period of a pendulum on its length for example). Often this dependence is revealed by making a graph of y versus x from the data. However, a certain amount of judgement is always involved in making a graph from experimental data since deviations in the measurements usually make it impossible to draw a smooth curve through all the data points. One usually tries to draw a smooth curve among the points in such a way that it appears that the deviations of the points from the line (positive and negative) add up to approximately zero. In other words, in the graph shown below

$$|d_1| + |d_3| + |d_4| + \ldots\ldots \simeq |d_2| + |d_5| + \ldots\ldots\ldots$$

where the deviations here and in the analysis to follow will be assumed to be deviations in y for precisely known values of x.



If a high degree of precision is required in the expression relating y to x, this method of balancing deviations "by eye" might not be sufficient. In this case a more scientific approach, based on statistics, is followed. It can be shown that the most probable disposition of the line representing the dependence of y on x is that for which the sum of the squares of the deviations of the points from the line is a minimum (hence the name "least squares")

$$\Sigma(d_i)^2 = d_1^2 + d_2^2 + d_3^2 + d_4^2 + \ldots\ldots = \text{a minimum}$$

This statement is called the "principle of least squares" and it is the basis of a method for finding the relationship between y and x which best fits the data points (for which the sum of the squares of the deviations is a minimum).

Actually the problem of determining the line which "best" fits a set of data points (x_i, y_i) is several different problems, depending on the type of curve which is to represent the relationship between x and y. If it has been predetermined from the data or from theory that y depends on x linearly so that $y = Ax + B$, the problem becomes one of picking out, from all possible straight lines, the one with values of slope A and intercept B such that the sum of the d_i^2 will be as small as possible. If (x_1, y_1) are the coordinates of the first data point, (x_2, y_2) the coordinates of the second and so forth, and if it is assumed that the deviations are only in the y measurement for precisely known x's, then

$$\Sigma(d_i)^2 = (Ax_1 + B - y_1)^2 + (Ax_2 + B - y_2)^2 + \ldots\ldots$$

If the "best" straight line is that which makes the sum of the squared deviations or a minimum.

$$\frac{d[\Sigma(di)^2]}{dA} = 0 = 2x_1(Ax_1 + B - y_1) + 2x_2(Ax_2 + B-y_2) + \ldots\ldots$$

$$\frac{d[\Sigma(di)^2]}{dB} = 0 = 2(Ax_1 + B - y_1) + 2(Ax_2 + B - y_2) + \ldots\ldots$$

are the conditions which should lend to the "best" values of A and B. These equations may be rewritten:

$$B\,\Sigma x_i + A\,\Sigma x_i^2 - \Sigma x_i y_i = 0 \qquad (1)$$

$$nB + A\,\Sigma x_i - \Sigma y_i = 0 \qquad (2)$$

where n is the number of points.

The method is illustrated below for a set of n = 5 points.

Point No.	1	2	3	4	5
x	1.00	1.90	2.60	3.20	4.00
y	0.90	3.00	4.00	5.50	6.90

A table is made as follows:

x	y	x^2	xy
1.00	0.90	1.00	0.90
1.90	3.00	3.61	5.70
2.60	4.00	6.76	10.40
3.20	5.50	10.24	17.60
4.00	6.90	16.00	27.60

$\sum x_i = 12.70 \quad \sum y_i = 20.30 \quad \sum x_i^2 = 37.61 \quad \sum x_i y_i = 62.20$

Substituting in (1) and (2),

$$12.70\ B + 37.61\ A = 62.60$$

$$5\ B + 12.70\ A = 20.30$$

Solving simultaneously, $B = -0.989$ $A = 1.988$

The equation of the straight line which best fits the data points is

$$y = 1.988\ x - 0.989$$

In other words the sum of the squares of the deviations of the points from the straight line is a minimum for a line of slope 1.988 and y intercept -0.989.

It is generally shown in books on statistics that the standard deviations in these values obtained for the slope A and intercept B may be found using the equations (3 and 4):

$$\sigma_A = \left[\frac{\sum d_i^2}{n\sum x_i^2 - (\sum x_i)^2}\right]^{1/2} = \left[\frac{\sum(Ax_i + B - d_i)^2}{n\sum x_i^2 - (\sum x_i)^2}\right]^{1/2}$$

$$\sigma_B = \left\{\frac{(\sum d_i^2)\ (\sum x_i^2)}{n^2\sum x_i^2 - n\ (\sum x_i)^2}\right\}^{1/2} = \left\{\frac{\left[\sum(Ax_i + B - y_i)^2\right]\left[\sum x_i^2\right]}{n^2\sum x_i^2 - n\ (\sum x_i)^2}\right\}^{1/2}$$

In cases where a <u>nonlinear</u> curve is to be fit to a set of data points in such a way as to make $\Sigma(di)^2$ a minimum, equations (1), (2), (3), and (4) no longer apply. Often one can get around this difficulty, however. For example, suppose some data points are to be fit with a parabola of the type $y = Ax^2 + B$. If the quantity $X = x^2$ is calculated for each of the points, the method may then be applied to quantities y and X, since y versus X will be a straight line ($y = AX + B$) even though y versus x is not.

The least squares method is not confined to finding the constants of a straight line, however; it can be applied to any kind of curve. For example, if one has a set of data points and wants to determine the constants of the "best fit" parabola $y = AX^2 + BX + C$, he can apply the conditions that minimize $\Sigma(di)^2$ with respect to variables A, B, and C and will obtain the equations:

$$\Sigma x_i^2 y_i = C\Sigma x_i^2 + B\Sigma x_i^3 + A\Sigma x_i^4$$

$$\Sigma x_i y_i = C\Sigma x_i + B\Sigma x_i^2 + A\Sigma x_i^3$$

$$\Sigma y_i = nC + B\Sigma x_i + A\Sigma x_i^2$$

which may be solved simultaneously for constants A, B, and C.

<u>References</u>:

1. Young, "Statistical Treatment of Experimental Data", section 14.

2. Baird, "Experimentation", Appendix 2

3. Barford, "Experimental Measurements", Chapter 3

4. Pugh and Winslow, "The Analysis of Physical Measurements", Chapter 10.

5. Bevington, "Data Reduction", Chapters 6 and 11

6. Gerhold, "Least-Squares Adjustment of Weighted Data to a General Linear Equation", American Journal of Physics, Vol. 37, p. 156.

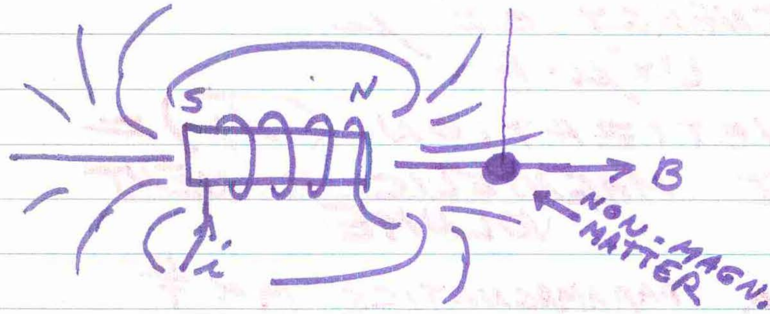
PHYS. IV

PHYS IV

PHYS IV

1-7-70

MOVING CHARGES → MAGNETIC FIELD
↳ SPINNING & ROTATING.



PARAMAGNETIC 1) VERY WEAK ATTRACTION
DIAMAGNETIC 2) " " REPULSION
FERROMAGNETIC 3) STRONG ATTRACTION
(NO STRONG REPULSION)

2) MECHANISIM OF DIAMAGNETIC EFFECT
IS IN ALL MATTER
1) PARAMAGNETIC CONTAINS PERMANENT
ATOMIC DIPOLES (LENZ' LAW)
3) FERROMAGNETIC - Fe, Ni, Co, ETC.

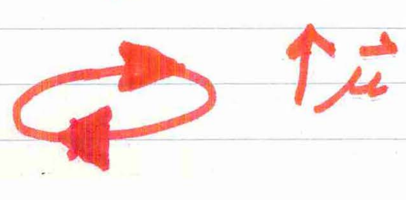
MONDAY - TEST

1-9-70

MAGNETIC DIPOLE MOMENT, $\vec{\mu}$

$|\vec{\mu}| = \mu\ (\text{LOOP}) = iA$

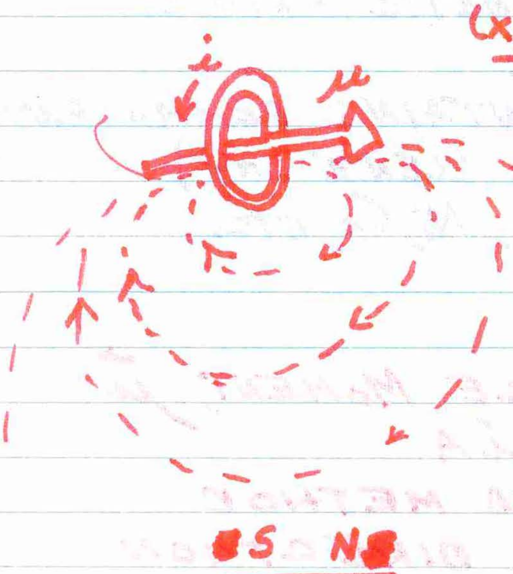
(USE RT. HAND METHOD
FOR FINDING DIRECTION
OF $\vec{\mu}$)



- PLACE $\vec{\mu}$ in EXT. $\vec{B}$,
  THEN $\tau = \mu \times B$
  ENERGY OF μ
  $U = -\mu \cdot B$
- MAGNETIZATION $(\vec{M})$ = $\frac{\text{NET MAGNETIC MOMENT}}{\text{VOLUME}}$

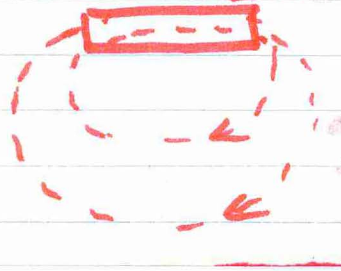
  FOR PARAMAGNETICS, $M \propto \frac{B}{T}$
  (CURIE'S LAW)
- THE FIELD B, DUE TO A DIPOLE
  $B_{(AXIS)} \simeq \frac{\mu_0 \mu}{2\pi x^3}$

  x - DISTANCE FROM DIPOLE TO FIELD POINT (x >> DIPOLE DIMENSIONS)

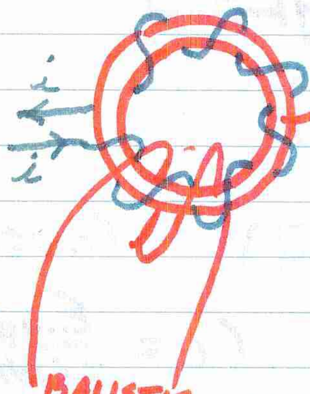


GAUSS'S LAW - THE NET FLUX THRU ANY CLOSED GAUSIAN SURFACE = 0 (MAGNETIC)

OR $\oint \vec{B} \cdot d\vec{S} = 0$



DONUT OF IRON



$B_0 = \mu_0 n i$ (WITHOUT IRON)

IF IRON IS PRESENT,
$\vec{B} = \vec{B}_0 + \vec{B}_{MAG}$

$B_{MAG} \propto M$

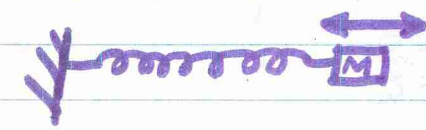
BALISTIC GALV · MEASURE q (PULSE) WHEN i IS TURNED ON, THEN SOLVE FOR $\vec{B}$

1-14-70

WEBER = $\frac{NT \cdot M}{AMP \cdot [redacted]} = \frac{JOULES}{AMP}$

ELECTROMAGNETIC OSCILLATIONS

MASS-SPRING SYSTEM

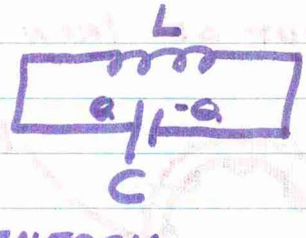


ENERGY ⟨ POTENTIAL = U_P, KINETIC = U_K

$E = U_P + U_K$ = CONSTANT IN TIME
$U = U_P + U_K$ = "

SIMILARLY,

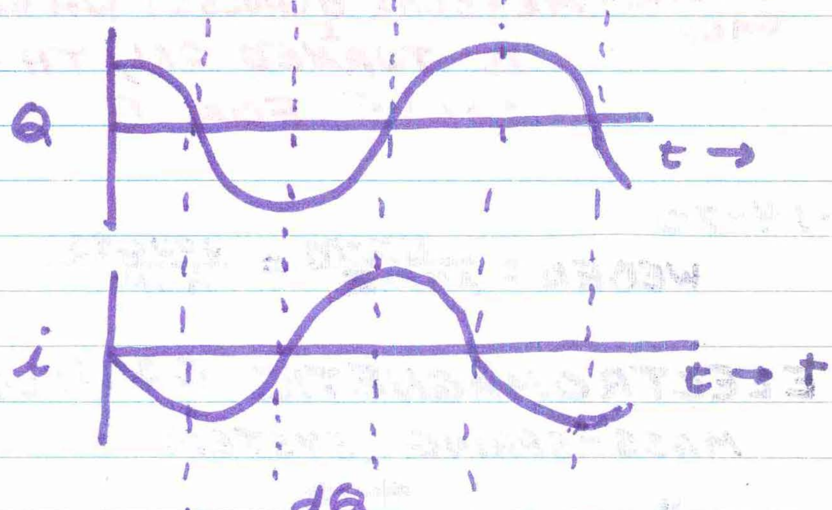
SYSTEM →



FORMS OF ENERGY

$$U_C = \frac{Q^2}{2C} = \frac{1}{2}QV \quad (C = Q/V)$$

$$U_L = \frac{1}{2}Li^2 \quad \left(L = \frac{\mathcal{E}_{ind}}{di/dt}\right)$$



BECAUSE $i = \frac{dQ}{dt}$

CONSERVATION:

$$U = U_{CAP} + U_{INDUCT} = U_E + U_P = \text{CONST.}$$

$$\frac{dU}{dt} = 0 = \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}Li^2\right)$$

$$0 = \frac{Q}{C}\left(\frac{dQ}{dt}\right) + Li\left(\frac{di}{dt}\right)$$

$$= \frac{Q}{C}(i) + Li\frac{d}{dt}\left(\frac{dQ}{dt}\right)$$

$$\Rightarrow \frac{d^2Q}{dt^2} + \frac{1}{LC} Q = 0$$

CHECK BY

KIRCHHOFF'S VOLTAGE LAW:

ΣV AROUND LOOP = 0

FOR L-C CIRCUIT:

$$\Sigma V = 0 = \frac{Q}{C} + L\frac{di}{dt}$$
$$= \frac{Q}{C} + L\frac{d}{dt}\left(\frac{dQ}{dt}\right)$$

SAME OUTCOME AS WITH ENERGY CONSIDERATION

SOLVING THE DIFFER. EQUATION

LET TRIAL SOLUTION:

$Q = Q(t) = A \sin(\omega t + \phi)$: A, ω, ϕ CONS.

$\frac{dQ}{dt} = \dot{Q} = \omega A \cos(\omega t + \phi)$

$\frac{d^2Q}{dt^2} = \ddot{Q} = -\omega^2 A \sin(\omega t + \phi) = -\omega^2 Q(t)$

SOLUTION IF $\omega^2 = 1/LC$

FREQ $\left(\frac{\text{CYCLES}}{\text{SEC}} = \text{HZ}\right) = 1/2\pi\sqrt{LC}$

$A = Q_{MAX} \Rightarrow$ BOUNDY CONDITION

1-18-70

B-FIELD PRODUCED BY TIME-VARYING E-FIELD

$$\oint_{\text{LOOP}} \vec{B}\cdot d\vec{l} = (\text{CONST}) \frac{d}{dt}\Phi_E$$

CONST > 0 = $\mu_0 \epsilon_0$

COMARE WITH:

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B$$

RECALL AMPERE'S LAW:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i$$ ← CURRENT IN LOOP

MAXWELL GENERALIZED:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i$$

BOTH CONDUCTION CURRENTS AND CHANGING E FIELDS ($\dot{E}$) ARE "SOURCES" OF $\vec{B}$

DISPLACEMENT CURRENT: (i_d)

$$\oint B\cdot dl = \mu_0 (i_d + i)$$

$$i_d \equiv \epsilon_0 \frac{d\Phi_E}{dt}$$ (DEFN.)

IF FIELD IS CHANGING, A $\vec{B}$ ARISES

SHOW MAG. OF i_d IN // PLATE CAPAC.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$\frac{dE}{dt} = \frac{dQ}{dt}\left(\frac{1}{A\varepsilon_0}\right) = \frac{i}{A\varepsilon_0}$$

BY DEF: $i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$

$$= \varepsilon_0 \frac{d}{dt}(EA)$$

$$= \varepsilon_0 A \frac{dE}{dt}$$

HOWEVER $\frac{dE}{dt} = i/A\varepsilon_0$

$\therefore\ i = i_d$

$\therefore$ DISPLACEMENT CURRENT IN CAPACITOR IS SAME AS CURRENT IN THE WIRE

REVIEW:

CASE - NO POLARIZABLE OR MAGNETIZABLE MATTER

GAUSS'S LAW: $\oint \vec{E} \cdot d\vec{S} = \frac{Q_{\text{IN SURF.}}}{\varepsilon_0}$

FOR MAGNETISM:

$$\oint \vec{B} \cdot d\vec{S} = 0$$

→ NO MAG. CHARGES EXIST

(CONT.)

(CONT.)

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(\epsilon_0 \frac{d\Phi_E}{dt} + i\right)$$

$$\oint \vec{E}\cdot dl = -\frac{d\Phi_B}{dt}$$

(CALLED FARADAY INDUCTION)

LET $i \to 0$, $Q \to 0$ (EMPTY SPACE)

$$\epsilon_0 \oint \vec{E}\cdot d\vec{s} = 0$$

$$\oint \vec{B}\cdot d\vec{s} = 0$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{E}\cdot dl = -\frac{d\Phi_B}{dt}$$

THESE ARE SOURCE OF WAVES (CAN BE DERIVED FROM THEM)

REMEMBER:

$$\Phi_E = \oint \vec{E}\cdot d\vec{s}$$

SEEK: $\vec{E} = \vec{E}(\vec{r},t) = \vec{E}(x,y,z,t)$

$B = B(\vec{r},t)$

1-19-70

SOME OPERATIONS WITH VECTORS

Let $A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

RECALL: $\hat{i}\cdot\hat{i} = 1$

$\hat{i}\cdot\hat{j} = 0$

$\hat{i}\times\hat{i} = 0$

$\hat{i}\times\hat{j} = \hat{k}$

$\hat{j}\times\hat{i} = -\hat{k}$

DEFINE $\vec{\nabla} = i\, d/dx + k\, d/dz + j\, d/dy$

∇ = dEL

LET $\vec{\nabla}$ OPERATE ON A SCALAR FIELD: $V = V(x, y, z)$

$$\vec{\nabla} V = i \frac{dV}{dx} + j \frac{dV}{dy} + \hat{k} \frac{dV}{dz}$$

= GRADIENT OF V

VECTOR PRODUCTS

→ 1) THE DIVERGENCE OF VEC. FLD.

let $\vec{F}(x, y, z)$

$$\text{DIV } \vec{F} = \vec{\nabla} \cdot \vec{F}$$

$$= (F_z \vec{k} + F_x i + F_y j) \left(i \frac{d}{dx} + j \frac{d}{dy} + \hat{k} \frac{d}{dz} \right)$$

$$\text{DIV } \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

→ 2) $\vec{\nabla} \times \vec{F} = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \times (F_x i + F_y j + F_z k)$

$$= i \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) + j \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) + k \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$$

$\vec{\nabla} \times \vec{F}$ CALLED CURL F

→ 3) $\vec{\nabla} \cdot \vec{\nabla} = \vec{\nabla}^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

e.g.) $\vec{\nabla}^2 \vec{B} = \frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 B}{\partial z^2}$

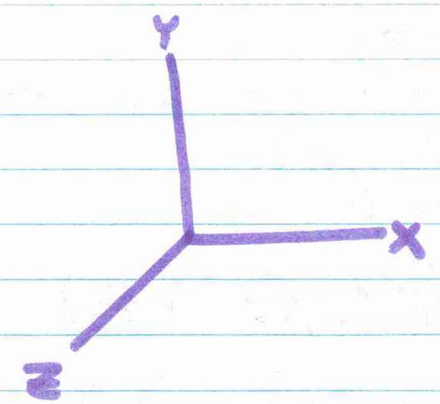
EXERCIZE: SHOW THAT THE

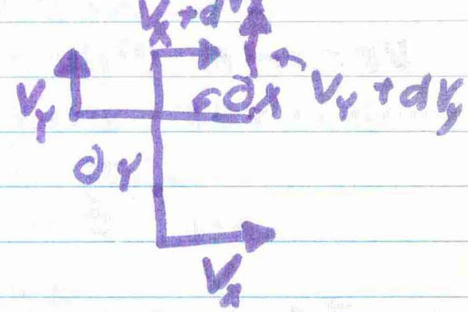
$$\text{CURL}(\text{CURL } \vec{G}) \equiv -\vec{\nabla}^2 \vec{G} + \vec{\nabla}(\vec{\nabla} \cdot \vec{G})$$

CURL $\vec{V}$; V = FLUID VELOCITY

SHOW: ANGULAR VELOCITY (ω) OF PORTION IS

$$\frac{1}{2}\,\text{curl}\,\vec{V}$$





ANGULAR VELOCITY OF ∂x IS

$$\frac{dV_y}{\partial x} = \frac{(\partial V_y/\partial x)\,\partial x}{\partial x} = \frac{\partial V_y}{\partial x}$$

ANGULAR VELOCITY OF ∂y IS

$$-\frac{\partial V_x}{\partial y}$$

AVERAGE ANGULAR VELOCITY

$$\omega_z = \frac{1}{2}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\therefore\ \omega_z = \frac{1}{2}\left(\text{curl}\,\vec{V}\right)_z$$

(Z COMPONENT OF CURL)

1) DIVERGENCE THEOREM

$$\oint \vec{A} \cdot ds = \int \vec{\nabla} \cdot \vec{A} \, d\iota$$

a) A = VECTOR FIELD
b) INTEGRATE AROUND ~~SURFACE~~ DIV. BOUNDED BY X
c) $\partial \iota$ = VOL ~~ELT~~ (= $dy \cdot dx \cdot dz$)
d) FLUX OF $\vec{A}$ (CONT.) = $\int \text{div} \vec{A} \, d\iota$

2) STOKE'S THEOROM

$$\oint \vec{U} \cdot dl = \int (\vec{\nabla} \times \vec{U}) \cdot ds$$

LOOP (↑ under $\oint$); SURFACE, S, SPANNING LOOP (↑ under $\int$)

LAWS AND STUFF:

1) $$\varepsilon_0 \oint \vec{E} \cdot d\vec{s} = q_{in} = \int \rho \, d\tau$$

VOL. BOUND BY S (↑ under $\int$)

ρ = CHARGE DENSITY.

DIV. THEOROM:

$$\varepsilon_0 \int_v \vec{\nabla} \cdot \vec{E} \, d\tau$$

~~HOLDING ρ CONSTANT~~?

⇒ AT EVERY POINT:

$$\vec{\nabla} \cdot \vec{E} = \rho / \varepsilon_0$$

2) $\vec{\nabla} \cdot \vec{B} = 0$ EVERYWHERE

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(\varepsilon_0 \frac{\partial \Phi_E}{\partial t} + i \right)$$

$$\Phi_E = \int \vec{E} \cdot dl$$

$$i = \int_S J \cdot dS$$

$$\therefore \oint \vec{B} \cdot d\vec{l} = \mu_0 \left(\varepsilon_0 \frac{d}{dt} \cdot \int_{S \text{ (SPANS LOOP)}} \vec{E} \cdot dl + \int_S \vec{J} \cdot d\vec{S} \right)$$

1-23-70

$$\oint_{\text{LOOP}} \vec{B} \cdot dl = \mu_0 \left(i + \varepsilon_0 \frac{\partial \Phi_E}{\partial t} \right)$$

$$= \mu_0 \left(\int \vec{J} \cdot dS + \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot dS \right)$$

SURFACE SPANNING LOOP

J = CURRENT DENSITY

$$= \mu_0 \int_{S'} \vec{J} \cdot dS + \mu_0 \varepsilon_0 \int_{S'} \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S}$$

O.K. IF $d\vec{S}$ IS STATIONARY

$$\oint_{\text{LOOP}} \vec{B} \cdot d\vec{l} = \mu_0 \int_{\text{SURFACE}} \left(J + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}$$

STOKE'S THEOROM:

$$\oint \vec{B} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{B}) \cdot dS$$

$(\vec{\nabla} \times \vec{B})$ = "CURL" OF $\vec{B}$

IF $\int_S$ ARE EQUAL, THE INTEGRADS ARE EQUAL EVERYWHERE.

$$\therefore \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = \vec{\nabla} \times \vec{B} \quad \text{AT ALL PTS.}$$

(BY ANALOGY, → $-\frac{\partial B}{\partial t} = \vec{\nabla} \times \vec{E} \Leftarrow \oint_{LOOP} \vec{E} \cdot d\ell = -\frac{d \Phi_B}{dt}$

$\operatorname{div} \vec{E} = \rho / \varepsilon_0$

$\operatorname{div} \vec{B} = 0$

MAXWELL'S EQUATIONS IN DIFFERINTIAL FORM

CASE: EMPTY SPACE (VACUUM)

$J = 0 \; ; \; \rho = 0$

$\therefore \operatorname{div} \vec{E} = \vec{\nabla} \cdot \vec{E} = 0$

$\operatorname{div} \vec{B} = 0$

$\text{CURL } \vec{B} = \vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$\text{CURL } \vec{E} = -\frac{\partial}{\partial t} \vec{B}$

TRICK: VECTOR OPERATIONS:

OPERATE WITH $(\vec{\nabla} \times)$ OF EQ. FOR CURL $\vec{E}$

$$\text{CURL CURL } \vec{E} = -\text{CURL } \frac{\partial \vec{B}}{\partial t} = -\frac{\partial}{\partial t} \text{curl } \vec{B}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial B}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial B}{\partial t}\right)$$

IDENTITY: $\text{curl curl } \vec{E} \equiv -\nabla^2 \vec{E} + \text{grad div } \vec{E}$

SINCE $\vec{E} = 0$

$$\therefore \nabla^2 \vec{E} = \frac{\partial}{\partial t} (\text{curl } \vec{B}) = \frac{\partial}{\partial t} \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

SIMILARLY: $\nabla^2 \vec{B} = -\frac{\partial^2 B}{\partial t^2}$

} WAVE EQUA.

TREAT CASE: CARTESIAN COORDINATES
PLANE WAVE SOLUTION

LET ELEC. & MAG. FIELD VECTORS LIE ALONG Y, Z AXES

$\therefore B_x = B_y = 0 \; ; \; E_x = E_z = 0$

LET ALL NON-ZERO FIELD COMPONENTS BE FUNCTIONS OF (x, t) ONLY (PLANES // TO YZ PLANE ALSO WORK)

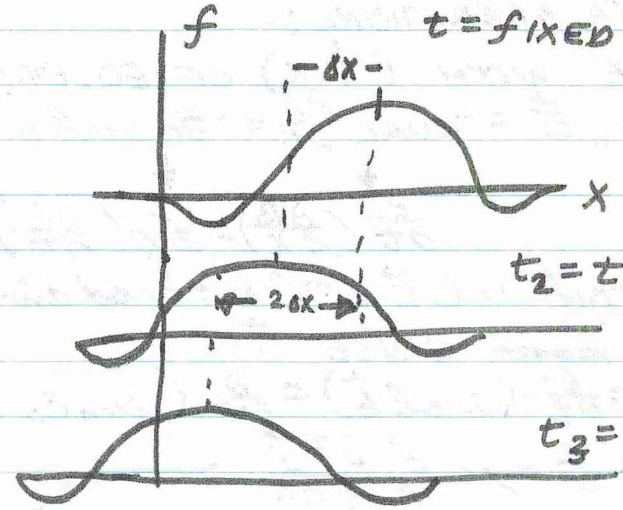
TRIAL SOLUTION TO E WAVE EQ.

Let $E = E_Y = f(x + vt)$

$$\frac{\partial f}{\partial x} = \frac{1}{V} \frac{\partial f}{\partial t}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{V^2} \frac{\partial^2 f}{\partial t} \qquad (f = |\vec{E}_Y| = E_Y)$$

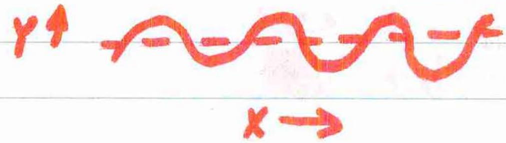
THUS: $\frac{1}{V^2} = \mu_0 \varepsilon_0$, IF TRIAL SOLUTION SATISFIES WAVE EQ.



$$\frac{\Delta x}{\delta t} = V$$

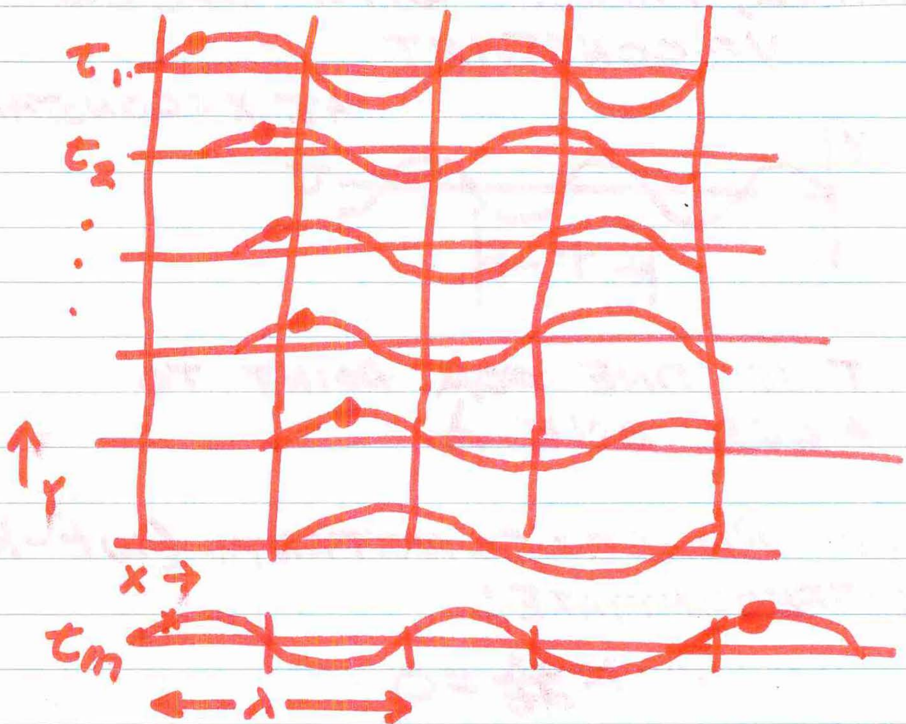
V IS TRIAL SOLN. = SPEED OF LIGHT

1-28-70



AT SOME FIXED TIME $Y = A \sin_? X$

LET Y VS. X OF SURFACE RIPPLE BE OBSERVED AT SUCCESSIVE INSTANTS OF TIME



WAVEVENGH (λ) DISTANCE BETWEEN TWO NODES, OR WHERE $Y^o = Y'$ AND $X^o = X'$ AND $\frac{dY^o}{dX^o} = \frac{dX'}{dY'}$.

PERIOD, $T = t_m - t_1$

VELOCITY; $v = \lambda / T$

FREQUENCY $= 1/T$

EXPRESS Y AS $Y(x,t)$

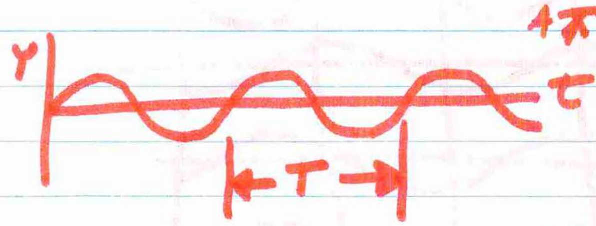
TRUE TRIAL:

$$Y = A \sin(\omega t - kx)$$

A POINT OF FIXED (CONSTANT) PHASE, MOVES WITH SPEED

V = CONSTANT

4π X = CONSTANT



T IS TIME FOR POINT TO MOVE ONE λ

LET PHASE = CONSTANT = $(\omega t - kx)$

DIFFERENTIATE:

$$\omega - k\frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k} = \text{CONSTANT} = V$$

$\sin(\omega t_1 - kx) = \sin[\omega(t_1+T) - kx)$

TRUE IF

$T = 2n\pi$, $n = \ldots {}^-2, {}^-1, 0, 1, 2 \ldots$

$\omega = 2\pi\nu = 2\pi/T$

$\omega T = 2\pi$

$k = 2\pi/\lambda$

$v = \lambda/T$

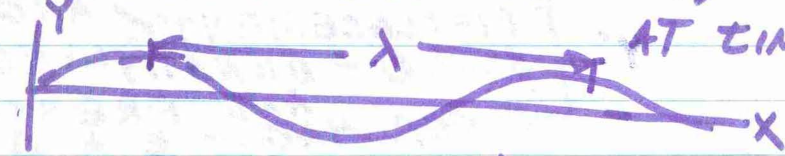
$\sin[\omega t - kx] = \sin[\omega(t_1+T) - kx_1)$

$= \sin(\omega t - kx + \omega T = {}^-A \sin(k(x - vt$

$f = f(x \pm vt)$

1-30-70

$Y = A \sin kx$ (OR $A \cos kx$)



$Y(x') = Y(x' + n\lambda)$

$\therefore A \sin kx' = A \sin(kx' + kn\lambda)$

$\sin A = \sin(A + 2n\pi)$ $n = 0, \pm1, \pm2 \ldots$

$\therefore k\lambda = 2n\pi$

OR $k = \frac{2\pi}{\lambda}$ = WAVE NUMBER

$\frac{1}{\lambda}$ = # OF WAVES/UNIT LENGTH

$Y = A \sin kx$

ADD TIME DEPENDENCE:

Let $Y' = Y'(x, t)$

$Y' = A \sin(kx \mp \omega t)$ | ω, k, A CONSTANTS |

POINT OF CONSTANT PHASE MOVES. PHASE = $(kx \mp \omega t)$ = CONST.

DIFFERINTIATE

$k\,dx \mp \omega\,dt = 0$

$\frac{dx}{dt} = \pm\frac{\omega}{k}$, = SPEED

SPEED $V = \lambda/T = \omega/k$

OR $\omega = k\,\lambda/T$

$\omega = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{T} =$

$V = \frac{\omega}{k}$ $\quad \omega = \frac{2\pi}{T} = 2\pi\nu$

Y' [DISPLACEMENT < ANG / LINEAR
SOUND - DENSITY AND PRESSURE
ELECROMAG. - E & B

SUPERPOSITION (ADDITION OR SUMMATION)

FORIER - ANY ~~MATHEMATICAL~~ PERIODIC FUNCTION MAY BE EXPRESSED BY:

$$x = \sum_{n=1,2,3\ldots}^{n=N} (a_n \sin nt + b_n \cos nt)$$

a_n; b_n ARE CONSTANTS

CASE ADD 2 WAVES OF SAME SPEED & FREQUENCY, BUT DIFFERENT PHASE.

$$Y_1 = Y_{MAX} \sin(kx - \omega t - \phi)$$
$$Y_2 = Y_{MAX} \sin(kx - \omega t)$$

ϕ = PHASE ANGLE, CONSTANT

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$Y = Y_1 + Y = 2 Y_{MAX} \sin\left(kx - \omega t - \frac{\phi}{2}\right) \cos\frac{\phi}{2}$$

→ MODIFIED TRAVELING WAVE

2-2-70

1) EX. OF ADDITION OF 2 WAVES TRAVELING IN SAME DIRECTION, SAME V

$$Y = \sin(kx - \omega t) + \sin(kx - \omega t - \phi)$$
$$= \left(2 \cos\frac{\phi}{2}\right) \sin(kx - \omega t - \phi/2)$$

WHICH IS ALSO A TRAVELING WAVE. THE AMPLITUDE DEPENDS UPON THE PHASE ANGLE ($= \phi$)

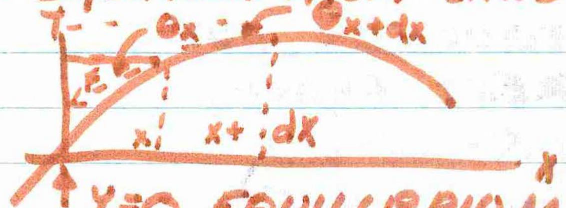
2) Ex. OF ADDITION OF 2 WAVES TRAVELING IN OPPOSITE DIRECTION

ξ IS SAME $\quad Y' = \sin(kx - \omega t) + \sin(kx + \omega t)$

$$= 2 \sin kx \cos \omega t$$

IS NOT A TRAVELING WAVE

WAVE EQUATION FROM LAWS OF MECHANICS



Y=0 EQUILIBRIUM

F = TENTION μ = LINEAR MASS DENSITY

m = MASS

$dm = \mu\, dx$

$F_{NET,Y} = (dm)\, a_y$

$F \sin\theta_{x+dx} - F \sin\theta_x = \mu\, dx\, a_y$

IF θ IS SMALL → $\theta \cong \sin\theta \cong \tan\theta$

$\tan\theta = \partial y / \partial x$

$\therefore\ F\left(\frac{\partial y}{\partial x}\right)_{x+dx} - F\left(\frac{\partial y}{\partial x}\right)_x = \mu\, dx\, a_y$

$F \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) \partial x$

$\therefore\ F \frac{\partial^2 y}{\partial x} \partial x = \mu\, \partial x\, \frac{\partial^2 y}{\partial t^2}$

$F \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2}$

PARTIAL DIFFERENTIAL EQUATION

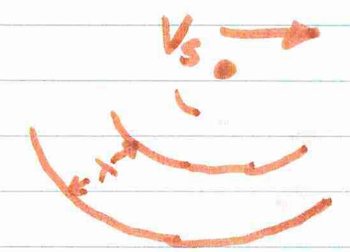
$y = f(x \pm vt)$

THIS SOLUTION SATISFIED DIFFERENTIAL EQ. IF

$v^2 = \frac{F}{\mu}$

DOPPLER EFFECT:

1) Ex) SOURSE MOVE TOWARD OBSERVOR



AS SOURCE MOVES, λ WILL BECOME MUCH SHORTER.

LET v = SOUND VELOCITY

$$\lambda = \frac{v - V_s}{f}$$

OBSERVOR'S FREQ = f'

$$f' = \frac{v}{\lambda'} = f \frac{v}{v - V_s}$$

2-4-70

EX. OF DOPPLER EFFET

① $\underrightarrow{v' = v}$ • ← OBSERVOR

$\vec{v}' = v$

$V_s = 2\pi r f$

$\simeq 19$ ft

V_0(SOUND) $\simeq 1.1 \times 10^3 \frac{ft}{SEC}$

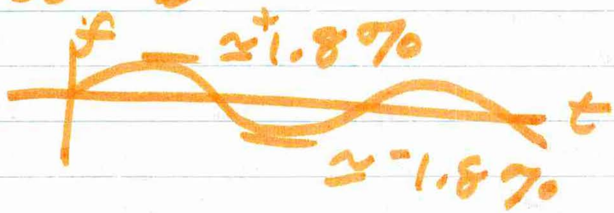
$$\left(\frac{\nu'}{\nu}\right)_{①} = \frac{V_0}{V_0 - V_s}$$

$$\frac{\Delta \nu'}{\nu} = \frac{\nu' - \nu}{\nu} = \frac{V_s}{V_0 - V_s} \simeq \frac{19}{1080}$$

2/8/70

TOTAL $\frac{\Delta\nu}{\nu} = 3.6\%$

$\nu_{OBS} = \nu_0'$



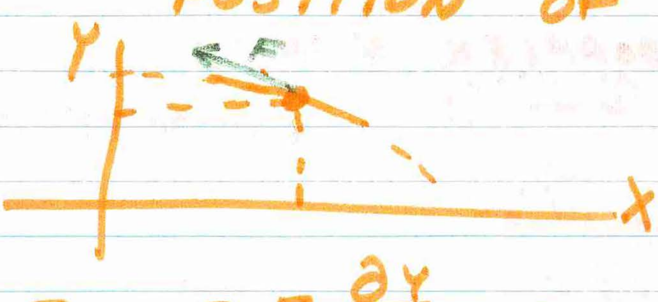
ENERY MOVES ALONG WAVES

POWER IN TRANSVERSE STRING WAVE.

Y = TRANSVERSE DISPLACMNT

F = TENSION

X AXIS = EQUILIBRIUM POSITION OF STRING



$Y = f(x, t)$

$F_{NET_Y} = F \frac{\partial Y}{\partial X}$ FOR $\theta = \tan^{-1} \frac{dy}{dx}$

FOR SMALL θ

POWER (P) PASSES THRU POINT X
(ENERGY/TIME) FOR WAVE
MOVING TO THE RIGHT

$$P = F_Y (\text{VELOCITY})_Y$$

$$= \left(F \frac{\partial y}{\partial x}\right) \frac{\partial y}{\partial t}$$

CASE: $Y = Y_{MAX} \cos(kx - \omega t)$

TAKING DIREVITIVES:

$$P = F(Y_m \bar{k} \sin kx - \omega t)(Y_m \omega \sin kx - \omega t)$$

$$= {}^{-}Y_m^2 F k\omega \sin^2(kx - \omega t)$$

⇒ DEPEND ON X AND t

AVERAGE POWER $(\bar{P})$
AVERAGE $\Delta t = T = 1/f = 2\pi/\omega$

$$\bar{P} = \frac{1}{T} \int_{t=t_1}^{t=t'+T} P\, dt \quad \left(\Rightarrow T = \int_{t'}^{t'+T} dt\right)$$

$$= \frac{1}{T}(-Y_m^2 F k\omega) \int_0^{2\pi} (\sin^2(kx - \omega t)\, dt$$

$\therefore T/2$

$$\Rightarrow \frac{-Y_m^2 F k\omega}{2} = \bar{P}$$

ALSO $v^2 = F/\mu$; $v = \frac{\omega}{k}$

$$\Rightarrow \bar{P} = \frac{-1}{2} Y_m^2 F \frac{\omega^2}{v}$$

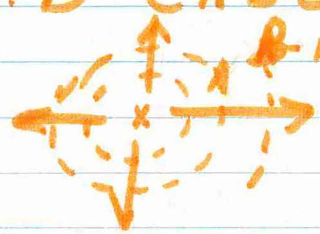
$$= -\frac{1}{2} Y_m^2 \omega^2 \mu v$$

AVERAGE POWER PROPORTIONAL TO 1) $AMPLITUDE^2$
2) $FREQ^2$ 3) MASS(μ) 4) SPEED

GENERALLY, ALL WAVES POWER IS DEPENDENT ON $(AMPL)^2$ AND $FREQ^2$

THERE IS ENERGY LOSS IN WAVES

3-D CASE:



$$\frac{POWER}{CROSS\ SECTION\ AREA} = I \ (= INTENSITY)$$

M.K.S. $I - \frac{JOULES}{SEC - m^2}$

SPHERICAL WAVE INTENSITY

- ASSUME - ENERY LOSS IS NEGLIGIBLE

$A_{SPHERE} = 4\pi r^2$

$P_A = P_B$

$I_A = 4\pi r_A^2 = I_B(4\pi r_b^2)$

$$\therefore \frac{I_B}{I_A} = \left(\frac{r_A}{r_B}\right)^2$$

2-11-70

MOMENTUM OF LIGHT WAVES

40-6)

m = mass

LIGHT

U = ENERGY
P = MOMENTUM

$t = 1$ DAY POWER $= \Delta U/\Delta t = 10^4$ WATTS

CONSERVATION OF U & P (MOMENTUM)

AFTER 1 DAY, SHIP HAS P OF mV

$P \propto U$ OR $P = \frac{U}{c}$

FOR ABSORPTION OR EMITION

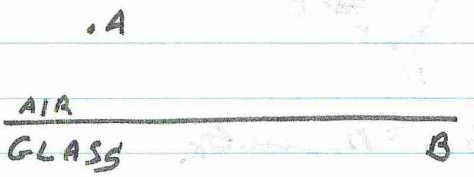
$$\Delta U = 10^4 \frac{\text{JOULE}}{\text{SEC}} (\Delta t \text{ SEC}) \quad (\Delta t = 1 \text{ DAY})$$

$$P_{LIGHT} = \frac{\Delta U}{c} = -m\Delta V$$

$$\Delta V \approx 4 \times 10^{-5} \frac{m}{SEC}$$

GEOMETRICAL OPTICS:

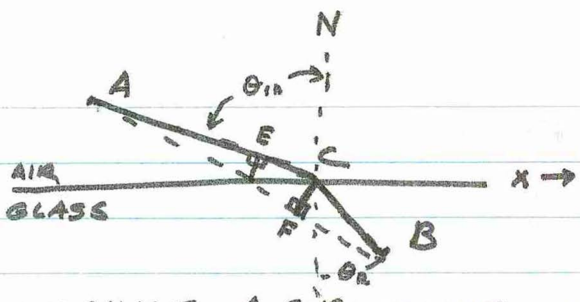
$\lambda \gtrless >$ DIMENSIONS OF SYSTEM



HOW DOES LIGHT GO FROM A to B?

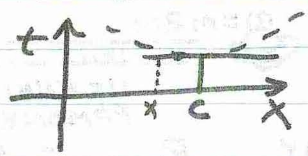
$$C_{GLASS} = \frac{1}{n}(C_{AIR}) \quad n = \text{INDEX OF REFRACTION}$$

PRINCIPLE: LIGHT RAY TAKES PATH A-B SUCH THAT, COMPARED TO NEARBY PATHS, TIME TAKEN IS A MINIMUM



ASSUME ACB TAKES THE LEAST TIME - t

t VS. X IS



THAT IS, Δt IS A SECOND ORDER ~~OF~~ INFINTESIMAL, FOR FIRST ORDER IN ΔX.

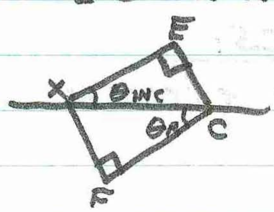
CHOOSE X REAL CLOSE TO C

TIME FOR LIGHT TO TRAVEL

ACB ≅ TIME AXB

TIME TO TRAVEL EC = TIME XF AS X → C

∴ $\overline{EC} = n \cdot \overline{XF}$



$\sin \theta_{IN} = n \sin \theta_R$

2-13-70

$n > 1$

$n = \frac{V_{IN\ AIR}}{V_{IN\ H_2O}}$

LET

V_0 = SP. OF LT. IN VACCUUM
V_1 = " " " " AIR
V_2 = " " " " GLASS

$$\frac{V_{AIR}}{V_{GLASS}} = \frac{V_1}{V_2} = n_{21} \; ; \quad V_0/V_2 = n_{20}$$

$$\frac{n_2}{n_1} = \frac{V_1}{V_0} = n_{01} = 1/n_{10}$$

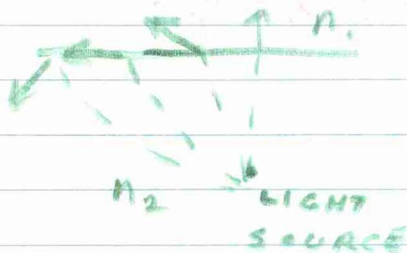
$$n_{10} = 1.0006$$

n = INDEX OF REFRACTION

$$\frac{V_1}{V_2} = n_{21} \qquad \frac{v_1 = \lambda_1 f_1}{V_2 = \lambda_2 f_2}$$

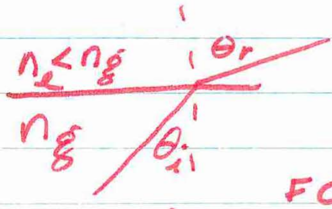
$$f_1 = f_2 \Rightarrow \frac{\lambda_1}{\lambda_2} = n_{21}$$

$\theta_i = \theta_R$



IF $n_2 > n_1$,
MAY GET TOTAL
INTERNAL REFLECTION

2-16-70



$$\frac{\sin\theta_r}{\sin\theta_i} = \frac{n_g}{n_\ell}$$

FOR CRITICAL ANGLE

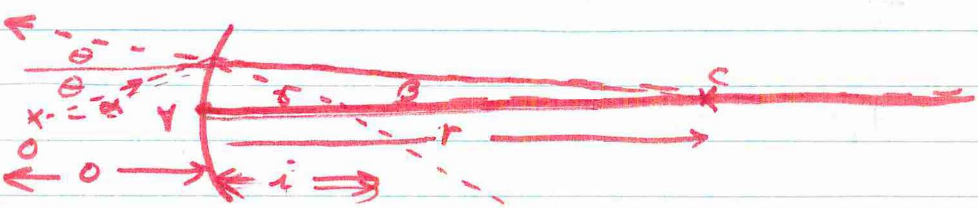
$$\frac{n_g}{n_\ell} = \frac{1}{\sin\theta_i}$$

TOTAL INTERNAL REFL. IFF $\sin\theta_i \geq \frac{n_\ell}{n_g}$

$$n_\ell \leq n_g \sin\theta_i$$

MIRRORS AND THINGS

CONSIDER CONVEX SPHERICAL MIRROR



Let $\alpha, \beta, \gamma,$ & θ BE SMALL

i.e, $\theta \approx \sin\theta \approx \tan\theta$

FIND $i, O,$ & r

$$\gamma = \frac{av}{i} \; ; \; \beta = \frac{av}{r} \; ; \; \alpha = \frac{av}{O}$$

$\Delta OaC \Rightarrow \alpha + \beta = \theta$

$\Delta Oai \Rightarrow \alpha + \gamma = 2\theta$

$$\gamma = 2\beta + \alpha$$

OR $$\frac{1}{i} = \frac{1}{O} + \frac{2}{R}$$

CONVENSIONS

R''	V
>0	<0

2-18-70

A) SETTING UP A MIRROR PROBLEM:

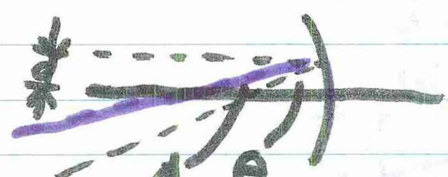
1) INCIDENT LIGHT L → RIGHT
2) REAL SIDE ON LEFT; VIRTUAL BEHIND MIRROR
3) r, i ARE + IF ON R SIDE
   " " " − " " VIR. "

B) NEAT FORMULA:

i = IMAGE DISTANCE
o = OBJECT DISTANCE
r = RADIUS OF MIRROR
f = FOCAL LENGTH

$$\frac{1}{i} + \frac{1}{o} = \frac{2}{r} = \frac{1}{f}$$

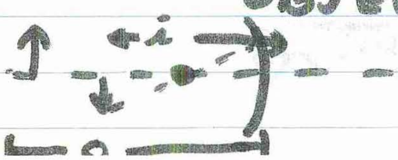
C) RELATION BETWEEN f & r



$\theta = 2\phi$
FOR SMALL α
$2f = r$

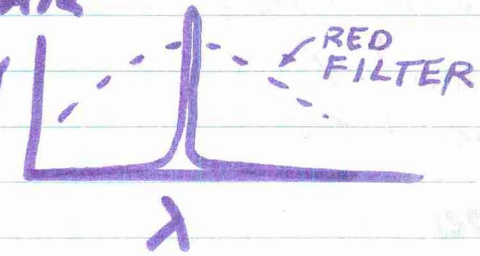
D) MAGNEFICATION, m

$$m = \frac{\text{IMAGE SIZE}}{\text{OBJECT SIZE}} = -\frac{i}{o}$$



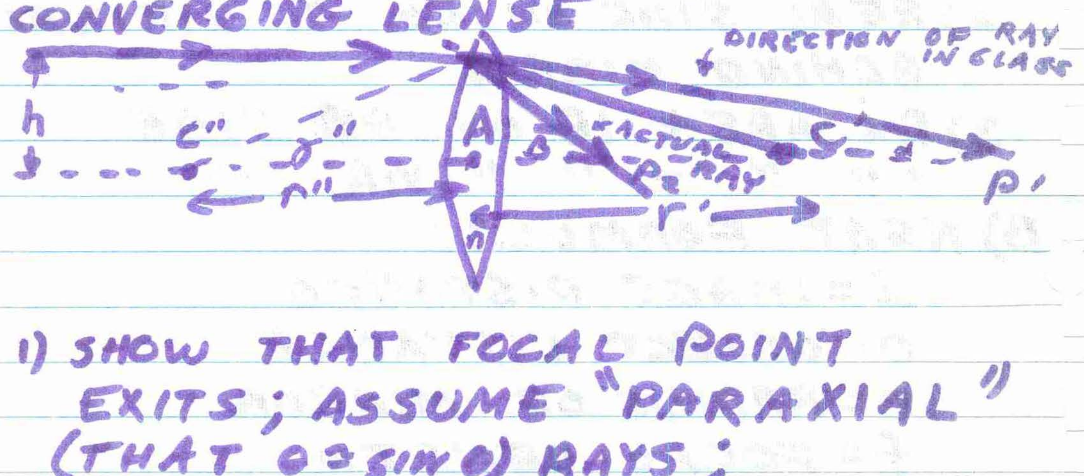
2-20-70

IN A LASAR



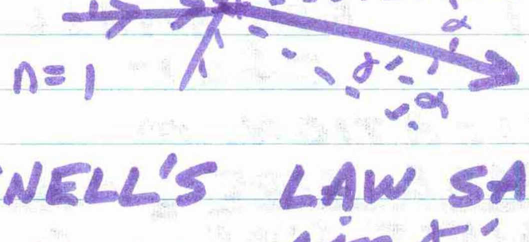
THIN LENSES, LIGHT, & JUNK

CONVERGING LENSE



1) SHOW THAT FOCAL POINT EXITS; ASSUME "PARAXIAL" (THAT θ ≈ SIN θ) RAYS;
THICKNESS << f, i, o
n = INDEX OF REF. FOR GLASS
C = CENTER OF CURVITURE

2) FIRST FACE



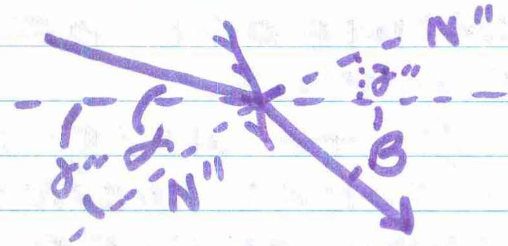
SNELL'S LAW SAYS:

$$n = \frac{\sin \gamma'}{\sin(\gamma' - \alpha)}$$

$\therefore \quad n \approx \frac{\gamma'}{\gamma' - \alpha}$ (ANGLES IN RADIANS)

EQ ① $\alpha = \left(\frac{n-1}{n}\right)\gamma'$

b) EXPANING SECOND SURFACE



SNELL'S LAW SAYS:

$$n = \frac{\sin(\gamma'' + B)}{\sin(\gamma'' + \alpha)}$$

EQ ② OR $\alpha = \frac{(1-n)\gamma''}{n} + \frac{B}{n}$

$\therefore \quad (n-1)\gamma' = (1-n)\gamma'' + B$

$B = (n-1)(\gamma' + \gamma'')$

ALSO, FROM BIG DIAGRAM:

$\tan B = \frac{h}{PA_2} \approx B; \ \tan \gamma' = \frac{h}{r'} \approx \gamma'$

$\tan \gamma'' \frac{h}{r''}$

SUBSTITUTING YIELDS:

$$\frac{1}{AP_2} = (n-1)\left(\frac{1}{r'} + \frac{1}{r''}\right)$$

(AP_2 INDEPENDENT OF h, THEREFORE ALL OF LIGHT RAYS FOCUS AT 1 POINT →

CALLED F (FOCAL LENGTH)
($=\overline{AP_2}$)

QUESTION: HOW TO RELATE
f, i, & o

ANS. USE KNOWLEDGE OF
RAY BEHAVIOR

1) RAY // TO AXIS; PASSES THRU FOCAL POINT
2) RAY THRU CENTER OF LENS (NOT REFACTED A LOT; JUST A TENY WEENY, ITSY BITSY, LITTLE BITTYE, MINUTE, INFINTESIMAL BIT

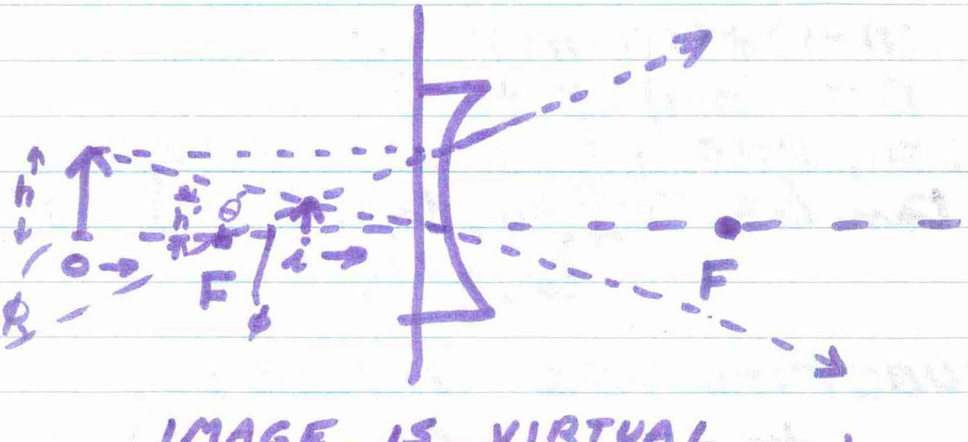


IMAGE IS VIRTUAL

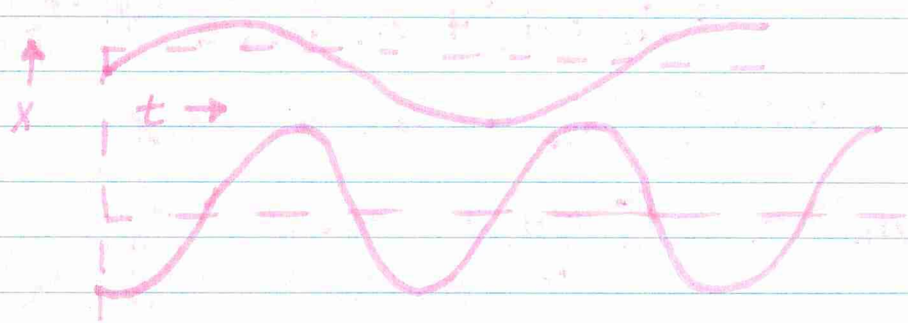
$\tan\theta = \frac{h}{o} = \frac{h'}{i}$; $\tan\phi = \frac{h}{f} = \frac{h-h'}{i}$

$\therefore \frac{1}{f} = \frac{1}{i} - \frac{1}{o}$

2-25-70

INTERFERENCE (SUPERPOSITION)

CONSIDER ADDING TWO WAVES. (SAME f & DIFFERENT PHASE CONSTANTS AND DIFFERENT AMPLITUDES)



LET EACH DISTURBANCE BE COMPLEX

$$\vec{A}_1(t) = A_1 e^{i(\omega t + \phi_1)}$$
$$\vec{A}_2(t) = A_2 e^{i(\omega t + \phi_2)}$$

LET REAL PART BE PHYSICAL QUANTITY (DISPL.; FIELD; ETC.)

$$Re[A_1(t) = A_1 \cos(\omega t + \phi_1)$$

RESULTANT (SUM) OF TWO

$$= R = (A_1 e^{i\phi_1} + A_2 e^{i\phi_2}) e^{i\omega t}$$
$$= \hat{R} e^{i\omega t}$$

$\hat{R}$ = "AMPLITUDE (CONSTANT)

$$= A_1 e^{+i\phi_1} + A_2 e^{+i\phi} = A_R e^{i\theta_R}$$

$\therefore\ R = A_R e^{i(\omega t+\phi_R)}$

FIND MAGNETUDE OF R (A_R)

1) FIND A_R^2 BY MULT. $\hat{R}$ BY IT'S COMPLEX CONJUGATE

$$A_R^2 = (A_1 e^{+i\phi_1} + A_2 e^{i\phi_2})(A_1 e^{-i\phi_1} + A_2 e^{-i\phi_2})$$

$$= A_1^2 + A_2^2 + A_1 A_2 (e^{i(\phi_1-\phi_2)} + e^{i(\phi_2-\phi_1)})$$

SINCE $e^{ix} + e^{-ix} = 2\cos x$

$$A_R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1-\phi_2)$$

$$\cos(\phi_2-\phi_1) = \cos(\phi_1-\phi_2)$$

I (NTENSITY) $\propto A^2$ (= AMPLITUDE)

3-2-70

REVIEW OF INTERFERENCE:

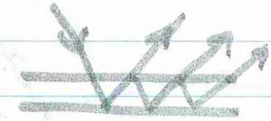
TWO WAVES FROM COHERENT SOURSES INTERFERE CONSTRUCTIVELY IF OPTICAL PATH DIFFERENCE = $m\lambda$ WHERE m IS AN INTEGER.

DESTRUCTIVE OCCURS AT $(m+\frac{1}{2})\lambda$ (180° OUT OF PHASE)

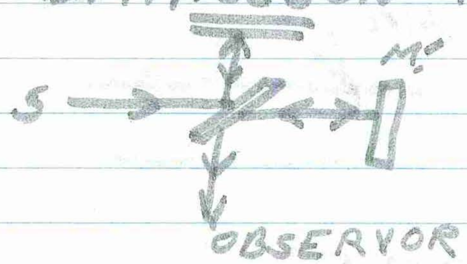
REFLECTED WAVE HAS π PHASE CHANGE IF IT TRAVELS IN MEDIUM (n_1) AND REFLECTS FROM n_2 ($>n_1$)

INTERFERENCE SOURCES

1) THIN FILM
2) TWO SLITS
3)

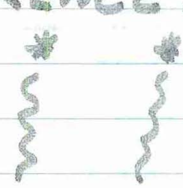


4) MIKLESON THINGIE

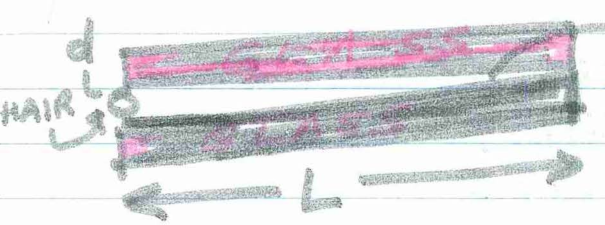


BY CHANGING M', ONE MAY DETERMINE ALL SORTS OF NEAT THINGS

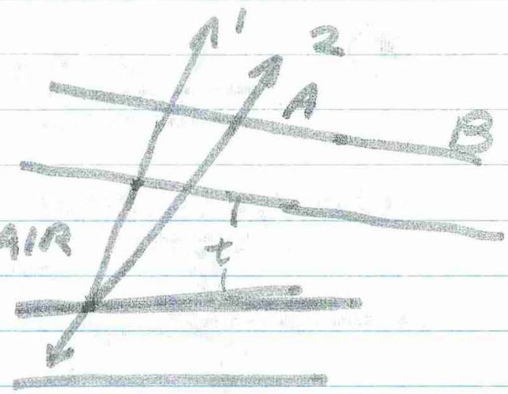
COHERENCE:



TWO ARE COHERENT IF SQUIGGLES ARE ALWAY THE SAME



$\frac{d}{L} = \theta$



AT A

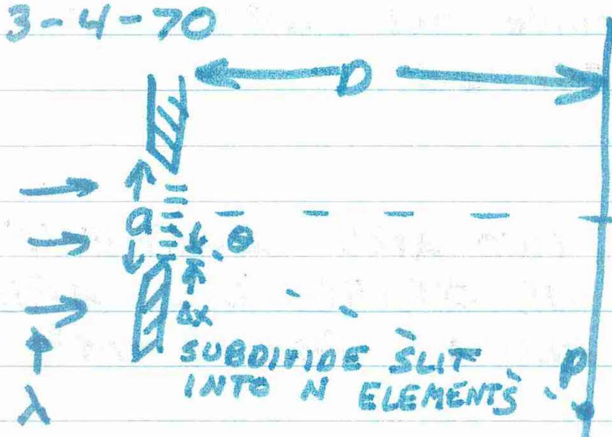
$(m_A + \frac{1}{2})\lambda = 2t_a$

AT B

$(m_B + \frac{1}{2})\lambda = 2t_B$

$m_B - m_a \Rightarrow \lambda = 2\Delta t$

3-4-70



$D \gg a$

INTENSITY?

FIND I AS F(θ) OR F(θ, λ, a)

PHASE DIFFERENCE AT P IS $\Delta\phi$: $\frac{\Delta\phi}{2\pi} = \frac{\text{PATH DIFFER}}{\lambda}$

$= \frac{\Delta x \sin\theta}{\lambda}$

1) CASE $\theta = 0$ $(\Delta\phi = 0)$

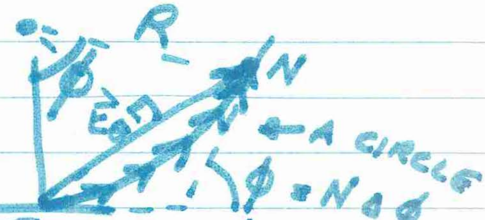
WAVELETS ARIVE AT P IN PHASE

$E_{\theta=0}$ = TOTAL AMPLITUDE = $N\Delta E$

WHERE ΔE = ELECTRIC FIELD AMP. AT EACH ELEMENT

ADD "PHASARS" (ROTATING VECTOR)

2) FOR $\theta \neq 0$, PHASE DIFFERS BY $\Delta\phi \neq 0$



E_{MAX} = ARC LENGTH; $\phi = \frac{E_{MAX}}{R}$ = [illegible]

$$\sin\frac{\phi}{2} = \frac{\frac{1}{2}E_\theta}{R}$$

$$E_\theta = 2R\sin\frac{\phi}{2}$$
$$= \frac{E_{MAX}}{\phi/2}\sin\frac{\phi}{2}$$

$$N\Delta E = E_{MAX}$$

$$E_\theta = E_{MAX}\frac{\sin\phi/2}{\phi/2}$$

$$I \propto E_\theta$$

$$I_\theta = I_{MAX}\frac{\sin^2\frac{\phi}{2}}{\phi/2}$$

3-6-70

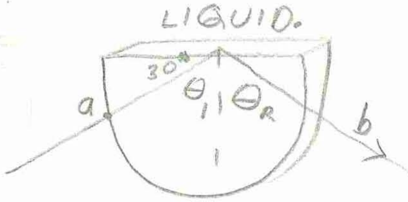
3-11-70

INTERFERENCE < WAVES / PARTICLES

DIFFRACTION "GRATING < 1 DIM. / 3D

BOB MARKS

5/15



$n=1.5$ (solid to vacuum)

n FOR THE LIQUID IS LESS THAN 1.5 ✓

$$\frac{\sin\theta_R}{\sin 60^\circ} = \frac{1}{1.5}(.866) = .575$$

$\theta_R = 35°$ 60° Law of reflect

$$\frac{\frac{n_{SOLID}}{n_{VAC}}}{\frac{n_{LIQ}}{n_{VAC}}} = \frac{n_{SOLID}}{n_{LIQ}} = \frac{\sin 60^\circ}{\sin 35^\circ} = \frac{.866}{.577}$$

$= 1.5$

$m_s = 1$ GRAM

$T = 30$ NTS

$A = .5$

$B = 1.0$

(.001)

$\mu = \frac{.001}{.5}$

$T = 30$

$l = .5$

$f =$

$C = \left(D^2 - B^2/4\right)$ Nt

$\left(C^2 + B^2/4\right) = D^2$

I) MAGNETIC PROPERTIES OF MATTER - 37

A) POLES AND DIPOLES

1) SOME DIPOLE EQUATIONS

a) TORQUE IN AN EXTERNAL FIELD

1) ELEC - $\tau = p \times E$

2) MAG - $\tau = \mu \times E$

b) ENERGY IN AN EXTERNAL FIELD

1) ELEC - $U = -p \cdot E$

2) MAG - $U = -\mu \cdot E$

c) FIELD AT DISTANT POINTS ALONG THE AXIS

1) ELEC - $E = \frac{p}{2\pi \epsilon_0 x^3}$

2) MAG - $B = \frac{\mu_0 \mu}{2\pi x^3}$

d) FIELD AT DISTANT POINTS ALONG ⊥ BISECT.

1) ELEC - $E = \frac{p}{4\pi \epsilon_0 x^3}$

2) MAG - $B = \frac{\mu \mu_0}{4\pi x^3}$

B) ELECTRON "SPIN" 1) ANGULAR MOMENTUM

a) $L_s = .52723 \times 10^{-34}$ JOULE-SEC

2) $\mu = N i A$

a) $i =$ EQUILILENT CURRENT IN LOOP

b) $N =$ UNITY FOR EACH LOOP

c) $A =$ AREA OF LOOP

3) ELECRON ORBITING A NUCLEUS

a) $V = \frac{q_e}{2\sqrt{\pi \epsilon_0 m r^3}}$

b) $\mu_e = L_s q_e / 2m_e$

B) GAUSS' LAW FOR MAGNETISM

1) $\Phi_B = \oint B \cdot dS = 0$

2) CONTRAST WITH $q = \varepsilon_0 \oint E \cdot dS$

C) PARAMAGNETISM - SLIGHT ATTRACTION

1) $U_T = \frac{3}{2} kT$ - MEAN K.E. OF TRANSLATION (k = BOLTZMAN'S CONSTANT)

2) $U_B = 2\mu B$ - MAGNETIC ENERGY

3) MAGNETIC FORCE

$$F_m = \mu \left(\frac{dB}{dx}\right)_{MAX}$$

4) MAGNETIZATION

a) $\vec{M} = \vec{\mu}/V$ (V = VOLUME)

b) $M = C\frac{B}{T}$ (CURIE'S LAW) C = CONSTANT

c) $M_{MAX} = \mu N/V$

E) DIAMAGNETISM - PRESENT IN ALL SUBSTANCES - SLIGHT REPULTION

1) $F_e = ma = m\omega_0^2 r$ (e ABOUT A NUCLEUS)

THE $\Sigma F_e \cong 0$

2) ACTED ON BY B, ω_0 CHANGES

a) $F_B = evB = e\omega rB$

b) $\omega = \omega_0 \mp eB/2m$

c) $\Delta\omega \Rightarrow \Delta\mu \Rightarrow$ MAGNETIC DIPOLES NO LONGER CANCEL

F) FERROMAGNETISM (Fe, Co, Ni, Gd, Dy) - STRONG ATTRAC.

1) CURIE TEMPERATURE - TEMP AT WHICH FERROMAGNETS BECOME PARAMAGNETS

2) THE ROWLAND RING

b) WITHOUT IRON INSIDE:

$B_0 = \mu_0 n i$ ($n = \frac{\#\text{ TURNS}}{\text{UNIT LENGTH}}$) ($d \ll r$)

c) WITH IRON CORE:

1) $B = B_0 + B_M$

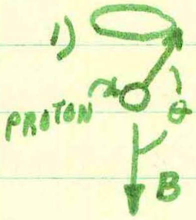
2) B_M = MAGNETIC INDUCTION DUE TO SPECIMIN

3) $B_m \propto M$

d) DOMAINS - LOCAL REGIONS IN WHICH THERE IS PERFECT ALIGNMENTS

e) HYSTERESIS - NOT RETRACABLE, SUCH AS ROWLAND RING

G) NUCLEAR MAGNETISM

1) 

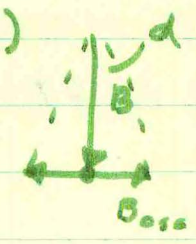
a) $\tau_p = \mu B \sin\theta$

b) $\omega_p = \frac{\mu B}{L_p}$

1) $\mu_p = 1.4 \times 10^{-26}$ AMP·M²

2) $L_p = .53 \times 10^{-34}$ J-S

~~c) $\therefore \nu_p = \frac{\omega_p}{2\pi} = 2.1 \times 10^{7}$ CPS~~

2) 

WITH FIELD $B_{osc} \perp B$

α IS VERY SMALL

a) $\omega_0 = \frac{\mu B}{L_p}$

b) $\therefore \mu_p = 1.410 \times 10^{-26}$ AMP·M²

H) THREE MAGNETIC VECTORS

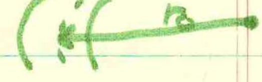
1) $\vec{M}$ - MAGNETIZATION - $d\mu = M\,dl$

a) $(A\,dl)$ = VOLUME OF SLICE

b) DEFINED AS THE MAGNETIC MOMENT PER UNIT VOLUME OF THE CORE MATERIAL

2) $\vec{B}$ - MAGNETIC INDUCTION

a) IN ROWLAND RING, WITH NO CORE; $\oint B \cdot dl = \mu_0 i$

b) OR: $B(2\pi r_0) = \mu_0 N_0 i_0$
WHERE r_0 = MEAN RADIUS 

c) ∴ AMPÈRE'S LAW (a) IS NOT VALID WHEN MAGNETIC MATERIALS ARE PRESENT. THEREFORE, MUST BE MORE CURRENT:

$$\oint B \cdot dl = \mu_0 (i + i_m)$$

OR $B(2\pi r_0) = \mu_0 (N_0 i_0) + \mu_0 (N_0 i_{m0})$

d) FOR ROWLAND RING:

$$\oint \left(\frac{B - \mu_0 M}{\mu_0}\right) \cdot dl = i$$

3) H - MAGNETIC FIELD STRENGTH = $\frac{B - \mu_0 M}{\mu_0}$

FOR R.R $H = n i_0$ ← a) AMPERE'S LAW BECOMES $\oint H \cdot dl = i$
(i IS TRUE CURRENT, NOT MAGNETIZING CURR.)

4) RELATIONS BETWEEN THESE VECTORS:

a) $B = K_m \mu_0 H$

1) (K_m = PERMEABILITY CONSTANT)

2) FOR PARA & DIAMAGNETIC MATERIALS

b) $M = (K_m - 1) H$ → (IN VACUUM, $k_m = 1$) → $B = \mu_0 H$
→ $M = 0$

c) K_m

1) PARAMAGNETIC > 1

2) DIAMAGNETIC < 1

---

5) TABLE (SUMMARY)

a) 1) MAGNETIC INDUCTION - B - ALL CURRENTS - HAS BOUNDARY
2) " FIELD STR. H - TRUE " - TAG. COMP. CONT.
3) MAGNETIZATION - M - MAG CURR. — = 0 IN VACUUM
4)

b) 1) DEF. OF B: $F = q v \times B = i l \times B$

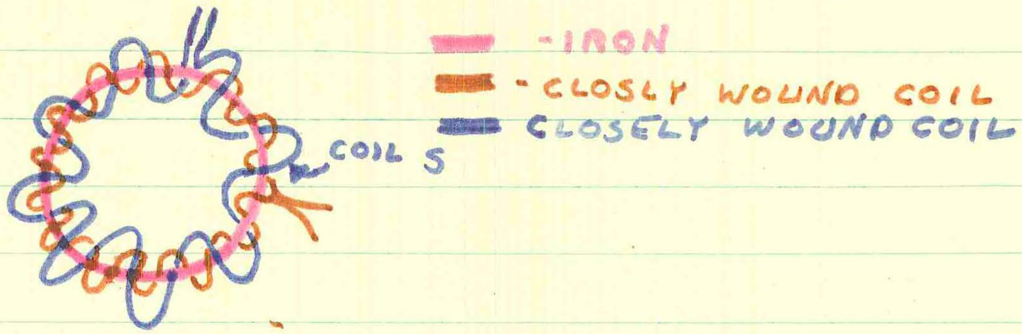
2) $B = \mu_0 H + \mu_0 M$

3) AMPERE'S LAW - $\oint H \cdot dl = i$ (TRUE CURRENT)

4) $B = K_m \mu_0 H$

5) $M = (K_m - 1) H$

I) MORE ON ROWLAND RING



A) WITH NO IRON, $B_0 = \mu_0 n i$ WHERE
n = # TURNS/UNIT LENGTH

B) WITH IRON INSIDE:

1) $B = B_0 + B_M$

2) B_M = MAGNETIC INDUCTION DUE TO IRON ($B \propto M$)

c) $\mathcal{E}_s = N_s \frac{\Delta \Phi_B}{\Delta t} = \frac{NBA}{\Delta t}$

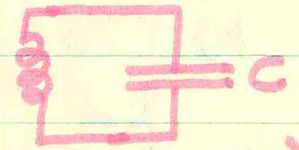
d) 1) $i_s = R_s^{-1} \mathcal{E}_s = \frac{NBA}{R \Delta t}$

2) OR $B = \frac{qR}{NA}$

II) ELECTROMAGNETIC OSCILLATIONS

A) LC OSCLLATIONS - SEE FIG. 38-1, pg 944

1) MUCH LIKE A MASS-STRING SYSTEM (K.E + U = CONSTANT)



2) ENERGY IN EACH

a) CAPAC - $U_E = q_m^2/2C$

b) INDUCTOR - $U_L = \frac{1}{2} L i^2$

3) POTENTIAL DIFFERENCE IN EACH

a) ACROSS CAPACITOR:

$V_C = q/C$

b) TO MEASURE i IN SYSTEM:

1) PUT IN SMALL RESISTANCE R, WHICH DOESN'T AFFECT SYSTEM

2) $V_R = Ri$

B) ANALOGY TO SIMPLE HARMONIC MOTION

1) a) SPRING (MECH) — ELECTROMAGNETIC

SPRING . . . $U_p = \frac{1}{2}kx^2$ - CAPAC. . . $U_E = q^2/2C$

MASS . . . $U_K = \frac{1}{2}mv^2$ - INDUCT. . . $U_B = \frac{1}{2}Li^2$

$v = dx/dt$ ——— $i = dq/dt$

b) q CORRESPONDS TO x

i " " v

C " " 1/k

L " " M

2) a) IN MECHANICAL SYSTEM:

$\omega = 2\pi v = \sqrt{k/m}$

b) ∴ IN ELECTROMAGNETIC:

$\omega = 2\pi v = \sqrt{1/LC}$

C) ELECTROMAGNETIC OSCILLATIONS - QUANITATIVE

1) DERIVATION OF FREQUENCY FUNCTION

a) $U = U_B + U_E = \frac{1}{2}Li^2 + q^2/2C$

b) $dU/dt = (d/dt)(\frac{1}{2}Li^2 + q^2/2C) =$
$Li\, di/dt + q/C\, dq/dt = 0$
(ASSUMING $R=0 \Rightarrow U =$ KONSTANT

c) $i = dq/dt \rightarrow di/dt = d^2q/dt^2$

d) $L\, d^2q/dt^2 + \frac{1}{C}q = 0$
(ANALAGOUS TO $m\, d^2x/dt^2 + kx = 0$
FOR MASS-SPRING SYSTEM)

e) SOLUTION OF DIFFERENTIAL EQUATION YIELDS:
$q = q_{MAX}\cos(\omega t + \phi)$

1) ω = ANGULAR FREQUENCY

2) ϕ = ARBITRARY PHASE ANGLE

2) FORMULAS FROM DERIVATION:

a) $dq/dt = i = -q_{MAX}\,\omega \sin(\omega t + \phi)$

b) $d^2q/dt^2 = di/dt = -q_{MAX}\,\omega^2 \cos(\omega t + \phi)$

c) SUBSTITUTION OF ORIGINAL DIFFERINTIAL EQUATION YIELDS:
$\omega = \sqrt{1/LC}$

d) $t = \frac{\pi}{4\omega}$

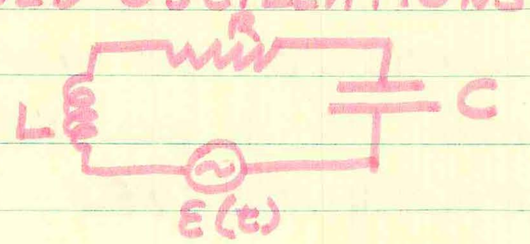
e) $U_e = \frac{1q^2}{2C} = \frac{q_{MAX}^2}{2C}\cos^2(\omega t + \phi)$

f) $U_b = \frac{1}{2}Li^2 = \frac{1}{2}L\omega^2 q_{max}^2 \sin^2(\omega t + \phi)$

3) FOR LRC CIRCUIT
$q = q_m e^{-Rt/2L}\cos \omega' t$
(INITITALLY $q = q_{MAX}$)

D) FORCED OSCILLATIONS AND RESONANCE

1)



$\mathcal{E} = \mathcal{E}_M \cos \omega' t$

2) MECHAN. VS ELECTRO. → $\mathcal{E}$ vs F

3) $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = \mathcal{E}_{MAX} \cos \omega'' t$

(ω'' = ANGULAR FREQ)

4) $q = \frac{\mathcal{E}_m}{G} \sin (\omega'' t - \phi)$

a) $(G = [(\omega'' L - 1/C)^2 + R^2 \omega''^2]^{\frac{1}{2}}$

b) $\phi = \cos^{-1}(R\omega''/G)$

5) $i = dq/dt = i_m \cos(\omega'' t - \phi)$

$(i_m = \omega'' \mathcal{E}_m / G)$

6) RESONANCE $(\omega'' = \omega)$

$i_m = \mathcal{E}_m / R$

E) LUMPED AND DISTRIBUTED ELEMENTS

1) EXAMPLES

a) LUMPED-MASS STRING SYSTEM.

1) K.E. ALL IN MASS

2) U ALL IN SPRING

b) DISTRIBUTED - IN ORGAN PIPE (VARIATIONS IN AIR DENSITY)

2) IN A SMALL AREA OF GAS:

$U_K = \frac{1}{2} \rho_0 V_g^2$

a) U_K = POTENTIAL ENERGY PER VOLUME

b) V_g = VELOCITY OF GAS

F) ELECTROMAGNETIC CAVITY OSCILLATOR

A) DISTRIBUTED ELECTROMAGNETIC OSCILLATOR, OPPOSED TO AN LC CIRCUIT, WHICH IS A LUMPED SYSTEM.

B) RESONANT FREQUENCYES ARE MUCH LARGER

C) ENERGIES IN E.C.O.

1) $U_E = \frac{1}{2} \epsilon_0 E^2$
(U_E = ENERGY PER UNIT VOLUME IN THE ELECTRIC FIELD)

2) $U_B = \frac{1}{2\mu_0} B^2$
(U_B = ENERGY PER UNIT VOLUME IN THE MAGNETIC FIELD)

3) $U_B + U_E$ = KONSANT ⇒ (ENERY OSCILLATES BETWEEN THE ELECTRIC AND MAGNETIC FIELDS)

4) E & B FIELDS NO LONGER OCCUPY SEPARATE REGIONS IN SPACE.

D) ω (= ANGULAR FREQUENCY) = $\frac{1.19c}{a}$

1) C = SPEED OF LIGHT

2) a = CAVITY RADIUS

E) $c = 1/\sqrt{\mu_0 \epsilon_0}$

F) CONDITION - E=0 ON CAVITY WALL AND HAVE NO TANGENTAL COMPONENTS ANYWHERE ON THE CAVITY WALL

1) ∴ NO $B(t)$ CAN EXIST INSIDE CAVITY WALL

2) ∴ NO $i(t)$ " " " " "

3) TANGENTAL COMP. OF B CAN EXIST ON SURFACE → $q(t)$ AND $i(t)$

G) ANALOGOUS PLUG AND CHUG TABLE

1) LUMPED SYSTEMS:

MECHANICAL SYST. (MASS & STRING)	ELECTROMAGNETIC SYST. (LC CIRCUIT)
$U_K = \frac{1}{2} m v^2$	$U_B = \frac{1}{2} L i^2$
$U_P = \frac{1}{2} k x^2$	$U_E = \frac{1}{2}(1/C) q^2$
$\omega = \sqrt{k/m}$	$\omega = \sqrt{1/LC}$

2) DISTRIBUTED SYSTEMS

MECHANICAL SYST. (ACOUSTIC CAVITY)	ELECTROMAGNETIC SYST. (ELECTROMAGNETIC CAVITY)
$U_K = \frac{1}{2} \rho_0 v_g^2$	$U_B = \frac{1}{2\mu_0} B^2$
$U_P = \frac{1}{2} B (\Delta\rho/\rho_0)^2$	$U_E = \frac{1}{2} \epsilon_0 E^2$
$\omega_1 = \frac{3.14\, v}{a}$	$\omega_1 = \frac{1.19\, c}{a}$
$v = \sqrt{B/\rho_0}$	$c = \sqrt{1/\varepsilon_0 \mu_0}$

H) INDUCED MAGNETIC FIELDS

1) A CHANGING E FIELD PRODUCES A B FIELD

2) a) $\oint E \cdot dl = -d\Phi_B/dt$

b) $\oint B \cdot dl = \mu_0 \varepsilon_0 \, d\Phi_E/dt$

3) AMPERE'S LAW: $\oint B \cdot dl = \mu_0 i$

a) $\therefore$ ENTIRE $\oint B \cdot dl = \mu_0 \varepsilon_0 \, d\Phi_E/dt + \mu_0 i$

b) OR $\oint B \cdot dl = \mu_0 (i_d + i)$

I) DISPLACEMENT CURRENT (i_d)

1) $i_d = \varepsilon_0 \, d\Phi_E/dt$

2) ACROSS CAPACITOR

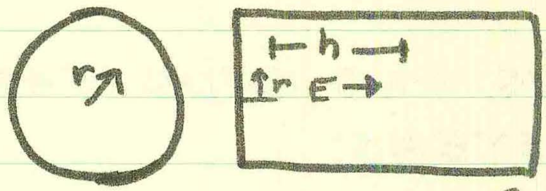
a) $E = q/\varepsilon_0 A \Rightarrow \frac{dE}{dt} = \frac{1}{\varepsilon_0 A}\frac{dq}{dt} = \frac{1}{\varepsilon_0 A} i$

b) $i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(EA) = \varepsilon_0 A \frac{dE}{dt}$

c) $i_d = \varepsilon_0 A \left(\frac{1}{\varepsilon_0 A} i\right) = i$

J) MAXWELL'S EQUATIONS & CAVITY OSCILLATIONS

1)



a) FOR SYSTEM: $\oint E \cdot d\ell = hE(r)$
WHERE $E(r)$ IS THE VALUE OF E AT r.

b) $E=0$ ON CAVITY WALLS ($E \perp d\ell$)

c) $E(r) = -\frac{1}{h} \, d\Phi_B/dt$

d) E IS MAX WHEN $B=0$

e) $B(r) = \frac{\mu_0 \epsilon_0}{2\pi r} \, d\Phi_E/dt = \frac{\mu_0}{2\pi r} i_d$

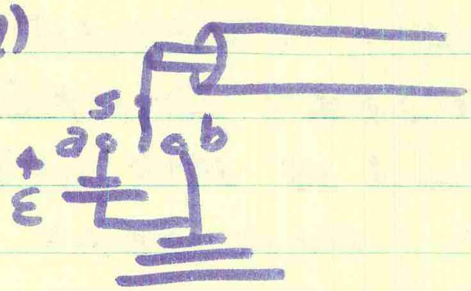
2) THIS B-E INTEPLAY OCCURS IN TRAVELING ELECTROMAGNETIC WAVES, SUCH AS RADIO & VISIBLE LIGHT RAYS

3) $B=0$ OUTSIDE CAVITY

III) ELECTROMAGNETIC WAVES

A) COAXIAL CABLE (TRANSMISSION LINE)

a)

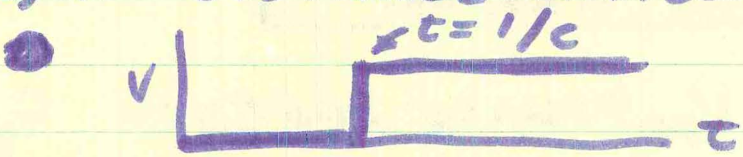


b) WHEN S IS ON b, THE CENTRAL & OUTER SURFACE HAVE SAME V

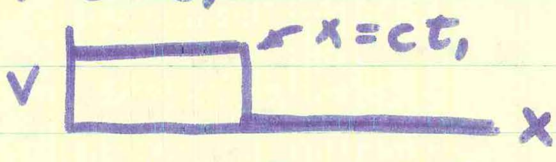
c) WHEN S IS ON a, THERE IS A POTENTIAL BETWEEN CORE & SURFACE

d) THIS POTENTIAL TRAVELS ALONG LINE AT c

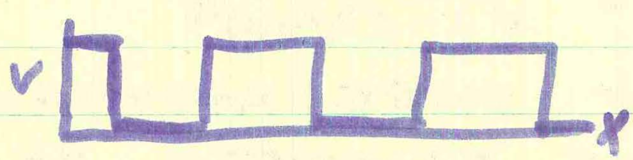
1) AT DISTANCE l ALONG LINE



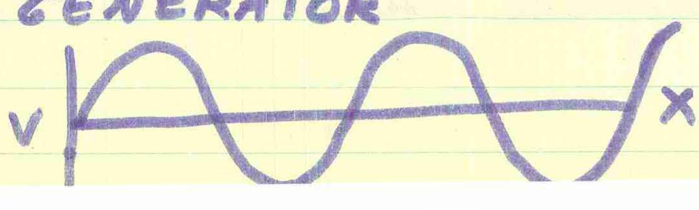
2) AT $t = t_1$



e) IF S ALTERNATES BETWEEN a AND b



f) S, a, b REPLACED BY SINOSOIDAL GENERATOR



B) A TRAVELING WAVE IN RESISTIVELESS TRANSMISION LINE: $\lambda = c/\nu$

a) COMMERCIAL; ν = 60 CYCLES / SEC

b) λ (COMMERCIAL) $\cong$ 3000 MILES

c) HAS NO RESONATE FREQUENCY

C) COAXIAL CABLE - FIELDS & CURRENTS

1) $i_d = \varepsilon_0\, d\Phi_E/dt \Rightarrow i_d$ IS MAX WHEN E=0

2) $B = \mu_0 i / 2\pi r$ FOR COAX. CABLE

3) RELATIVE MAGNETUDES

a) E & B ARE IN PHASE IN A TRAVELING WAVE - REACH MAX. AT SAME POINT

b) E & B ARE 90° OUT OF PHASE IN A STANDING WAVE

D) WAVEGUIDE

1) 

a) V_{PH} (= PHASE SPEED) = $c / \sqrt{1-(\lambda/2a)^2}$

b) V_{gr} (= GROUP SPEED) = $c \sqrt{1-(\lambda/2a)^2}$

c) AS $a \to \infty$, $V_{PH} = V_{gr} = c$

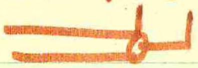
2) GUIDE WAVELENGTH (= λ_g)

a) $\lambda_g = V_{PH}/\nu = V_{PH}/c/\lambda = \lambda \cdot V_{PH}/c$

b) $\lambda_g = \dfrac{\lambda}{\sqrt{1-(\lambda/2a)^2}}$

c) THE GUIDE ~~PATTERN~~ λ IS THAT EXIBEDED BY CERTAIN FIELD PATTERNS. IT IS LARGER THAN FREE-SPACE λ

E) RADIATION

1)  - ELECTRIC DIPOLE, WHOSE V, AND THUS p, VARY SINOSOIDALLY WITH TIME

a) ELECTRIC LINES OF FORCE BREAK OFF IN CLOSED LOOPS, AND GO INTO SPACE c.

b) THESE ARE ELECTROMAGNETIC WAVES, RADIATION

2) FACTS & JUNK

a) $c = \nu\lambda \Rightarrow c = \omega/k$ (k = WAVE #)

b) $\omega = 2\pi\nu$; $k = 2\pi/\lambda$

F) TRAVELING WAVES & MAXWELL'S EQUATIONS

1) IN AN ELECTROMAGNETIC WAVE

a) $B = B_m \sin(kx - \omega t)$

b) $E = E_m \sin(kx - \omega t)$

2) $\frac{dE}{dx} = -\frac{dB}{dt}$

3) $\omega/k = E_m/B_m = c \Rightarrow E = cB$

4) $-\frac{\partial B}{\partial x} = \mu_0 \varepsilon_0 \, \partial E/\partial t$

5) $E_m/B_m = k/\mu_0 \varepsilon_0 \omega = 1/\mu_0 \varepsilon_0 c$

G) THE POYNTING VECTOR (S)

1) $\vec{S} = \frac{1}{\mu_0} E \times B$ (MKS S — $\frac{\text{WATT}}{m^2}$)

2) ENERY MOVES IN DIRECTION OF $\vec{S}$

3) dU/dx IN A BOX ($(dx)A = d\,$VOLUME)

a) $dU = \left[\frac{1}{2}\varepsilon_0 E cB + \frac{1}{2\mu_0}\frac{BE}{c}\right] A\,dx$

b) $= \left[\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2\right] A\,dx$

c) $= EBA/\mu_0 c$

4) $S = \frac{dU}{dt\,A} = \frac{EBA\,dx}{\mu_0 (dx/c) c A} = \frac{1}{\mu_0} EB$

5) $\bar{S}$ (AVERAGE) $= \frac{1}{2\mu_0} E_M B_m$

IV) THE DIFFERENTIAL FORM OF MAXWELL'S EQUATIONS & THE ELECTROMAGNETIC WAVE EQUATION (Pg 1215)

A) MAXWELL'S EQUATIONS

1) $\varepsilon_0 \oint E \cdot dS = q$

2) $\oint B \cdot dS = 0$

3) $\oint B \cdot dl = \mu_0 \left(i + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$

4) $\oint E \cdot dl = -d\Phi_B / dt$

5) E AND B ARE USUALLY UNKNOWN, AND THUS IT IS HARD TO COMPUTE THESE EQUATIONS IN GENERAL CASES

a) $m = \int \rho \, d\tau$

1) m = MASS

2) ρ = DENSITY OF BODY

3) τ = VOLUME

b) IF ρ = CONSTANT, THEN $\rho = \frac{m}{\tau}$

c) IT IS DESIREABLE TO CONVERT MAXWELL'S EQUATIONS TO DIFFERENTIAL FORM

B) THE OPERATOR DEL (∇)

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

1) USING ELECTROSTATIC FIELD E

a) $E = iE_x + jE_y + kE_z = -\left(i\frac{\partial V}{\partial x} + j\frac{\partial V}{\partial y} + k\frac{\partial V}{\partial z} \right)$

b) $E = -\nabla V$

2) APPLICATIONS OF THE DEL OPERATOR

a) ON A SCALAR FIELD ψ

1) $\nabla \psi$ (= VECTOR FIELD)

2) CALLED GRADIENT OF ψ

b) ON A VECTOR FIELD – $U = U_x i + U_y j + U_z k$

1) DOT PRODUCT: $\vec{\nabla} \cdot \vec{U}$ CALLED DIVERGENCE U

2) CROSS PRODUCT: $\vec{\nabla} \times \vec{U}$ CALLED CURL U

c) EXAMPLES OF ABOVE

1) GRAD $\psi = \nabla \psi = i \frac{\partial \psi}{\partial x} + j \frac{\partial \psi}{\partial y} + k \frac{\partial \psi}{\partial z}$

2) DIV $U = \nabla \cdot U = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}$

3) CURL $U = \nabla \times U = i\left(\frac{\partial U_z}{\partial y} - \frac{\partial U_y}{\partial z}\right) + ~~j\left(\frac{\partial U}{\partial y} - \frac{\partial U}{\partial z}\right)~~$

$+ j\left(\frac{\partial U_x}{\partial z} - \frac{\partial U_z}{\partial x}\right) + k\left(\frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}\right)$

d) GRAD & CURL ARE VECTORS; DIV IS SCAL.

e) $\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

c) MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM-I

1) a) $\oint E \cdot ds = dx\,dy\,dz\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right)$

b) $\oint E \cdot ds = dx\,dy\,dz$ DIV E

c) $q = \rho\, dx\,dy\,dz$

→ d) ε_0 DIV $E = \rho$ ←

2) BY ANALOGY: → DIV $B = 0$ ←

D) MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM-II

1) a) $\oint B \cdot dl = dx\,dy\left(\frac{\partial B}{\partial x} \cdot \hat{j} - \frac{\partial B}{\partial y} i\right)$ -

$= dx\,dy\left[\frac{\partial}{\partial x}(B \cdot j) - \frac{\partial}{\partial y}(B \cdot i)\right)$

$= dx\,dy\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right.$

b) ABOVE FROM:

$\oint B \cdot dl = B \cdot (-j\,dy) + B \cdot (i\,dx) + \left(B + \frac{\partial B}{\partial x} dx\right) \cdot j\,dy + \left(B + \frac{\partial B}{\partial y} dy\right) \cdot (-i\,dx)$

c) 1) $i = J \cdot dS = J \cdot (k\,dx\,dy) = dx\,dy\, J_z$

2) $\frac{d\Phi_E}{dt} = \frac{\partial E}{\partial t} \cdot dS = \frac{\partial E}{\partial t} \cdot (k\,dx\,dy)$

3) OR $\frac{d\Phi_E}{dt} = \frac{\partial E_z}{\partial t} dx\,dy$

4) COMBINING & CANCELING YIELDS:

$$\frac{1}{\mu_0}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = J_z + \varepsilon_0 \frac{\partial E_z}{\partial t}$$

→d) CURL $B = \mu_0 (J + \varepsilon_0 \frac{\partial E}{\partial t})$ ←

→e) CURL $E = -\partial B/\partial t$ ←

E) THE WAVE EQUATION

1) MAXWELL'S EQ. IN DIFF. FORM

a) ε_0 DIV $E = \rho$

b) DIV $B = 0$

c) CURL $B = \mu_0 (J + \varepsilon_0 \frac{\partial E}{\partial t})$

d) CURL $E = -\partial B/\partial t$

2) IN FREE SPACE, $\rho = J = 0$

MAXWELL'S EQUATIONS BECOME:

a) DIV $E = 0$

b) DIV $B = 0$

c) CURL $E = -\partial B/\partial t$

d) CURL $B = \mu_0 \varepsilon_0 \, \partial E/\partial t$

3) DERIVATION

a) CURL CURL $E = -$CURL $\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}$ CURL B

b) CURL $B = \mu_0 \varepsilon_0 (\partial E/\partial t)$

c) ∴ CURL CURL $E = -\mu_0 \varepsilon_0 (\partial^2 E/\partial t^2)$

d) CURL $E \equiv -\nabla^2 E +$ GRAD DIV E

e) DIV $E = 0 \Rightarrow$ GRAD DIV $E = 0$

f) CURL CURL $E = \nabla^2 E$

g) ∴ $\nabla^2 E = \mu_0 \varepsilon_0 (\partial^2 E/\partial t^2)$

h) BY ANALOGY: $\nabla^2 B = \mu_0 \varepsilon_0 (\partial^2 B/\partial t^2)$

i) ABOVE TWO VECTORIAL DIFFERENTIAL EQUATIONS HAVE SIX SCALAR COMPONENT EQUATIONS. FOR SIMPLIFICATION:

$E_x = E_z = 0$ AND $B_x = B_y = 0$

j) FOR NON-VANISHING COMPONENTS E_Y & E_Z

1) $\frac{\partial^2 E_Y}{\partial X^2} + \frac{\partial^2 E_Y}{\partial Y^2} + \frac{\partial^2 E_Y}{\partial Z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_Y}{\partial t^2}$

2) $\frac{\partial^2 B_Z}{\partial X^2} + \frac{\partial^2 B_Z}{\partial Y^2} + \frac{\partial^2 B_Z}{\partial Z^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_Z}{\partial t^2}$

k) FOR FURTHER SIMPLIFICATION, LET E_Y AND B_Z BE FUNCTIONS OF ONLY x & t

l) $\therefore$ 1) $\frac{\partial^2 E_Y}{\partial X^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_Y}{\partial t^2}$

2) $\frac{\partial^2 B_Z}{\partial X^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_Z}{\partial t^2}$

m) SOLUTIONS TO ABOVE DIFFERENTIALS:

1) $E_Y = E_m \sin(kx - \omega t)$

2) $B_Z = B_M \sin(kx - \omega t)$

n) INTERPRET ABOVE AS ELECTROMAGNETIC WAVE TRAVELING IN +X DIRECTION WITH SPEED $c = \omega / k$

o) SUBSITUTION & CANCELATION YIELDS:

$c = \omega / k = 1 / \sqrt{\mu_0 \varepsilon_0}$

V) WAVES IN ELASTIC MEDIA

A) TYPES OF WAVES (MECHANICAL)

1) a) TRANSVERSE - MOTION OF SYSTEM IS $\perp$ TO DIRECTION OF WAVES

   b) TUDINAL - MOTION OF SYSTEM IS BACK & FORTH (SPRING)

2) DIMENSIONALLY
   a) 1 DIMEN. - (SPRING)
   b) 2 DIMEN. - (WATER)
   c) 3 DIMEN - (SOUND & LIGHT) PROPOGATE IN WAVE FRONTS

3) BY SHAPE (3D)
   a) PLANE WAVES - |||||
   b) ~~RA~~SPHERICAL WAVES -

B) EQATIONS & JUNK

1) $Y = Y_{MAX} \sin \frac{2\pi}{\lambda} X$
   AT A GIVEN TIME

2) AT TIME t
   $Y = Y_{MAX} \sin \frac{2\pi}{\lambda}(x - vt)$

3) T = PERIOD (= TIME FOR WAVE TO TRAVEL ONE λ) = λ / v

4) $\therefore\ Y = Y_{MAX} \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right)$

5) K (= WAVE #) = $2\pi / \lambda$

6) ω (= ANGULAR FREQUENCY) = $\frac{2\pi}{T}$

7) $Y = Y_{MAX} \sin (kx - \omega t)$
   ~~8)~~ FOR A WAVE TRAVELING TO RIGHT

8) $Y = Y_{MAX} \sin (kx + \omega t)$
   FOR A WAVE TRAVELING TO LEFT

9) V (PHASE VELOCITY) = $\frac{\lambda}{T} = \frac{\omega}{k}$

10) AT $t = 0$, $\Phi \neq 0$ -

$Y = Y_m \sin(kx - \omega t - \phi)$ FOR WAVE TRAVELING TO THE RIGHT

11) AT A FIXED POINT, SUCH AS $x = \frac{\pi}{k}$

$Y = Y_m \sin(\omega t + \phi)$

C) THE SUPERPOSITION PRINCIPLE -

1) DEFN. - TWO OR MORE WAVES CAN TRANSVERSE THE SAME SPACE INDEPENDENTLY OF ONE ANOTHER

2) ANY PERIODIC FUNCTION MAY BE EXPRESSED BY THE FOURIER SERIES:

$$Y(t) = A_0 + A_1 \sin \omega t + A_2 \sin 2\omega t \ldots + B_1 \cos \omega t + B_2 \cos 2\omega t \ldots$$

a) $\omega = 2\pi/T$

b) A's & B's CONSTANTS ACCORDING TO THE FUNCTION

D) WAVE SPEED - V

1) $V = \sqrt{F/\mu}$ IS A STRETCHED STRING

2) $\lambda = V/f$

a) f = FREQUENCY

b) μ = MASS/UNIT LENGTH

E) POWER & INTENSITY IN WAVE MOTION

1) $P = \left(-F \frac{\partial Y}{\partial x}\right) \frac{\partial Y}{\partial t}$

F = TENTION IN THE STRING

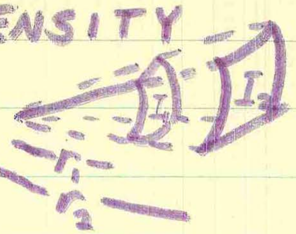
2) FOR A SINE WAVE IN A STRING

a) $P = Y_m^2 k \omega F \cos^2(kx - \omega t)$

b) $\bar{P} = \frac{1}{T} \int_t^{t+T} P\,dt = \frac{1}{2} Y_m^2 k \omega F = 2\pi^2 Y_m^2 \nu^2 \frac{F}{V}$

$= 2\pi^2 Y_m^2 \nu^2 \mu V$

3) INTENSITY



$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

F) WAVE INTERFERENCE - WAVES MAY BE ADDED ALGEBRAICALLY AND GRAPHICALLY.

G) STANDING WAVES

$Y = 2Y_M \sin kx \cos \omega t$

1) NODES OCCUR AT:

a) $kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

b) $x = \frac{\lambda}{4}, 3\lambda/4, \frac{5\lambda}{4} \ldots$

2) ANTINODES:

a) $kx = \pi, 2\pi, 3\pi \ldots$

$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots$

3) ON REFLECTION FROM A FIXED END, A WAVE UNDERGOES A PHASE CHANGE OF 180°

4) ON A FREE END, NO PHASE CHANGE OCCURS

H) RESONANCE

1) IN A FIXED STRING

$\lambda = \frac{2l}{n}$ $n = 1, 2, 3, 4 \ldots$

2) NATURAL FREQUENCIES OF OSCILLATION ARE

$$f = \frac{n}{2l}\sqrt{\frac{F}{\mu}}$$

VI) SOUND WAVES

A) CLASSIFICATIONS

1) ACCORDING TO FREQUENCY

a) BELOW 20 HZ - INFRASONIC

b) 20-20,000 HZ - AUDIBLE

c) ABOVE 20,000 HZ - ULTRASONIC

2) TYPE OF WAVE

a) TRANSVERSE - WAVE MOVES $\perp$ TO BODY

b) LONGITUDINAL - (SOUND) - WAVE MOVES // TO BODY

B) PROPIGATION & SPEED OF LONGITUDINAL WAVES

1) V (= SPEED IN 1 DIMENSIONAL SYSTEM) $= \sqrt{B / \rho_0}$

a) B = (BULK MODULUS OF ELASTICITY) $= -V\Delta P / \Delta V$

b) ρ_0 = DENSITY OF FLUID

2) IT IS POSIBLE TO EXPRESS B ~~AS THE~~ IN TERMS OF INITITIAL PRESSURE P_0

$V = \sqrt{\gamma P_0 / \rho_0}$

(γ = CONSTANT = RATIO OF SPECIFIC HEATS FOR THE GAS)

C) TRAVELING LONGITUDINAL WAVES

1) FOR A WAVE TRAVELING TO RIGHT

a) $Y = Y_m \cos \frac{2\pi}{\lambda} (x - vt)$

b) $Y = Y_M \cos (kx - \omega t)$

c) 1) $p = -B \, dY/dx$ (PRESSURE)

2) $\frac{dY}{dx} = -k Y_M \sin (kx - \omega t)$

3) $\therefore p = B k Y_m \sin (kx - \omega t)$

2) P (= PRESSURE AMPLITUDE) = $k \rho_0 v^2 y_m$

a) ∴ $p = P \sin(kx - \omega t)$

b) DISPLACEMENT IS 90° OUT OF PHASE WITH p

D) VIBRATING SYSTEMS & SOURSES OF SOUND

1) FIXED STRING CAN RESONATE AT FREQUENCIES

$$f_n = \frac{nv}{2l} = \frac{n}{2l}\sqrt{F/\mu}\ ;\quad n = 1, 2, 3 \ldots$$

a) v IS SAME FOR ALL FREQUENCIES

b) $f = \sqrt{\frac{F}{\mu}}\,\frac{1}{2l}$ IS CALLED FUNDIMENTAL FREQUENCY

2) FOR "ORGAN" TYPE PIPE (RES. FREQ.)

$$f_n = \frac{nv}{2l} \qquad n = 1, 2, 3 \ldots$$

a) v = SPEED OF LONGITUDINAL WAVES

b) n = # OF HALF λ'S

c) l = LENGTH OF THE COLOMN

d) EVEN NUMBERED HARMONICS ARE NOT PRESENT WHEN ONE END OF THE PIPE IS CLOSED

E) BEATS

1) OCCURS WHEN 2 SLIGHTLY DIFFERENT f ARE SOUNDED SIMULTANEOUSLY

2) QUANTITATIVELY ($y_{m_1} = y_{m_2}$)

a) $y_1 = y_m \cos 2\pi f_1 t \cdot y_2 = y_m \cos 2\pi f_2 t$

b) $y = 2y_m \cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t \cdot \cos 2\pi\left(\frac{f_1 + f_2}{2}\right)t$

c) AMPLITUDE VARIES WITH TIME:

$$f_{AMP} = (f_1 - f_2)/2$$

d) RESULTING VIBRATION'S FREQUENCY:

$$f = (f_1 + f_2)/2$$

e) BEAT HAS MAX AMP WHEN:

$$\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t = \pm 1$$

F) THE DOPPLER EFFECT

1) OBSERVOR MOVING TOWARD SOURCE AT SPEED V_0

a) AT REST, $\frac{Vt}{\lambda}$ WAVES IN TIME t

b) MOVING, ADDITIONAL $\frac{V_0 t}{\lambda}$ WAVES

c) OR, APPARENT # WAVES/SEC ($= f'$)

1) $f' = \frac{Vt/\lambda + V_0 t/\lambda}{t} = \frac{V + V_0}{\lambda} = \frac{V + V_0}{V/f}$

2) OR $f' = f\,\frac{V + V_0}{V} = f\left(1 + V_0/V\right)$

d) IF OBSERVOR IS MOVING AWAY:

$$f' = f\left(\frac{V - V_0}{V}\right) = f\left(1 - \frac{V_0}{V}\right)$$

e) COMBINING YIELDS:

$$f' = f\left(\frac{V \pm V_0}{V}\right)$$

2) WHEN SOURSE IS MOVING ~~AWAY~~ TOWARD ~~THE~~ OBSERVOR, λ IS SHORTENED, $\therefore f$ TO OBSERVOR INCREASES

a) $f' = V/\lambda' = \frac{V f}{(V - V_S)} = f\left(\frac{V}{V - V_S}\right)$

b) IF SOURCE IS MOVING AWAY:

$$f' = f\left(\frac{V}{(V + V_S)}\right) \rightarrow \text{DECREASED } f$$

c) COMBINING YIELDS:

$$f' = f\left(\frac{V \pm V_0}{V \mp V_S}\right)$$

IF BOTH SOURSE AND OBSERVOR ARE MOVING

VII) NATURE AND PROPAGATION OF LIGHT

A) MEASURMENTS OF WAVELENGTH

1) $1\,\mu = 10^{-6}\,m$
2) $1\,m\mu = 10^{-9}\,m$
3) $1\,A = 10^{-10}\,m$

B) ENERGY AND MOMENTUM OF LIGHT

1) $S = \frac{1}{\mu_0} E \times B$
   a) S = POYNTING VECTOR
   b) B = MAGNETIC FIELD
   c) E = ELECTRIC FIELD

C) MOMENTUM OF LIGHT

1) IF LIGHT IS COMPLETELY ABSORBED
$$p = \frac{U}{c}$$
2) IF LIGHT IS ENTIRELY REFLECTED
$$p = \frac{2U}{c}$$
3) $F = \frac{p}{t}$

D) THE SPEED OF LIGHT

1) $c = 2.997925 \times 10^{8}\,\frac{m}{sec} \pm 3 \times 10^{2}\,\frac{m}{sec}$
2) INSIDE A CAVITY, LIGHT WAVES' FREQUENCY MAY BE MEASURED ANALOGOUS TO SOUND WAVES:
   a) $\lambda_g = \frac{2l}{n}$ $\quad n = 1, 2, 3, 4 \ldots$
   b) TO FIND λ IN FREE SPACE
$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}}$$
   c) $c = \lambda f$ IN FREE SPACE

E) MOVING SOURSES AND OBSERVORS

1) EINSTEIN POSTULATED THAT RELATIVE VELOCITY FORMULA:

$$V = V' + U$$

WASN'T RIGHT, BUT THAT:

$$V = \frac{V' + U}{1 + V'U/C^2}$$ HOLDS

a) V' = SPEED OF WAVE PROPAGATED

b) U = VELOCITY OF OBSERVOR

2) FOR $V' = C$; $V = C$

F) THE DOPPLER EFFECT IN LIGHT

$$f' = f \frac{1 - u/c}{\sqrt{1 - u/c}}$$

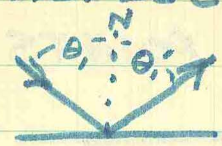
VIII) REFLECTION AND REFRACTION

A) GENERAL LAWS

1) THE REFLECTED & REFRACTED RAYS LIE IN THE PLANE FORMED BY THE INCIDENT RAY.

2) FOR REFLECTION:

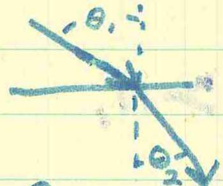
a)



b) $\theta_1 = \theta_1'$

3) FOR REFRACTION:

a)



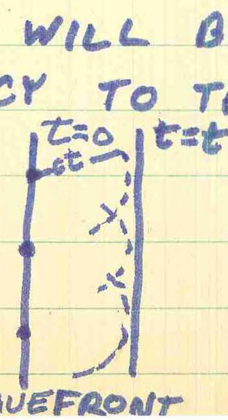
b) $\frac{\sin\theta_1}{\sin\theta_2} = n_{21}$ (INDEX OF REFRACTION)

① IN COMPARISON WITH VACUUM, $n > 1$

② n VARIES SLIGHTLY WITH λ

B) HUYGEN & LIGHT

1) HUYGEN'S PRINCIPLE: ALL POINTS ON A WAVEFRONT CAN BE CONSIRED AS POINT SOURCES FOR THE PRODUCTION OF SPHERICAL SECONDARY WAVELETS. AFTER TIME t THE NEW POSITION OF THE WAVEFRONT WILL BE THE SURFACE OF ~~TH~~ TANGENCY TO THESE SECONDARY WAVELETS



2) HUYGEN & LAW OF REFRACTION:

a) $v_1 \lambda_2 = v_2 \lambda_1$

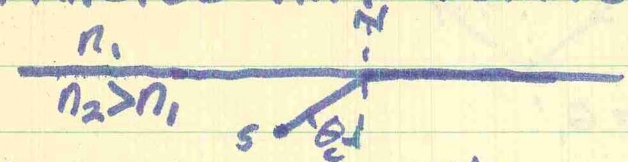
b) $n_{21} = \frac{v_1}{v_2}$

c) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

d) $\lambda_2 = \lambda_1 \frac{v_2}{c} = \frac{\lambda}{n_2}$

C) TOTAL INTERNAL REFLECTION

1) THE CRITICAL ANGLE - IS REACHED WHEN THE REFRACTED RAY POINTS ALONG SURFACE

a) 

θ_c = CRIT. ANGLE

b) IS OBTAINED WHEN:

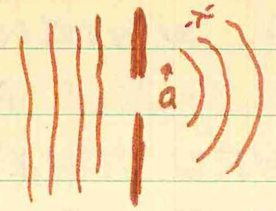
$\sin\theta_c = \frac{n_2}{n_1}$

2) OCCURS ~~WHEN~~ ⇔ $n_2 > n_1$

D) FERMAT'S PRINCIPLE

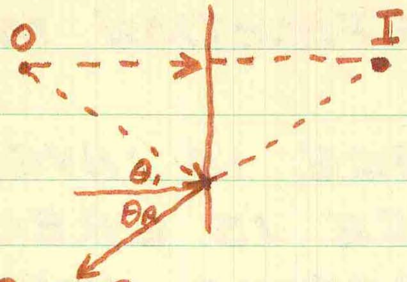
A LIGHT RAY TRAVELING FROM ONE POINT TO ANOTHER WILL CHOOSE A PATH, THAT COMPARED WITH NEARBY PATH'S, IS EITHER A MAXIMUM, A MINIMUM, OR STATIONARY

IX) REFLECTION & REFRACTION; SPHERICAL WAVES AND SPHERICAL SURFACES

A) 

AS $\frac{a}{\lambda} \to 0$, SYSTEM → A POINT SOURSE

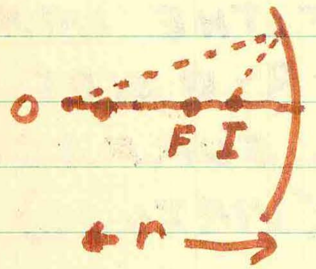
B) SPHERICAL WAVES - PLANE MIRROR

1) 

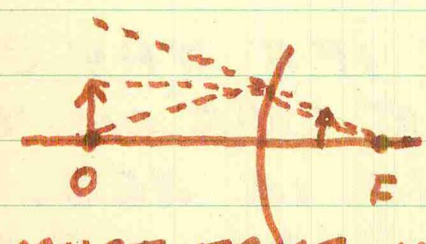
2) a) $\theta_1 = \theta_R$

b) $O = {}^{-}I$

C) SPHERICAL WAVES - SPHERICAL MIRROR

1) 

$$\frac{1}{o} + \frac{1}{i} = \frac{2}{r} = \frac{1}{f}$$

2) 

3) MUST TAKE INTO CONSIDERATION ABOUT THE IMAGE:

a) REAL OR VIRTUAL

b) INVERTED OR NOT

c) ENLARGED OR SMALLER

4) WORKING GEOMETRICALLY:

a) MIRROR REFLECTS DIRECTLY AT IT'S "Y AXIS" CENTER

b) A RAY REFECTED, WHICH WAS // TO AXIS, WILL GO THRU FOCAL POINT

c) A RAY PASSING THRU THE FOCAL POINT ~~PASSES~~ REFLECTS // ~~THRU~~ TO AXIS

5) LATERAL MAGNEFICATION m

$m = -i/o$

a) $m < 0 \Rightarrow$ IMAGE IS INVERTED

b) $m > 0 \Rightarrow$ IMAGE IS ERECT

D) SPHERICAL REFRACTING SURFACE

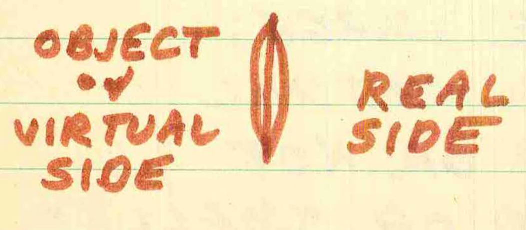
1) $\frac{n_1}{o} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$

2) SIGN CONVENTION

a) i IS POSITIVE IF THE IMAGE (REAL) IS ON THE R-SIDE OF THE REFRACTING SURFACE. IS NEG. IF IMAGE (VIRTUAL) IS ON VIR-SIDE

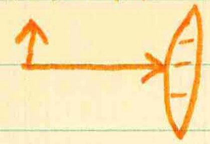
b) r IS POSITIVE IFF THE CENTER OF CURVATURE LIES ON THE "REAL" SIDE, AND IS NEG. IF C. OF. C. IS ON "VIRTUAL" SIDE

c)

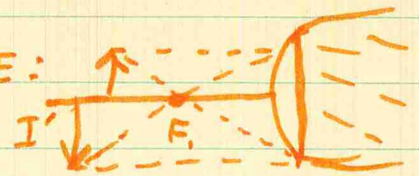


E) THIN LENSES

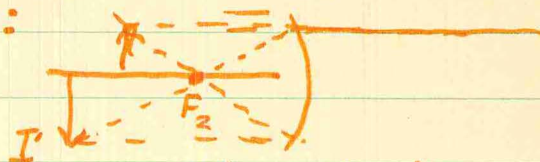
1) TREAT AS TWO PROBLEMS: EX



a) CASE ONE:



b) PART TWO:



c) $\frac{1}{o} + \frac{1}{i} = (n-1)\left(\frac{1}{r'} - \frac{1}{r''}\right)$

2) SINE CONVENTIONS

a) $i > 0$ IFF IMAGE (REAL) LIES ON REAL SIDE

b) $r > 0$ IFF THEY LIE ON REAL SIDE

3) $\frac{1}{f} = (n-1)\left(\frac{1}{r'} - \frac{1}{r''}\right)$

(LENS MAKER'S FORMULA)

Vectors (PRACTICE)

Using the notation for unit vectors $\hat{i}, \hat{j}, \hat{k}$ in Cartesian coordinates ($\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$),
verify by direct multiplication, term by term, that
$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$

Verify by explicit multiplication, as above, the vector IDENTITY:

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\,\vec{B} - (\vec{A} \cdot \vec{B})\,\vec{C}$$

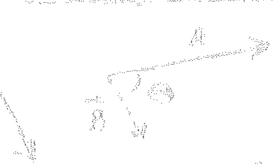
$$\left[= \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \right]$$

Note: if $\vec{A} = \vec{B} = \vec{\nabla}$, use of this identity proves the relation for curl curl $\vec{E}$ used in derivation of electromagnetic wave equation.

By direct mult. we mean, for example:
Given $\vec{F} = 5\hat{i} + 2\hat{j}$, $\vec{G} = -\hat{j} + 3\hat{k}$,
then

$$\begin{aligned} \vec{F} \times \vec{G} &= (5\hat{i} + 2\hat{j}) \times (-\hat{j} + 3\hat{k}) \\ &= 5\hat{i} \times (-\hat{j}) + 2\hat{j} \times (-\hat{j}) + 5\hat{i} \times 3\hat{k} + 2\hat{j} \times 3\hat{k} \\ &= -5\hat{k} + 0 + 15(-\hat{j}) + 6\hat{i} \\ &= 6\hat{i} - 15\hat{j} - 5\hat{k} \end{aligned}$$

"Dot product"



$$\vec{A} \cdot \vec{B} \equiv AB\cos\theta \qquad (1)$$

If $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$
show that
$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$ is equivalent to

Derivation of significance of Poynting [illegible]

Given Maxwell's equations in differential form, and recalling expressions for the energy density associated with E and B fields, we need only straightforward mathematical manipulation (with vector operators), and an easily verified vector operator identity, to **show** that $\vec{S}$ represents rate of flow of energy in empty space.

Given: $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$, $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Vector Identity: $\vec{\nabla} \cdot (\vec{K} \times \vec{L}) = \vec{L} \cdot (\vec{\nabla} \times \vec{K}) - \vec{K} \cdot (\vec{\nabla} \times \vec{L})$. VER[illegible]

Now form the scalar (dot) products indicated using eq'ns above:

$$\vec{E} \cdot (\vec{\nabla} \times \vec{B}) = \mu_0 \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{\mu_0 \epsilon_0}{2} \frac{\partial}{\partial t} E^2$$

$$\vec{B} \cdot (\vec{\nabla} \times \vec{E}) = -\frac{1}{2} \frac{\partial}{\partial t} B^2 \qquad [\text{Note: } \vec{E} \cdot \vec{E} = E^2]$$

By identity, $\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = -\frac{1}{2} \frac{\partial}{\partial t} (\mu_0 \epsilon_0 E^2 + B^2)$

Since, by def'n, $\vec{S} = \vec{E} \times \vec{B} / \mu_0$

$$\vec{\nabla} \cdot \vec{S} = \frac{\partial}{\partial t} \left(-\frac{\epsilon_0}{2} E^2 - \frac{1}{2\mu_0} B^2 \right) = -\frac{\partial}{\partial t} (\text{energy density}, u)$$

Integrating over a volume τ,

$$\int_\tau \vec{\nabla} \cdot \vec{S} \, d\tau = -\int_\tau \frac{\partial u}{\partial t} d\tau = -\frac{\partial}{\partial t} \int_\tau u \, d\tau$$

($\int_\tau u \, d\tau$ = U total energy)

$d\tau$ is an element of vol ($= dx\,dy\,dz$).

By the Divergence Thm ("Gauss"), the left-hand side above becomes $\oint \vec{S} \cdot \vec{dA}$ (closed surface), the flux of $\vec{S}$ out of the volume τ, where $\vec{dA} \equiv$ el't. of surface area.

[illegible] (out) of $\vec{S}$ is rate of loss of energy from enclosed [illegible]

CHAPT. 37

1) $\mu_e = 6.4 \times 10^{21}$ AMP-M

$\mu = N i A$

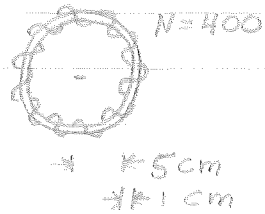
$i = \frac{\mu}{NA}$; $N = 1$

$= \frac{6.4 \times 10^{21} \text{ AMP-M}^2}{\pi (6.37 \times 10^6 m)^2}$

$= .0494 \times 10^5$ AMPS

$= 4.94 \times 10^7$ AMPS

4) a)



$B_0 = \mu_0 n i$

$i = \frac{B_0}{\mu_0 n}$

$= \frac{2 \times 10^{-4} \text{ WEBER-}m^2 (2\pi)(.055)}{m^2 (1.26 \times 10^{-6} \text{ HENRY})(400)}$

$= .00137 \times 10^2 \frac{\text{WEBER}}{\text{HENRY}}$

$= .137$ AMPS

b) $B_M = 400 B_0$

$B_0 = 2 \times 10^{-4} \frac{\text{WEBER}}{m^2}$

$\therefore B = 8.02 \times 10^{-2} \frac{\text{WEBER}}{m^2}$

$R_s = 8\,\Omega$ $N_s = 50$

$B = \frac{qR}{NA} \Rightarrow q = \frac{BNA}{R}$

$q = \frac{\text{WEBER } m^2 (8.02 \times 10^{-2}) \pi (.055)^2 (50)}{m^2 \quad 8\,\Omega}$

$= .0477 \times 10^{-2} \frac{\text{WEBER}}{\Omega}$

$= 4.77 \times 10^{-4}$ COULOMBS

5) $\mu_{FE} = 1.8 \times 10^{-23}$ AMP-M^2 $L = 5cm$, $A = 1cm^2$

a) $\mu_B = N \mu_{FE}$ $D = 7.9 g/cm^3$

$= \frac{7.9 g (5 cm^3)(6.02 \times 10^{23} \text{ ATOMS})(1.8 \times 10^{-23} \text{ AMP-M}^2)}{cm^3 \quad g \quad \text{ATOM}}$

$= 4.29 \times 10^2$ AMP M^2

b) $\mu_B = 7.6$ AMP-M^2

$\tau = \mu \times B$

$= (7.6 \text{ AMP-M}^2)(1.5 \times 10^4 \text{ GAUSS}) \frac{1 \text{ WEBER}}{m^2 (10^3 \text{ GAUSS})} (\sin 90°)$

$= 11.4$ WEBER-AMPS

CHAPT. 19 ; Pg 492

1) $Y = Y_m \sin(kx - \omega t)$

a) $Y = Y_m \sin k(x - \frac{\omega}{k} t)$

$= Y_m \sin k(x - vt)$

b) $Y = Y_m \sin \omega(\frac{k}{\omega} x - t)$

$= Y_m \sin \omega(\frac{x}{v} - t)$

c) $Y = Y_m \sin 2\pi (\frac{k}{2\pi} x - \frac{\omega}{2\pi} t)$

$= Y_m \sin 2\pi (x/\lambda - \nu t)$

d) $Y = Y_m \sin 2\pi (x/\lambda - t/T)$

2) a) FOR VIS. LIGHT $4 \times 10^{-7} m < \lambda < 7 \times 10^{-7} m$

$V_{LIGHT} = 3 \times 10^8 \frac{m}{SEC}$

$VK = \omega \qquad K\lambda = 2\pi$

$\frac{\omega}{V} = \frac{2\pi}{\lambda}$

$\frac{2\pi f}{V} = \frac{2\pi}{\lambda}$

$f = \frac{2\pi V}{\lambda}$

$f_0 = \frac{2\pi(3 \times 10^8)}{4 \times 10^{-7}} = 4.71 \times 10^{15}/2\pi \text{ HZ}$

$f_f = \frac{2\pi(3 \times 10^8)}{7 \times 10^{-7}} = 2.69 \times 10^{15}/2\pi \text{ HZ}$

b) $1.5 \times 10^6 < f < 3.00 \times 10^8$

$\frac{f}{V} = \frac{1}{\lambda}$

$\lambda = \frac{V}{f}$

$\lambda_0 = \frac{3 \times 10^8}{1.5 \times 10^6} = 2 \times 10^2 = 200 \text{ m}$

$\lambda_1 = \frac{3 \times 10^8}{3 \times 10^8} = 1 \text{ m}$

c) $5 \times 10^{-9} < \lambda < 10^{-11} m$

$f = \frac{V}{\lambda}$

$f_0 = \frac{3 \times 10^8}{5 \times 10^{-9}} = 1.67 \times 10^{17}$

$f_1 = \frac{3 \times 10^8}{10^{-11}} = 3 \times 10^{19}$

5) $f = 500\ Hz$; $V = 350\ \frac{m}{sec}$

$\lambda = V/f = \frac{35}{50} = .7\ m$

$\left(\frac{60}{360}\right).7 = .117\ cm$

$\lambda = V/f = .7$

$Vt = X \Rightarrow (350)(10^{-3}) = .35$

$\frac{.35}{.7} = .5 = \frac{PHASE}{360}$

PHASE = 180°

~~14)~~ A. TIME t

$Y_1 = .03 \sin(k_1 X - \omega t)$

$Y_2 = .04 \sin(k_2 X - \omega t + \frac{\pi}{2}$

CHAPT. 21

18) $f = \frac{nV}{2\ell}$

$\ell = \frac{nV}{2f} = \frac{n(330)}{2(660)} = \frac{n}{4}$

$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ \& \ 1$ METER

20)  $f = \frac{V}{2\ell}$

$\ell = \frac{V}{2f} = \frac{331}{2(300)} = .550\ m$

ROBERT MARKS
85

0

1\. [illegible] how many poles you [illegible] bar magnet into four equal pieces and why?

N [ ] S

N[ ]S N[ ]S N[ ]S N[ ]S

(1) THERE WILL BE 8 POLES. EACH ATOM ALLIGNED WITHIN THE MAGNET ACTS AS A SMALL MAGNET. ERGO, THE MAGNET COULD THEORETICALLY BROKEN DOWN INTO N ATOMIC MAGNETS, WITH A TOTAL OF 2N POLES

2\. A 1 inch long bar magnet has a dipole moment of 0.01 amp-m². Explain or calculate the net magnetic flux passing through a spherical surface of radius 4.0 m centered on the magnet (or [illegible] how to go about such a calculation). $\mu_0 = 4\pi \times 10^{-7}$ [illegible]

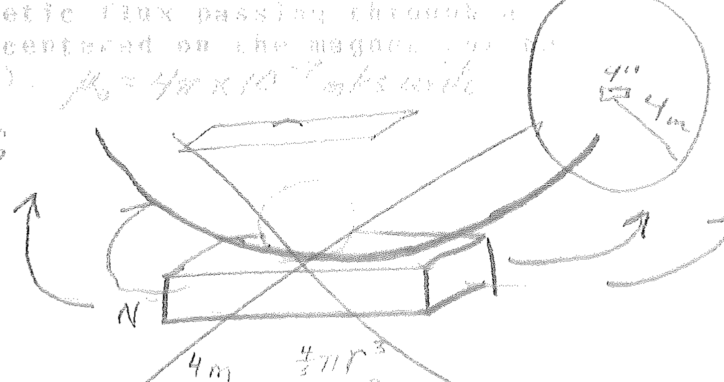
$\mu = .01$ AMP-M²

1

$$\Phi_B = \oint B \cdot dS = 0 \quad \text{Always}$$

$$= \oint\oint B \cdot (dh\, dw)$$

$$= \int\int B \cdot dh\, dw$$



3\. At atmospheric pressure oxygen liquifies at 90°K and freezes at 55°K. Explain whether it is more or less paramagnetic at 77°K than at 80°K, and why.

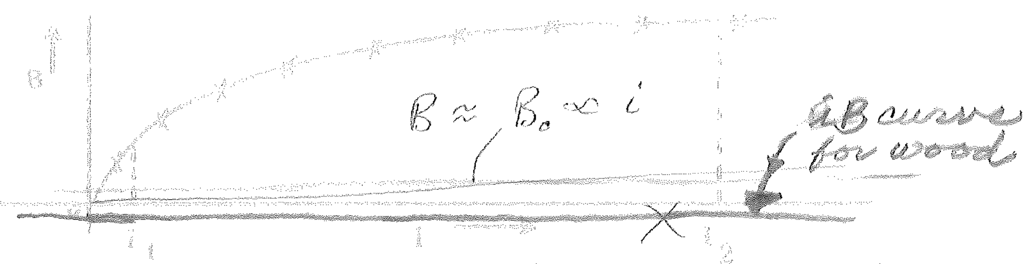
?

70° ?

20 BECAUSE THE HIGHER THE TEMP, THE MORE ATOMIC VIBRATION, WHICH (IN TURN FURTHER) RANDOMIZES THE orientation of ATOMIC DIPOLES.

4\. A toroid wound on an iron ring, carries a current i. By means of a secondary coil linking the iron, and a ballistic galvanometer the magnetic induction B in the iron is determined. Suppose a plot of observed pairs of the physical quantities i and B looks like this:

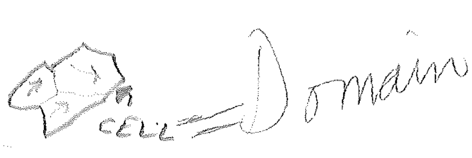
B



a) Explain what occurs in the iron for currents near i_1.

10 THE CURRENT IN THE COIL SETS UP A MAGNETIC FIELD, WHICH IN TURN LINES UP THE "CELL" DIPOLES WITHIN THE ATOM.

5



b) Explain what occures in the iron for currents near i_2.

IN THAT THERE IS A FINITE NUMBER OF CELLS WITHIN THE IRON, INCREASED MAGNETIC FIELD (FROM THE INCREASED CURRENT IN THE WINDINGS) ONLY HAS THOSE FEW UNLINED CELLS TO ALIGN ~~GN.~~

10

c) On this plot sketch what might be observed if the iron were replaced by a "non-magnetic" material (e.g., wood). Explain.

WOOD BEING DIAMAGNETIC WOULD HAVE NEARLY A CONSTANT B FOR ANY CURRENT, FOR THERE ARE NO "CELLS" WHICH CAN BE PERMANETLY ALIGNED (B=CONSTANT; $B \cong 0$; $B_m < 0$) $B_0 > 0$

5

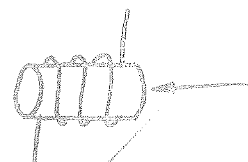
30pt 5) In arriving at a useful definition of inductance L for an arbitrary size and shape coil we wrote the relation $N\phi_B = Li$ for cases where no magnetic material was present. State qualitatively what changes, if any, would occure in L for a coil which contained a core of

$L \propto \frac{N\Phi_B}{\Delta i}$

a) paramagnetic material.

SLIGHTLY INCREASE

10



Core.

b) Diamagnetic material.

SLIGHTLY DECREASE

10

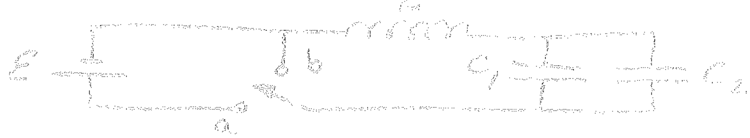
On what physical grounds do you base your answers?

SINCE $M \propto L$.
A PARAMAGNETIC MATERIAL (COMPARED TO A VERY WEAK FERROMAGNETIC) WOULD INCREASE M, THUS INCREASE L. THE SAME REASONING (~~USING~~ ...) WOULD LEAD ONE TO THE FACT THAT A DIAMAGNETIC MATERIAL, (~~MAGNETICALLY OPPOSED~~ TO A PARAMAGNETIC MATERIAL) WOULD SLIGHTLY DECREASE M, THUS L

9

JMK 0 (45) BOB MARKS

1. In the following circuit $\mathcal{E}$ is a fixed e.m.f., C_1 a fixed capacitance, C_2 a variable capacitance, and L a fixed inductance. Assume resistance is negligible.



Opt A) When the switch is thrown from a to b the various charges, currents and voltages are seen to oscillate at some frequency f. Derive an expression for the value that C_2 must have, in terms of the given parameters.

3

$C_{EQ} = C_1 + C_2$ ✓

$U_p = q_m^2 / 2(C_1 + C_2)$

$U_L = \frac{1}{2} L i^2$

$\frac{\partial U}{dt} = \frac{\partial q}{dt} \frac{q}{2C_1 + C_2} + \frac{1}{2} L \frac{di}{dt}$

$\frac{q}{(C_1 + C_2)} = -L \frac{di}{dt}$

$C_2 = \frac{q}{-(\frac{di}{dt} L)} - C_1$

Opt B) Let t = 0 be the time at which the switch is thrown (above). Derive an expression for the first instant $t = t_1$ at which the (energy) stored in C2 vanishes. Express t_1 in terms of the parameters given on the circuit.

1

energy $\propto q^2$

when is $q^2 = 0$?

$T = 2\pi / \omega$

$q = q_{max} \sin(\omega t + \phi)$ (cos)

$\sin^{-1} \frac{q}{q_{max}} = \omega t + \phi$

let $\phi = 0 \Rightarrow \omega$ $(\sin^{-1} \frac{q}{q_{max}}) \omega^{-1} = t$

$q = q_{max} \cos(\omega t + \phi)$

$i = -\omega q_{max} \sin(\omega t + \phi)$

$\frac{di}{dt} = -\omega^2 q_{max} \cos(\omega t + \phi)$

Opt C) If f = 1000/π Hz, $\mathcal{E}$ = 1.50V, L = 5.0mh, and C_1 = 10microfarad, compute C_2.

0

7

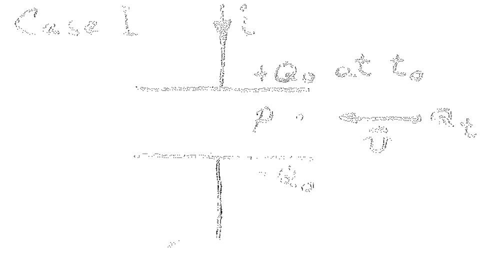
Opt D) State the effect(s) on the circuit behavior [redacted] of non-zero resistance

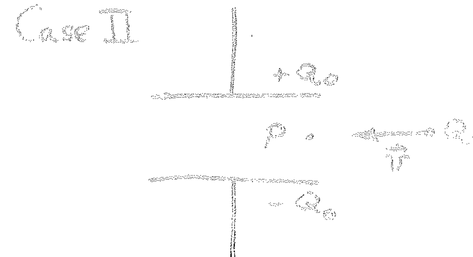
10

$q = q_{max} e^{-\frac{Rt}{2L}} \cos \omega' t$

THE CHARGE & CURRENT BECOME DAMPED

2. Consider two identical parallel plate capacitors with circular plates. One of these is being charged with a current i and the other has a steady charge q_0 on its plates. Suppose at some time t_0 the charging capacitor also holds a charge q_0. A test charge q_t is fired into the space between the the plates of each capacitor with a velocity $\vec{v}$ as shown.





the test charge arrives at point 2, just inside the capacitors at time t_0 and in Case I experiences a net force F_I and in Case II a force F_{II}.

pt A) Which of the following is true? $F_I < F_{II}$, $F_I = F_{II}$, $F_I > F_{II}$.

20pt B) Give the reason(s) for your above answer.

THE INDUCED CURRENT IN CASE I WOULD BE ~~GREATER~~. RELATIVELY, THE CHARGE IN CASE TWO WOULD LESSEN THE INDUCED CURRENT

0

20pt 3) Prove the identity $\vec{\nabla}\times(\vec{\nabla}a)\equiv 0$ for an arbitrary scalar field function $a(x,y,z)$.

$\vec{\nabla}a = \text{GRAD } a = i\frac{\partial a}{\partial x} + j\frac{\partial a}{\partial y} + k\frac{\partial a}{\partial z}$

9

$\vec{\nabla}\times(\nabla a) = \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\times\left(i\frac{\partial a}{\partial x} + j\frac{\partial a}{\partial y} + k\frac{\partial a}{\partial z}\right)$

$= 0 + \frac{\partial^2 a}{\partial x \partial y}k - \frac{\partial^2 a}{\partial x \partial z}j + \frac{\partial^2 a}{\partial x \partial y}k - \frac{\partial a^2}{\partial y \partial z}i + \frac{\partial^2 a}{\partial x \partial z}j + \frac{\partial^2 a^2}{\partial y \partial z}i = 0$

5pt 4) For the vector field function $\vec{A}(x,y,z) = \hat{i}y^2z^3 + \hat{j}x^2yz + \hat{k}x^3y^2z$ compute $\vec{\nabla}\times\vec{A}$.

$\vec{\nabla}\times\vec{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\times\left(iy^2z^2 + \hat{j}x^2yz + kx^3y^2z\right)$

3

$= -k(2xyz) - j(3x^2y^2z) + k(2yz^3) - i(x^3 2yz)$
$+ j(3z^2y^2) + i(x^2y)$

check signs

5pt 5) Compute the divergence of the answer to problem 4.

0

$\text{DIV } \vec{A} = 0 + (x^2z) + (x^3y^2)$ $\quad$ $div(\vec{\nabla}\times\vec{A}) = ?$
$= x^2z + x^3y^2$

20pt 6) Consider the equation $\oint\vec{S}\cdot d\vec{A} = -\frac{\partial U}{\partial t}$. $\vec{S}$ is the ~~Rxxxxx~~ Poynting vector, $d\vec{A}$ an element of area, and U the total energy stored in a volume containing both an $\vec{E}$ and a $\vec{B}$ field. How does this equation express the principle of conservation of energy?

$\oint S\cdot dA = -\frac{dU}{dt}$

IT SAYS IN EFFECT THAT THE AMOUNT OF ENEGY LEAVING THE VOLUME $(= -\frac{\partial U}{\partial t})$ IS THE AMOUNT OF ENERGY passing through area ~~OUTSIDE THE VOLUME~~ $(\oint\vec{S}\cdot dA)$ IN THE FORM OF ELECTROMAGNETIC ~~RADIATION,~~ WAVES. ERGO, THE ENERY DEPLETION INSIDE THE VOLUME IS EQUAL TO THE AMOUNT EMITTED

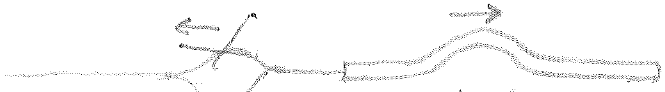
19

BOB MARKS

40

AVE = 47

[illegible]



PART OF THE WAVE WILL BE REFLECTED, THE REMAINDED WILL GO ON INTO THE DENSER ROPE, WITH A SMALLER AMPLITUDE, AND A DECREASED FREQ

relative velocity?

[illegible]

~~NONE OF THE WAVE WILL BE REFLECTED, THE AMPLITUDE AND FREQUENCY WILL INCREASE~~

[illegible]

$f(x-vt) = A_1 \sin(x-vt)$ - WAVE TO THE RIGHT

$g(x+vt) = A_2 \sin(x+vt)$ - WAVE TO THE LEFT

WITH K = 1

$$Y = A_1 \sin(x-vt) + A_2 \sin(x+vt)$$

$$\frac{\partial Y}{\partial x} = A_1 \cos(x-vt) + A_2 \cos(x+vt)$$

$$\frac{\partial^2 Y}{\partial x^2} = -A_1 \sin(x-vt) - A_2 \sin(x+vt)$$

$$\frac{\partial Y}{\partial t} = -VA_1 \cos(x-vt) + VA_2 \cos(x+vt)$$

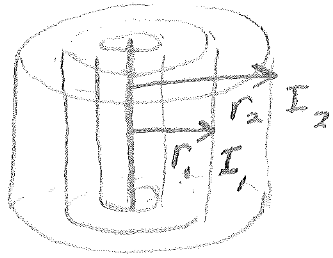
$$\frac{\partial^2 Y}{\partial t^2} = -V^2 A_1 \sin(x-vt) - V^2 A_2 \sin(x+vt)$$

$$\left[-A_1 \sin(x-vt) - A_2 \sin(x+vt)\right] = \left[\frac{1}{V^2}\right]\left[-V^2(A_1 \sin(x-vt) + A_2 \sin(x+vt)\right]$$

3

10

2



$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2} = ~~\frac{A_2}{A_1}~~ = \frac{A_1}{A_2}$$

why?

0

$$f' = \frac{v}{\lambda'} = f\left(\frac{v}{v - v_s}\right)$$

$$\frac{f' - f}{f} = \frac{f'}{f} - 1 = \frac{v}{v - v_s} - 1 = ~~\frac{1070}{1070-44}~~ - 1$$

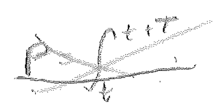
$$= \frac{1070 - 1026}{1026} = \frac{44}{1026}$$

$$= 1.0429$$

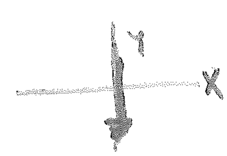
8

λ / T ✓

~~$\frac{1}{2} X_m^2 k \omega$~~

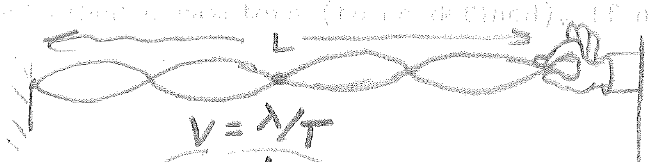


DOWN Y AXIS ✓



BOTH. THERE IS NO MASS DIMENSION IN THE EQUATION. THUS, THIS WAVE, FROM THE EQUATION, IS INDEPENDENT OF MASS

[illegible] end of a stretched rope of length L [illegible]
[illegible] mode of the rope is set in motion, [illegible] disturbance in the rope?
[illegible] (if needed).



$V = \lambda / T$

$V = \frac{L}{2T}$

8

~~$Y = 2Y_{MAX} \sin kx \cos \omega t$: Y_M = AMPLITUDE~~

~~$\sin \frac{2\pi}{\lambda} x \cos kvt$~~

~~$= 2Y_{MAX} \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi v t}{\lambda}$~~

~~$= 2Y_{MAX} \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi t}{T\lambda}$~~

✓

b) Consider that after doubling the tension in this rope the propagation speed becomes V'. Compute V'/V

b $\quad V = \sqrt{\frac{T}{\mu}}$; $V_1' = \sqrt{\frac{2T}{\mu}} \Rightarrow \frac{V_1}{V} = \frac{\sqrt{2T}/\mu}{\sqrt{T}/\mu} = \sqrt{2}$ ✓

T = TENSION

c) If a [illegible] sensitive detector with a sufficiently broad frequency response were placed near the rope in (a) above, might it detect a sound?

[illegible] State, in a sentence, why or why not.

Read

17

$f = \frac{nV}{2L} = \frac{(2)\, L}{2T\, (L)} < 20$

3/6

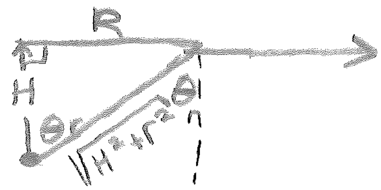
NO,
BECAUSE
INFRASONIC ≤ 20 HZ – Sound?
AUDIBLE SOUND ≅ 20 TO 20,000 HZ
ULTRASONIC > 20,000

AVE-60

BOB MARKS (36)

10

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1}{n}$$ ✓



$$\frac{R}{\sqrt{H^2+R^2}} = \frac{1}{n}$$ ✓

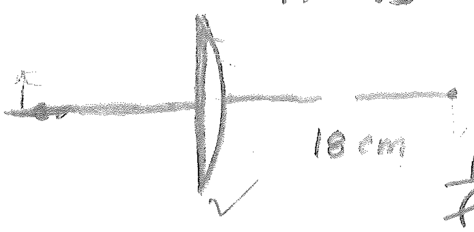
15

$$R^2 = \frac{H^2}{n} + \frac{R^2}{n}$$ Careless

$$R^2(n-1) = \frac{H^2}{n}$$

$$R = \frac{H}{\sqrt{n(n-1)}} \Rightarrow D = \frac{2H}{\sqrt{n(n-1)}}$$

[illegible] A magnifying lens, plano convex, has an index of refraction of [illegible] light is focused 18-cm from the lens. What is the magnitude of [illegible] radius of curvature?



3

$$\frac{1}{o} + \frac{1}{i} = \frac{2}{r} = \frac{1}{f}$$ (crossed out)

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_I} - \frac{1}{r_{II}}\right)$$

$$= (.5)\left[\left(\frac{1}{36}\right) - 0\right]$$ ✓

$$f = 729 \text{ cm}$$

[illegible]

$$-\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$$

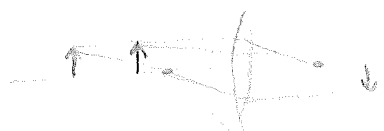
$$\frac{1}{i} = \frac{1}{6} + \frac{1}{18} = \frac{4}{18} \Rightarrow i = \frac{9}{4} = 2.25 \text{ cm}$$ ON THE RIGHT SIDE OF THE LENSE

D

v

b) Find the speed and direction of [illegible] of the image when the [illegible]

$$\frac{1}{i} = \frac{1}{o} - \frac{1}{f}$$



$$\frac{1}{i} = \frac{f-o}{of}$$

$$i = \frac{of}{(f-o)}$$

~~$$\frac{di}{do} = \frac{f}{f-o} - \frac{of}{(f-o)^2}$$~~

$$i = of(f-o)^{-1}$$

$$\frac{di}{do} = \frac{f}{f-o} + \frac{of}{(f-o)^2}$$

two sign errors!

18

$$\frac{do}{dt} = 44 \Rightarrow \frac{di}{dt} = 44\left(\frac{f}{f-o} + \frac{of}{(f-o)^2}\right)$$

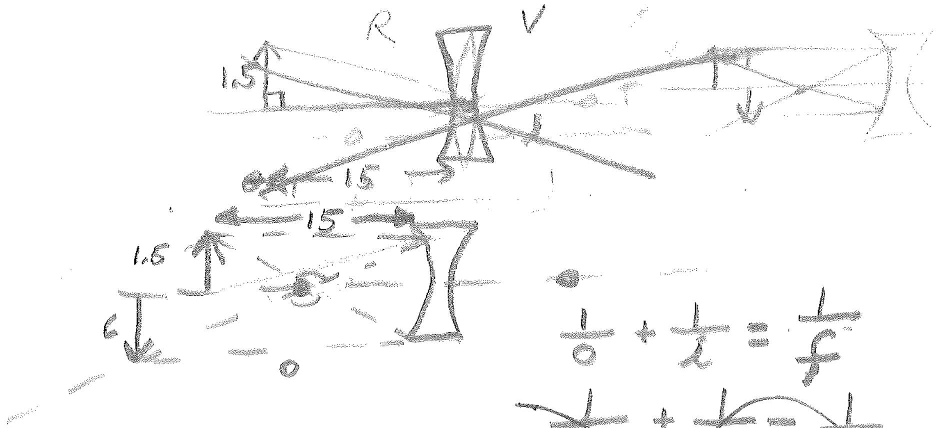
$$= 44\left(\frac{6}{6-18} + \frac{6(18)}{144}\right)$$

$$= 44\left(-\frac{1}{2} + \frac{3}{4}\right) = \frac{1}{4}(44) = 11 \frac{cm}{SEC}$$

TOWARD THE MIRROR FROM THE RIGHT SIDE

An object which is [illegible] cm long [illegible] forms an image which is 6 cm long.

a) Compute the focal length of the lens if the image [illegible]



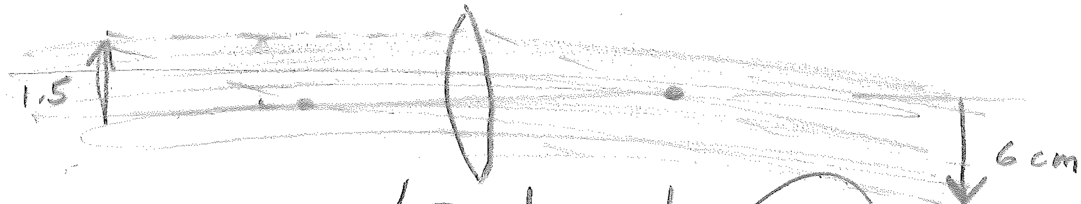
$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$

$\frac{1}{1.5} + \frac{1}{6} = \frac{1}{f}$

.667 + .167

$\frac{1}{f} = .833 = 1.2$ cm

b) Compute the focal length of the lens if the image is erect



$\frac{1}{o} - \frac{1}{i} = \frac{1}{f}$

$.667 - .167 = \frac{1}{f}$

$.500 = \frac{1}{f} \Rightarrow f = 2$ cm

69

Physics IV test #5, 9/3/70   Name BOB MARKS

Instr: WJB (JMK) Hour: (4) 5

1. A single slit diffraction pattern is observed on a screen placed 2.0 m from the slit. The distance between the first minimum on one side of the central maximum and the first minimum on the other side is 2.0 cm. The wavelength of the light is 5000 A. Find the width of the slit in mm.

$D = 2m \quad Y = 1\times10^{-2}m \quad \lambda = 5\times10^{-7}m \quad m = 0$

AT ~~MINIMUM~~; $d\sin\theta \cong (m+\frac{1}{2})\lambda$ max,

$\theta \simeq \tan\theta \simeq \sin\theta \simeq \frac{Y}{D}$

$\frac{dY}{D} = \frac{1}{2}\lambda \Rightarrow d = \frac{\lambda D}{2Y}$ — gives $\frac{1}{2}\theta$!

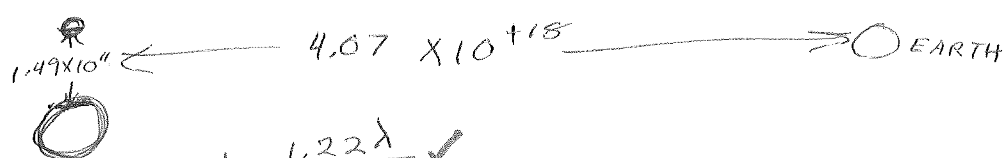
$d = \frac{(5\times10^{-7})(2)}{2(10^{-2})} \Rightarrow d = 5\times10^{-5}m = .05\times10^{-3}m = .05$ mm

0.1 mm.

(18)

2. The nearest star is approximately 4.3 light years (4.07×10^{18}m) away from the earth. Suppose this star had a planet with an orbit radius about the same as that of the earth - 93 million miles (1.49×10^{11} m). What aperture diameter is necessary for a telescope to be able to just resolve the images of the star and planet? How many times larger or smaller is this than the 200" Palomar telescope? Assume the images are formed with light of wavelength 5000 A.

$\sin\theta = 1.22\frac{\lambda}{d}$



$d = \frac{1.22\lambda}{\sin\theta}$ ✓

$\tan\theta \simeq \sin\theta \simeq \frac{1\times4.9\times10^{10}}{4.07\times10^{18}} = 3.66\times10^{-8}$

$\therefore d = \frac{(1.22)(5\times10^{-7})}{3.66\times10^{-8}} = 1.67\times10^{1} = 16.7$ m ✓

$1'' \simeq 2.46$ cm

$200''\left(\frac{2.46 cm}{1''}\right)\left(\frac{10^{-2}m}{cm}\right) = 4.92m$ 2.54

$X = \frac{16.7}{4.92} = 3.4$ 3.29

THE TELESCOPE WOULD HAVE TO BE ~~ABO~~ A LITTLE LESS THAN $3\frac{1}{2}$ TIMES AS LARGE

(24)

9
18
24
—
69

3. (a) Compute the minimum thickness of a transparent material (n = 1.3[illegible]) needed to coat a glass flat (n = 1.62) in order to prevent reflection of normally incident light of wavelength 6500 Å in air?

$2dn = (m + \frac{1}{2})\lambda$

$m = 0$

$d = \frac{\lambda}{4n}$ ✓

$= \frac{(6.5 \times 10^{-7})}{4(1.62)} = 10^{-7} m$

(9)

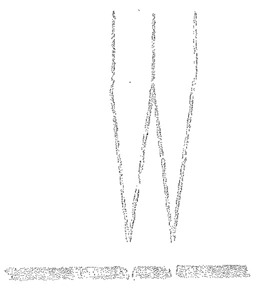
Why???

(b) Would the above film prevent the reflection of similar light incident obliquely? Explain.

~~$d = \frac{(m+\frac{1}{2})\lambda}{4n}$~~

~~YES. THE NORMAL INCIDENT LIGHT IS THE MOST CRITICAL CONDITION WHICH CAN BE SET FOR SUCH A SYSTEM, THUS, OBIQUE LIGHT WOULD BE ALSO BE PREVENTED FROM REFLECTING.~~

4.



Sharp edges of two razor blades, taped together, make straight 'scratches' in an opaque coating on a glass plate (see sketch). Red light incident normally on the plate passes through the two straight slits and forms several red slit images - with centers one centimeter apart on a screen placed 3 meters from the plate. Estimate the thickness of each razor blade from this information.

LET $\lambda_R = \lambda$ OF RED LIGHT

AT MIN $\Rightarrow d \sin\theta = (m + \frac{1}{2})\lambda$

LET X BE WIDTH OF CENTER (MOST INTENSE) IMAGE (IN cm) OR ANY IMAGE

$\theta \cong \sin\theta \cong \frac{Y}{D}$

$Y = (X + \frac{1}{2}) cm$?

$d = \frac{\lambda}{2\sin\theta} \cong \frac{\lambda D}{2Y} = \frac{3\lambda_R}{2X+1}$

ASSUMING $\lambda_R \cong 800 \times 10^{-9} m$; $X \cong 1 cm$

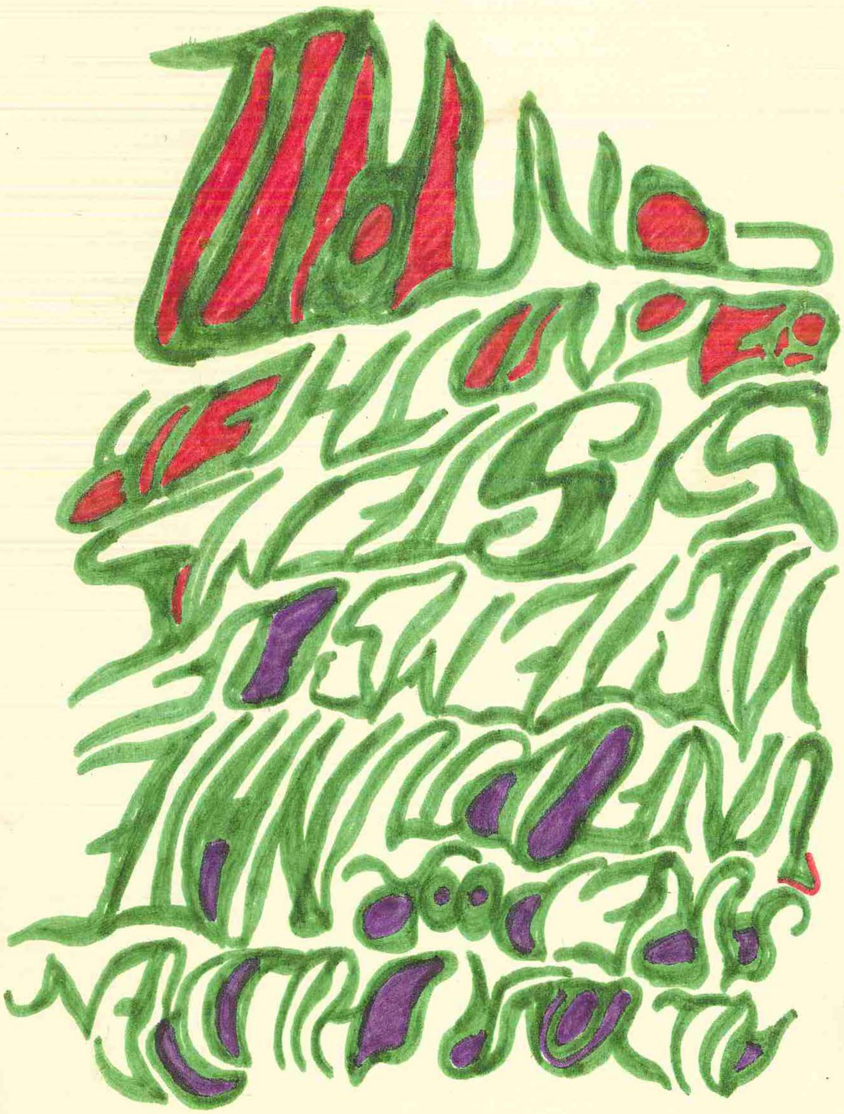
$d \cong \frac{3(8 \times 10^{-7})}{2 \times 10^{-2}} = 12 \times 10^{-5} = .12 \times 10^{-3}$ units?

(18)

free question - Comment on what has or has not been of value to you in your physics courses, if you care to take the time. Please be critical, constructive and brief. (write on back.)

another: All good things must come to an end. True. False. (mark one)

PHYS. II

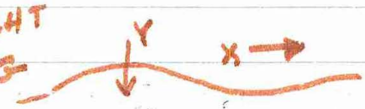


PHYS IV

4-3-70

1) $\nabla \vec{E} = \frac{1}{C^2} \frac{\partial^2 E}{\partial t^2}$ · LIGHT

$\frac{\partial Y^2}{\partial X^2} = \frac{\mu}{T} \frac{\partial^2 Y}{\partial t^2}$ · STRING

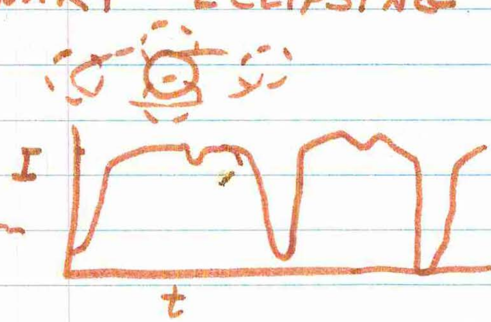


∴ LIGHT WAVES ARE ELECTROMAGNETIC WAVE

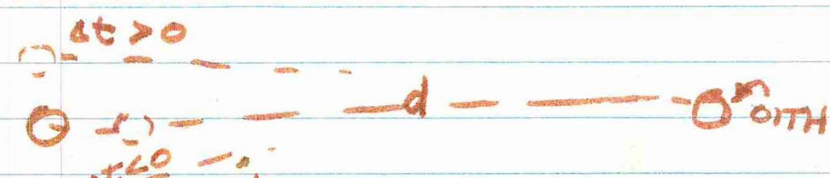
2) ETHER DEVISED AS MEDIA OF ELEC.-MAG. WAVES

3) MUST CONSIDER V OF SOURSE WHEN MEAS. E·M. THINGIIES

4) MOTION OF LIGHT SOURCE
BINARY ECLIPSING STAR



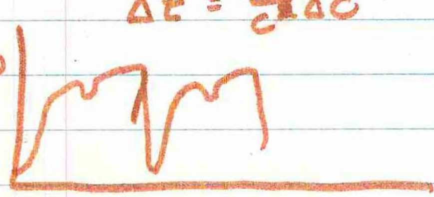
T~3DAYS



$t = \frac{d}{c}$

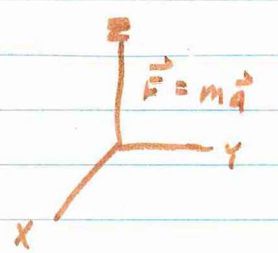
$\Delta t = \frac{-d}{c^2} \Delta c$

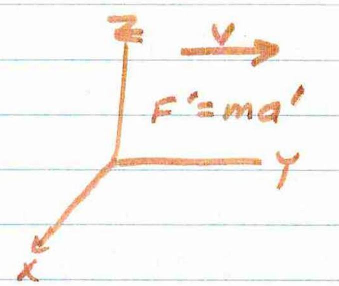
THESE CONSIDERATIO
YIELD



∴ THIS MEASURE FOR MEASURING LIGHT SPEED IS ERRONEOUS. C IS INDEPENDENT OF THE VELOCITY OF SOURSE.

UNIFICATION OF NEWTON'S LAWS & MAXWELL'S EQUATIONS UNDER ONE REFERENCE FRAME.





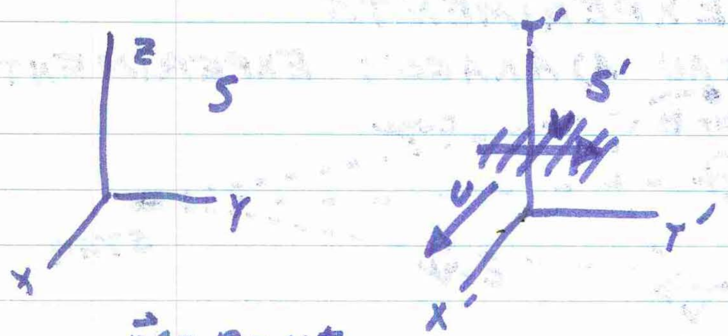
$\left.\begin{array}{l}\vec{r} = \vec{r} - vt \\ t' = t\end{array}\right\}$ NEWTONIAN TRANSFORMATION

TAKE A MAXWELL EQUATIONS

$\mathrm{curl}\ \vec{H} = \frac{1}{c}\frac{\partial E}{\partial t}$

x_{comp} $\frac{\partial H_z}{\partial y'}$.

4-6-70



$\vec{r}' = r - vt$
$t' = t$ ⇔ SUGGESTED BY NEWTON

$x' = x - vt$
$y' = y;\ z' = z;\ t' = t$

MAXWELL:

EX) $\text{curl}\ \vec{H} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$

X-COMP IN S'

$$\frac{\partial H_3}{\partial y'} - \frac{\partial H_2}{\partial z'} = \frac{1}{c}\left(\frac{\partial E_1}{\partial t_1} + \frac{\partial E_1}{\partial x'}\frac{\partial x'}{\partial t}\right) \quad (\Leftarrow -v)$$

$$\therefore \frac{\partial H_3}{\partial y'} - \frac{\partial H_2}{\partial z'} = \frac{1}{c}\left(\frac{\partial E_1}{\partial t_1} - v\frac{\partial E_1}{\partial x'}\right)$$

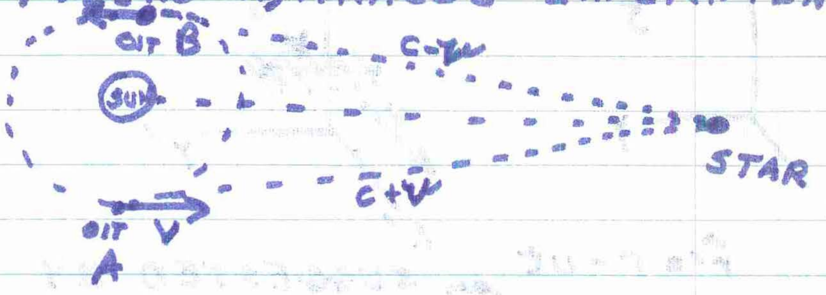
NEWTON

$$\frac{\partial H_3}{\partial y'} - \frac{\partial H_2}{\partial z'} = \frac{1}{c}\frac{\partial E_1}{\partial t}$$

MAXWELL'S EQUATION IS NOT INVARIENT UNDER NEWTONIAN TRANSFORMATION

RELATIVISTIC HISTORICAL EXPERIMENTS

1) FIZEAU A) ARAGO'S EXPERIMENT



FOR A SINGLE LENSE

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$(n = c/v_{\text{IN GLASS}})$

$V_g = V_c$ IN GLASS IN LAB

AT A; $\frac{1}{f_a} = \left(\frac{c}{V_g + V} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

AT B; $\frac{1}{f_b} = \left(\frac{c}{V_g - V} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{f_a}{f_b} = \frac{\frac{c}{V_g - V} - 1}{\frac{c}{V_g + V} + 1}$$

$C = 3 \times 10^8$ m/SEC

$V_g = 2 \times 10^8$ m/SEC

$V_E = 3 \times 10^4$ m/SEC

HOWEVER; ALL FOCAL LENGTHS CAME OUT THE SAME

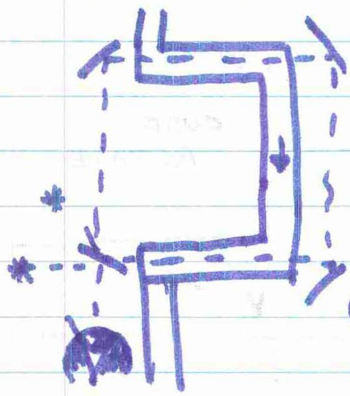
B) FRESNEL - SAYS ARAGO'S ERROR DUE TO ETHER

V = VELOCITY OF LIGHT IN A MOVING MEDIA

$$V = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)v$$

⇑ CALLED DRAGGING COEFFICIENT

C) FIZEAU'S INFEROMETER CHECH OF ABOVE



COULD CHANGE V OF FLUID INSIDE (LIGHT GETS DRAGGED BY THE WATER)

PREDICTED - .438
EXPERIMENT - .5 ± 1
(DRAG CO-EFFICIENT)

MADE ON ASSUMPTION ETHER EXISTS, AN EXP. CHECKS

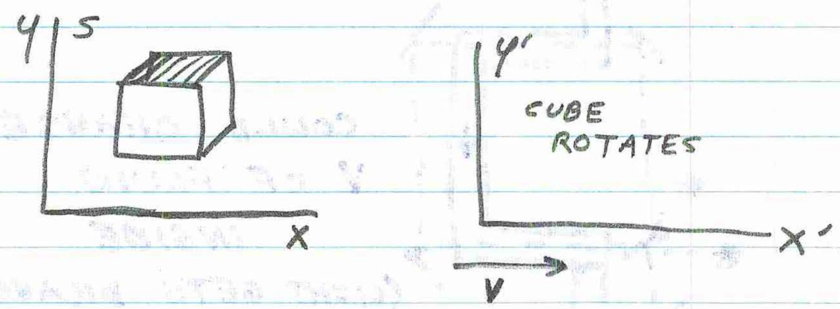
2) MICHELSON-MORLEY

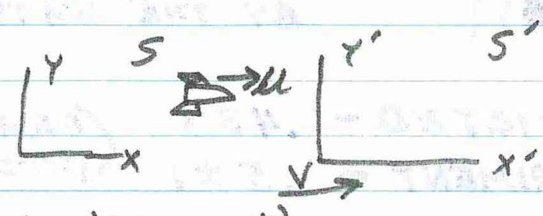
4-16-70

TIME DILATION & SHORTENED LENGTH WITH INCREASED VELOCITY

TWIN PARADOX

4-17-70





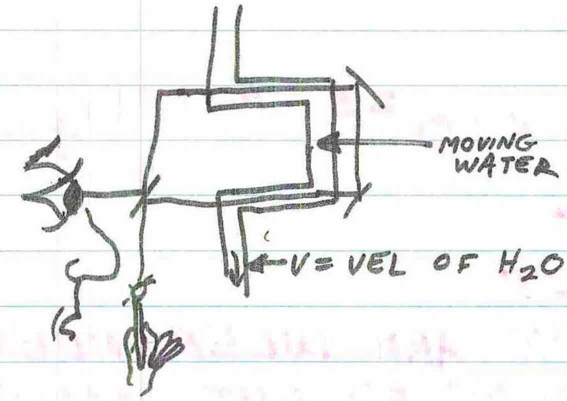
$$x' = \gamma(x - vt)$$
$$t' = \gamma\left(t - \frac{v}{c}x\right)$$

$$\frac{dx' = \gamma(dx - v\,dt)}{dt' = \gamma\left(dt - \frac{v}{c^2}dx\right)}$$

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - V}{1 - \frac{V}{c^2}\frac{dx}{dt}} = \frac{u - V}{1 - vu/c^2}$$

$$U' = \frac{U + V}{1 - VU/c^2}$$ ← EINSTEIN VELOCITY TRANSFORMATION

E.V.T. $\overset{\text{IMPLIES}}{\Longrightarrow}$ FIZEAU EXP



c/n = VELOCITY OF LIGHT IN REST FRAME OF THE WATER (S)

c' = VELOCITY MEASURED IN LAB

$$c' = \left(\frac{c}{n} + v\right) \Big/ \left(1 + \frac{v\,c}{c^2 n}\right)$$

$$= \left(\frac{c}{n} + v\right)\left(1 - \frac{v}{nc} + \left(\frac{v}{nc}\right)^2 - \left(\frac{v}{nc}\right)^3 + \dots\right)$$

$$\simeq \left(\frac{c}{n} + v\right)\left(1 - \frac{v}{nc}\right)$$

$$\simeq \frac{c}{n} + v - \frac{v}{n^2} - \frac{v^2}{nc}$$

$$\simeq \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right) - \frac{v^2}{nc}$$ ← NEGLIGIBLE

$v\left(1 - \frac{1}{n^2}\right)$ → MEASURED

$\frac{v^2}{nc}$ → NOT MEASURABLE

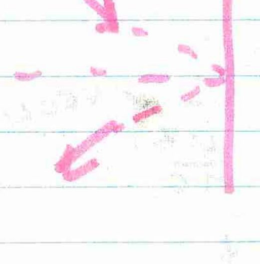
DOPPLER EFFECT

4-20-70
4-22-70
4-27-70

$N_0 = 6.025 \times 10^{23}$ moles/gm mole

I IDEAL GAS

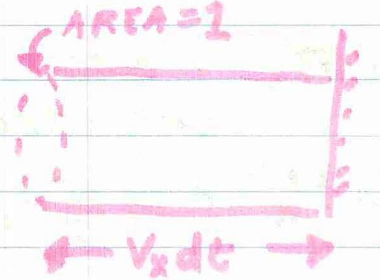
1) $PV = nRT$
2) PARTICLES ARE INDEPENDENT (NO FORCES, EXCEPT COLLISION)
3) MOLECULES ARE 'SMALL' COMPARED WITH THE DIMENSIONS OF THE PROBLEM
4) MOLECULE COLLISIONS ARE COMPLETELY ELASTIC (K.E. CONSERVED)



net $\Delta P = 2M V_x$

$= \int F \, dt$

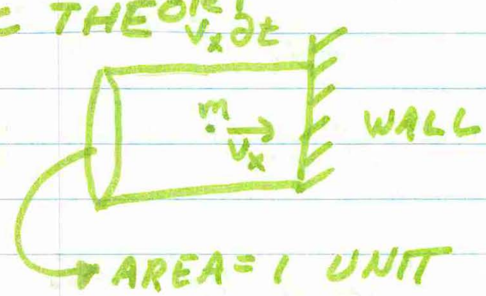
FOR SINGLE PARTICLE



$V_x dt$ = DIST. MOLECULE MOVES IN dt

4-30-70

KINETIC THEORY



$pV = RT$ $(n = 1)$

1) $N_v = \frac{\#}{\text{UNIT VOL}}$ WITH VEL. X-COMP V_x

2) VOL OF CLYL $= V_x \partial t$

3) # MOLEC IN CYL. WITH VX COMP $V_5 = N_v V_x \partial t$

4) $\frac{\text{\# MOLEC. STRIKING WALL}}{\text{UNIT TIME}} = N u_x \partial t / 2 \partial t$

$= \frac{N_v V_x}{2}$

5) MOMENTUM CHANGE / $\frac{\text{UNIT AREA}}{\text{UNIT TIME}}$

$= \frac{N_v V_x}{2} \cdot 2 m V_x = m N_v V_x^2$

6) Δ MOMENTUM = ~~∂F~~ $\int F \partial t$

7) $P_{v_x} = m N_v V_x^2$ (PRESSURE)

$$= \sum_{v_x} m N_v V_x^2 = m \sum_{v_x} N_v V_x^2$$

$$= m N_v \overline{V_x^2} \qquad N_v = \frac{\#\text{PAR.}}{\text{VOL}}$$

$$P_{v_y} = m N_v V_y^2 ; \quad P_{v_x} = m N_v V_z^2$$

$$\overline{V}^2 = \sum_{i=1}^{3} V_i^2 \Rightarrow V_x^2 = \frac{1}{3} V^2$$

$$\Rightarrow \therefore P = \frac{1}{3} m N \overline{V^2} \Leftarrow$$

$$p V_m = \frac{1}{3} m N V_m \overline{V^2}$$

$$N V_m = \frac{\#\text{ MOLEC}}{\text{VOL}} \cdot \frac{\text{VOL}}{\text{MOLE}} = N_0 \left(\text{AVA } \#\right)$$

$$\therefore p V_m = \frac{1}{3} m N_0 \overline{V^2} = RT$$

$m N_0$ = TOTAL MASS

$\frac{1}{2} m \overline{v}^2$ = AVER K.E. OF 1 MOLEC

$$= \frac{3}{2} \frac{R}{N_0} T$$

$\frac{R}{N_0} = k$ = BOLTZMAN CONSTANT

$$\overline{K} = \frac{3}{2} kT$$

$$\frac{1}{2} m \overline{V_x^2} = \frac{1}{2} kT$$

DEGREE OF FREEDOM

$\frac{1}{2} kT$ AT RM. TEMP $\simeq \frac{1}{40}$ eV

$$C_V = \frac{\partial U}{\partial T} \qquad pV_m = \frac{1}{3} M \overline{V^2} = RT$$

$$C_V = \frac{\partial}{\partial T}\left(\frac{3}{2} RT\right) = \frac{3}{2} R$$

(MONOTOMIC

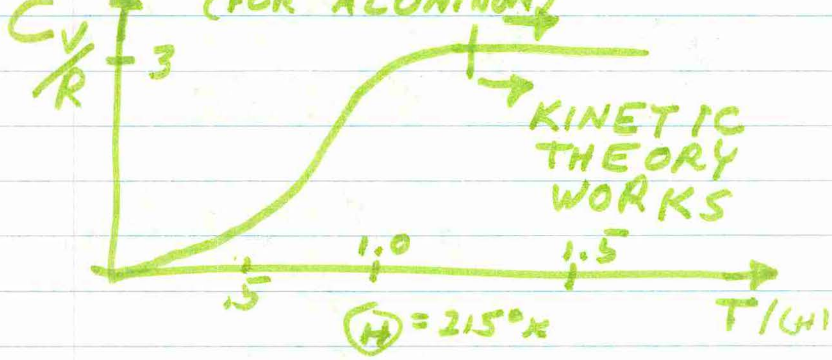
$$C_V = \frac{\partial}{\partial T}\left(\frac{5}{2} RT\right) = \frac{5}{2} R$$

(DIATOMIC

SOME SOLIDS ⇒ $C_V = \frac{\partial}{\partial T}\left(\frac{6}{2} RT\right) = 3R$

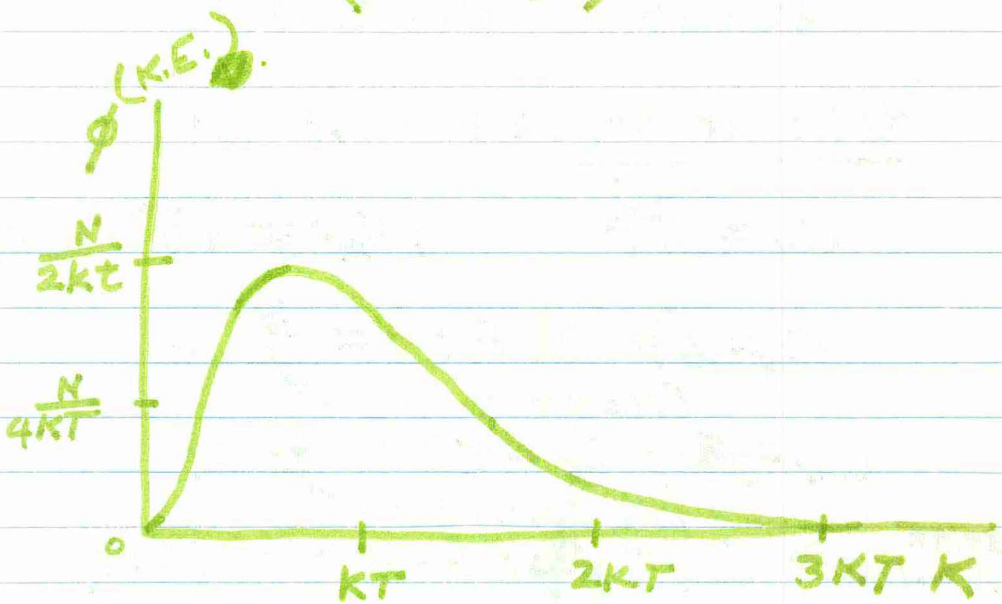
(LAW OF DU LONG & PETIT)

(FOR ALUMINUM)



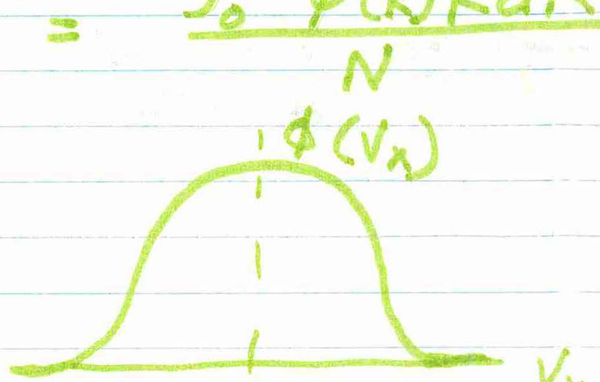
MAXWELL'S DISTRIBUTION
(K.E. DISTRIBUTION)

$$\phi(K) = 2N\left(\frac{K}{\pi (kT)^3}\right)^{\frac{1}{2}} e^{-K/kT}$$



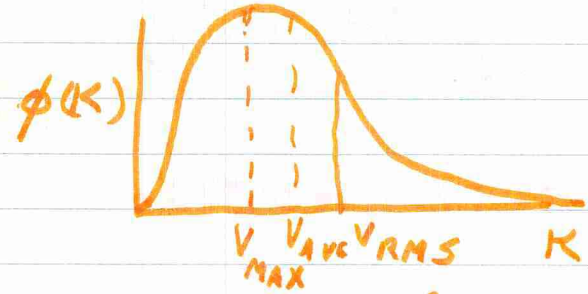
$$\bar{K} = \frac{\int_0^\infty \phi(K)\, K\, dK}{\int_0^\infty \phi(K)\, dK}$$

$$= \frac{\int_0^\infty \phi(K)\, K\, dK}{N}$$

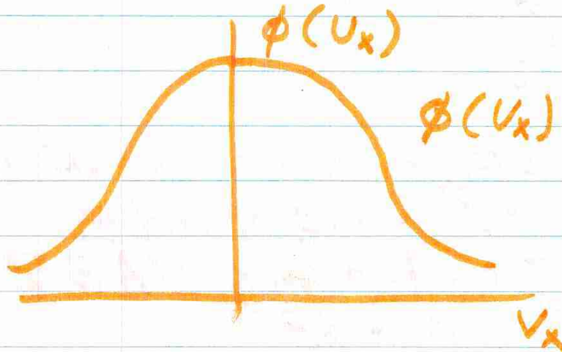


5-1-70

MAXWELL'S DIST. OF K



$$\phi(K) = 2N\left(\frac{K}{\pi(kT)^3}\right)^{\frac{1}{2}} e^{\frac{-K}{kT}}$$



$$\phi(U_x) = N\left(\frac{m}{2\pi kT}\right) e^{\frac{-mV_x^2}{2kT}}$$

$$\bar{K} = \frac{\int_0^\infty \phi(K)\,K\,dK}{\int_0^\infty \phi(K)\,dK}$$

TO CHECK MAXWELL BY ZERTMAN & KO

UNFOLD & LOOK

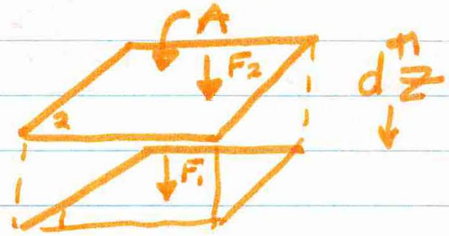
(CONT.)

$v_x = 2r/t$; $d = r\omega t$

$= r\omega \cdot 2r / v_x$

$v_x = 2r^2 \omega / d$

BOLTZMANN'S DISTRIBUTION THINGIES



$F_1 = F_2 +$ WT. OF GAS IN BLOCK

$= F_2 + mgNA\,dz$

$P_1A_1 = P_2A_2 + mgNA\,\partial z$

$P_2 = P_1 + dP$

$dp = (F_2 - F_1)/A$

$= -mgA\,N\,dz/A$

$dp = -mgN\,dz$

GO TO $pV_M = RT = p\left(\frac{N_0}{N}\right)$

$\Rightarrow p = kNT$

$dp = kT\,dN = -mgN\,dz$

$N = -\frac{dp}{dz}\frac{1}{mg}$

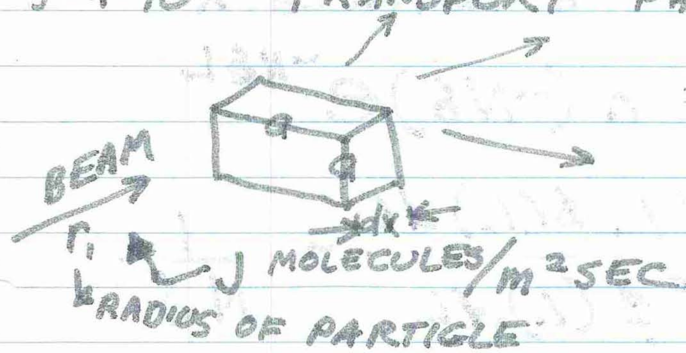
$p = kt\left(-\frac{dp}{dz}\right)\frac{1}{mg}$

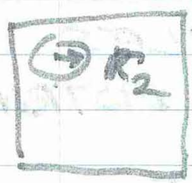
$$dp = kT\,dN = -Nmg\,dz$$

$$\frac{dN}{N} = \frac{-mg}{kT}\,dz$$

$$N = N_1 e^{\frac{-mg}{kT}(z - z_1)}$$

5-4-70 TRANSPORT PHENOMENA





EFFECTIVE AREA

$\sigma = \pi(r_1 + r_2)^2$

PARTICLE DENSITY = N

AREA PRESENTED TO BEAM

$= N \cdot a^2 dx\ \pi(r_1 + r_2)^2$

FRACTIONAL AREA OCCUPIED BY TARGET PARTICLES $= N \cdot a^2 dx\,\pi(r_1 + r_2)^2$

$N \cdot dx\,\pi(r_1 + r_2)^2 = dJ/J$

$$\int_{J_0}^{J} dJ/J$$

SOLVING YIELDS:

$$J = J_0 e^{-N\pi(r_1+r_2)^2 x}$$

$\pi(r_1+r_2)^2 = \delta$ = CROSS SEC. AREA

$n = n_0 e^{-N\delta x}$ = # PARTICLES AFTER A DISTANCE X

$$\phi(L) = \frac{dn}{dx} = \frac{dn}{dL} = n(-N\delta)e^{-N\delta L}$$

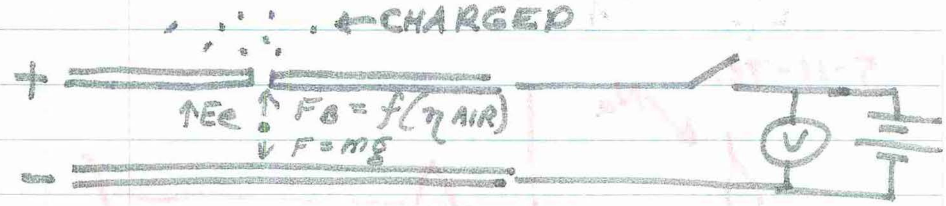
$$\bar{L} = \frac{\int_0^\infty L\,\phi(L)\,dL}{\int_0^\infty \phi(L)\,dL} = \frac{1}{N\delta}$$

IF TARGET MOLECULES ARE MOVING (MAXWELL'S DISTRIBUTION)

$$\bar{L} = \frac{1}{\sqrt{2}\,N\delta}$$

CHAPT. (3)

I) ELECTRIC CHARGE (e OF ELECTRON)



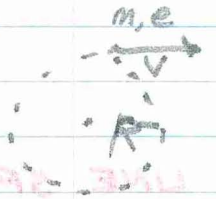
VELOCITY OF PARTICLE IS CONSTANT BECAUSE OF AIR ($F_B = F_g$)

CLOSE SWITCH, FORCE Ee UPWARD

DONE BY MILLIKAN

$e = 1.6 \times 10^{-19}$ COULOMB

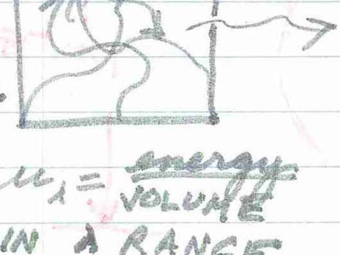
II) MASS OF ELECTRON



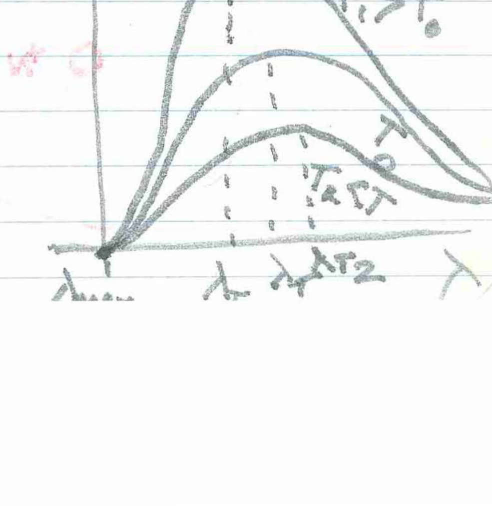
$$Be = \frac{mv}{R}$$

$$\frac{e}{m} = \frac{v}{BR}$$

III) BLACKBODY RADIATION



$T\lambda_{MAX} = K_{ONS.}$



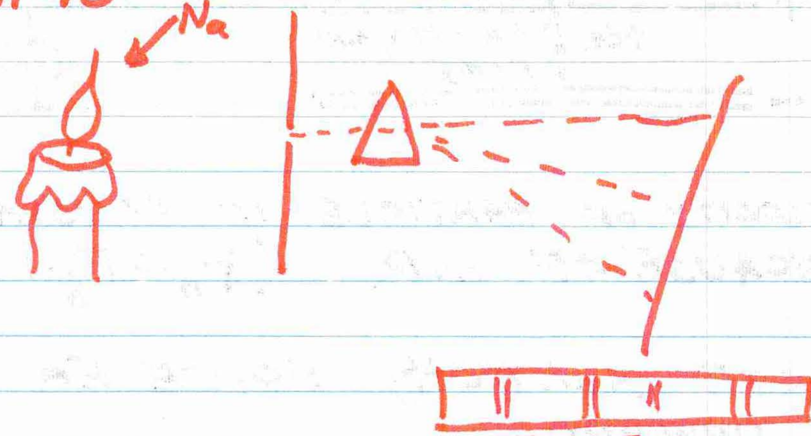
$\mu_\lambda = \frac{energy}{VOLUME}$ IN λ RANGE $\lambda \to \lambda + d\lambda$

EXPERIMENTAL OBSERVATIONS

$T\lambda_{max} = \text{CONSTANT}$

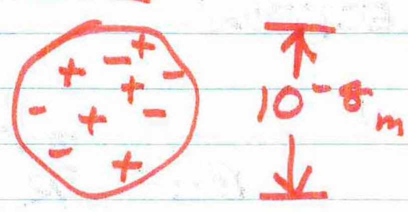
$\int_0^\infty u_\lambda \, d\lambda \sim T^4$

5-11-70



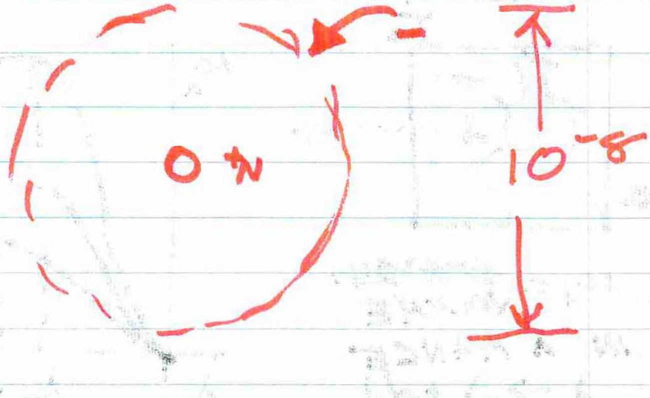
LINE SPECTRA

<u>THOMPSON</u> ATOM MODED

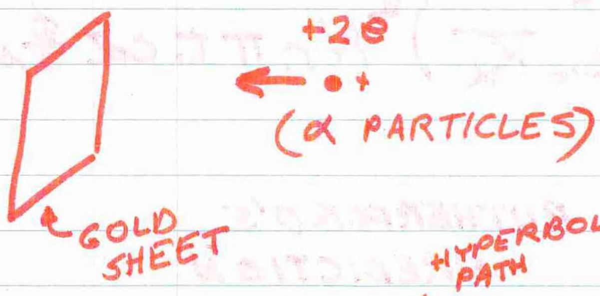


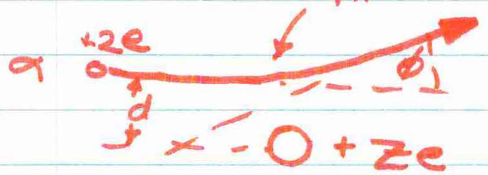
DOESN'T PREDICT LINE SPECTRA

<u>RUTHERFORD</u> ATOM MODEL



RUTHERFORD'S ALPHA-SCATTERING EXP. SUPPORTED ASSERTATION WITH MAGNETIC SCATTERING

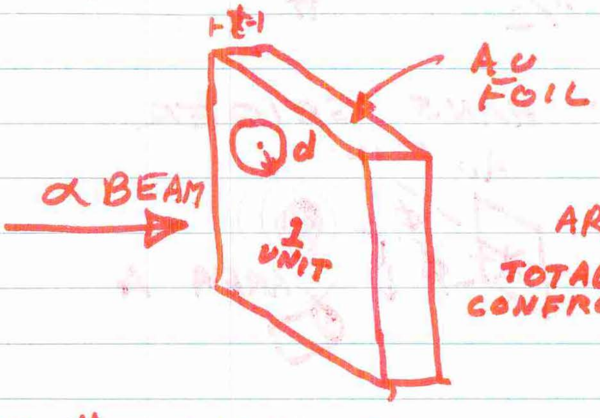




$$F = \frac{2Ze^2}{r^2 4\pi\epsilon_0}$$

d - IMPACT PARAMETER

$$\cot\frac{\phi}{2} = \frac{mv^2 d}{2Ze^2 \cdot 4\pi\epsilon_0}$$



AREA $= \pi d^2$

TOTAL AREA CONFRONTING BEAM $= N\pi d^2 t$

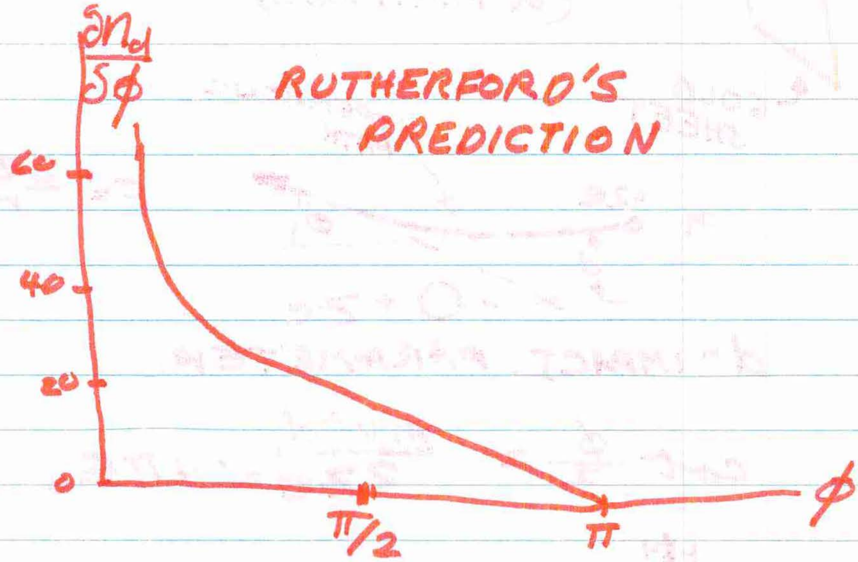
RATIO # α PARTICLES WITH IMPACT PARAMETERS $\leq d$ $(= n_d)$ TO TOTAL # α's $(= n)$

$$\Rightarrow \frac{n_d}{n} = \frac{N\pi d^2 t}{\text{UNIT AREA}} = N\pi d^2 t$$

$$n_d = nN\pi d^2 t$$

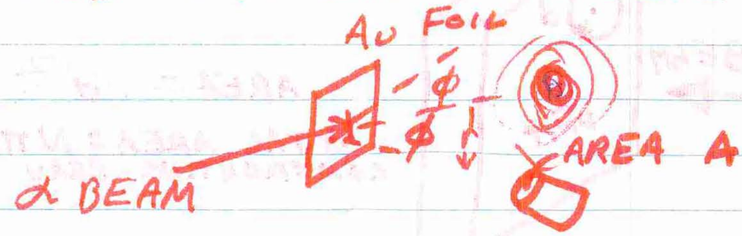
$$\delta n_d = 2nN\pi t d\,\delta t$$

$$\frac{|\delta n_d|}{\delta \phi} = \left(\frac{Ze^2}{4\pi\epsilon_0 K_\alpha}\right)^2 N n \pi t \cot\frac{\phi}{2}\sec^2\frac{\phi}{2}$$



RUTHERFORD'S PREDICTION

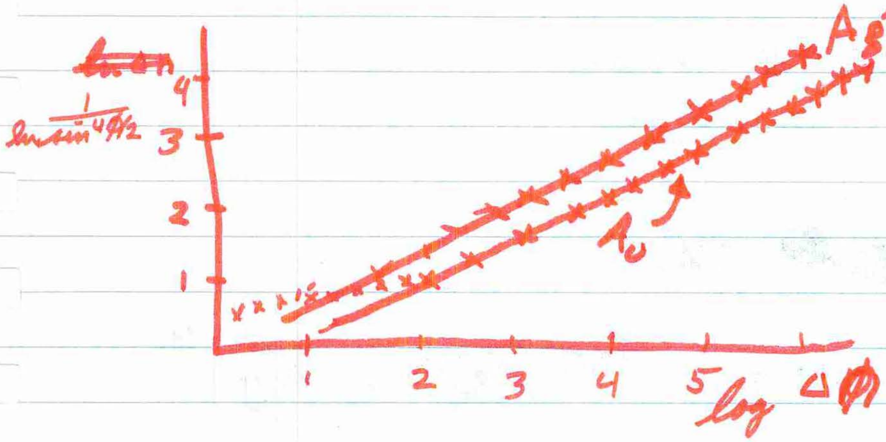
PERFORMED BY HANS GEIGER



FRACTION OF ALL α's SCATTERED AT $\phi = f$

$$f = \frac{A}{2\pi r^2 \sin\phi}$$

$$\Delta n = |\delta n_d| f = \left(\frac{ze^2}{4\pi\epsilon_0 K}\right)^2 \left(\frac{nNtA}{4R^2}\right) \cdot \sin^4 \phi/2$$

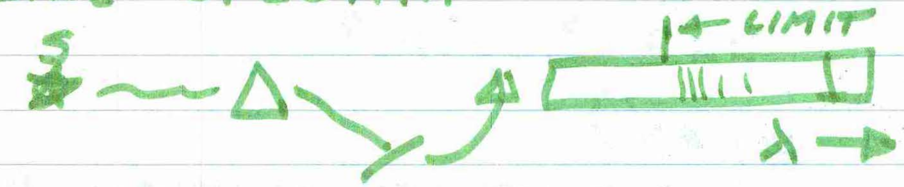


RUTHERFORD'S MODEL ACCEPTED

$R_{NUCLEUS} \simeq 10^{-14}$ m

5-18-70

LINE SPECTRA



ASSUMPTIONS

(1) PERMITTED ORBITS

$MVR = n\, h/2\pi$ (n=INTEGER)

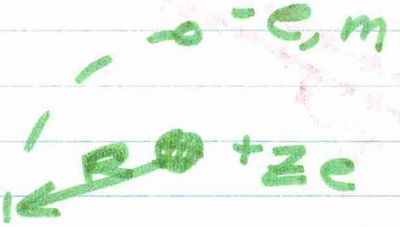
$\left(\frac{h}{2\pi} = \hbar\right)$

ELECTRONS IN THESE ORBITS COULDN'T RADIATE

2) RADIATION

$$hf = E_{INITIAL} - E_{FINAL}$$

HYDROGEN ATOM



$$F = -\,Ze^2/R^2 = \frac{mv^2}{R}$$

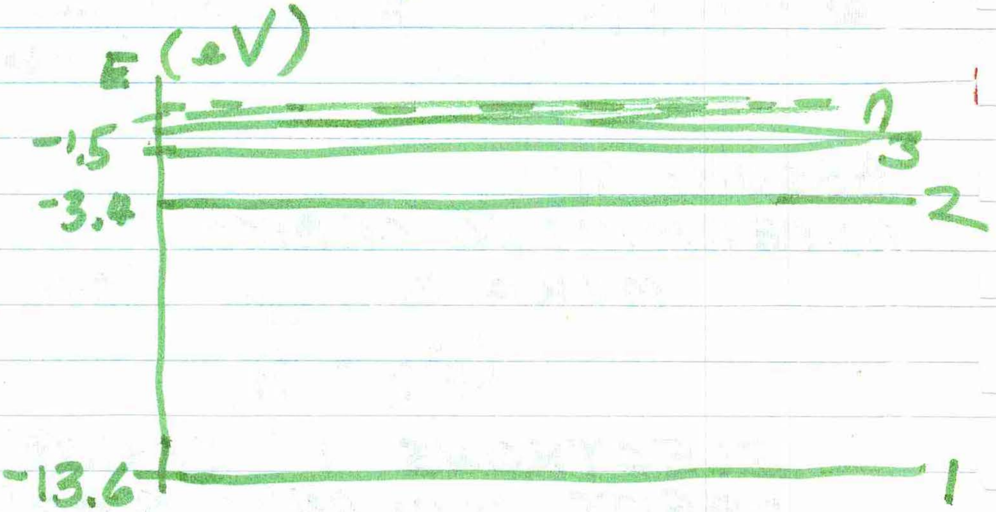
$$f \sim \frac{1}{r^{3/2}}$$

$$E = K + U \sim -1/r \Rightarrow$$ BOHR ASSUMPTION

$$E_n = -\frac{2\pi^2 (Ze^2)^2 m}{h^2 n^2}$$

FOR H:

$$E_n = 13.6/n^2$$



ELECTRONS MAY ONLY OCCUR ON THESE LINES, WHICH WILL ONLY OCCUR WHEN ELECTRONS CHANGE ORBITS

$$h f_{PHOTON} = E_2 - E_1 = 10.2\ EV$$
(GOING $n=1$ TO $N=2$)

$$h f_{MAX} = 13.6\ eV$$
↖ f IN ULTRAVIOLET

FIRST SEEN BY LYMEN

BALMER SAW LINES MANY YEARS BEFORE BOHR IN VISIBLE SPECTRUM.

PASHIN IN INFRARED
BRACKET BEYOND INFRARED

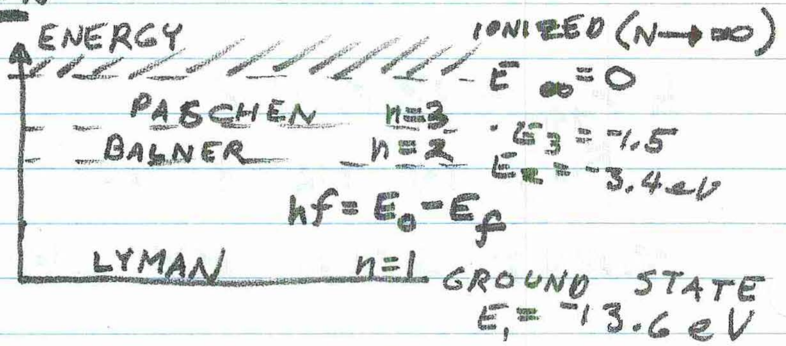
WILSON & SUMMERFELD

$$\oint p\,dq = n\hbar$$

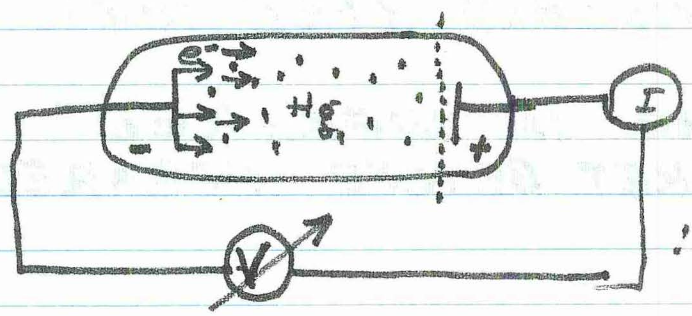
(ANOTHER ASSUMPTION)

5-21-70

HYDROGEN

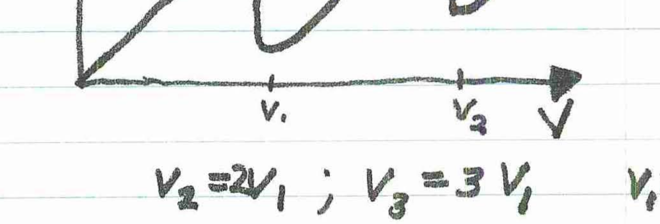


FRANCK-HERTZ SAID ELECTRONS SHOULD WORK OTHER WAY (ABSORPTION)



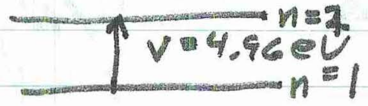
$K_e = e \cdot V$

$\therefore V = K/e$

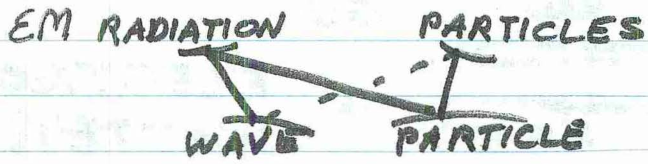


$V_2 = 2V_1$; $V_3 = 3V_1$ $\quad V_1 \approx 4.9$ eV

BOHR



WAVE-PARTCLE DUALITY OF LIGHT



deBROGLIE RELATION

GRAD STUDENT SAID $\lambda = h/p$

GRABBED BY DAVISSON & GERINER

POUND & REBKA (CHANGE IN FREQ OF PHOTON DUE TO GRAVITY)

JOSEPHESON TOLD P. & R. TO CONTROL Temp.

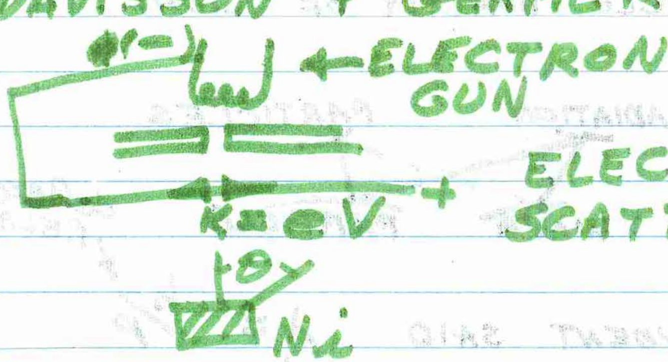
5-25-70

DE BROGLIE ⇒ $\lambda = h/p$

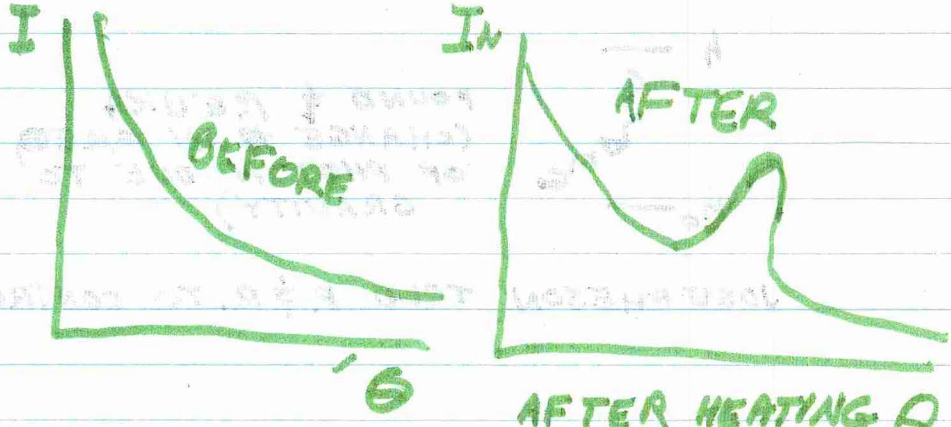
$p = \sqrt{2mK}$

SCHRÖDINGER ⇒

⇒ DAVISSON & GERMER



ELECTRON SCATTERING



AFTER HEATING NICKLE θ

THOUGHT IT WAS BRAGG SCATTERING:

$\lambda = 2d \sin \theta$

FOR NICKLE $\lambda = 1.65\,\text{Å}$

TRIED FOR DIFFERENT p & IT STILL WORKS, MAY USE ANY ELEMENT & ANY PARTICLE.

MATTER COULD ACT LIKE WAVES.

MOVING WAVE HAS

λ, f, v, $k = 2\pi/\lambda$; $f = \omega/2\pi$

FINITE WAVE

PHASE VELOCITY $\Rightarrow \omega/k$

GROUP VELOCITY $\Rightarrow d\omega/dk$

- MATTER WAVES ($v \neq$ [illegible]

CLASSICAL

$\Delta x \Delta k \approx 1$

INSERT DeBROGLIE & PLANK

$\Delta x \Delta p \approx h$

HEISENBERG UNC. PRI.

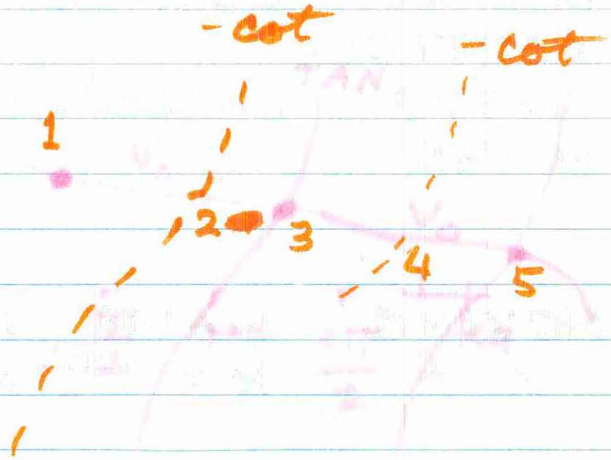
$\Delta x \Delta p \approx \hbar$

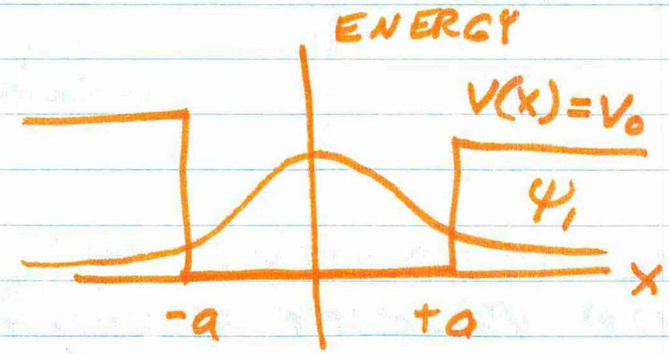
$(\Delta E \Delta t \approx \hbar)$

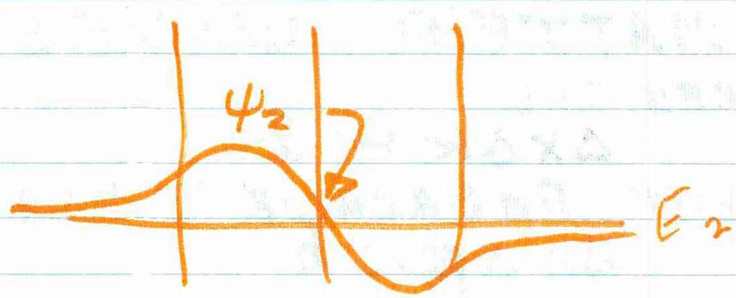
6-1-70

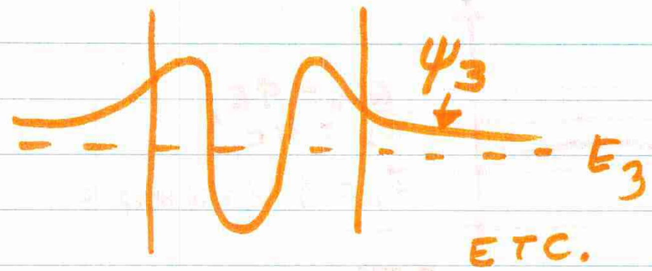
TAN ka

-COT ka









ETC.

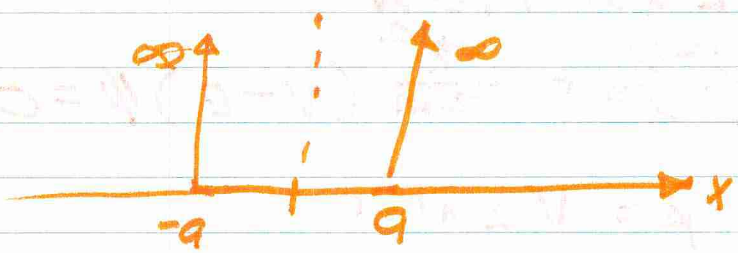
$|\psi|^2$ — ψ IS COMPLEX CONJ.
IS A PROBABILITY
PARTICLE WILL OCCURE
AT POINT

$$k = \sqrt{2mE}/\hbar$$

$$ka = \sqrt{2mE}\, a/\hbar$$

(E ↑) TOTAL ENERGY = KINETIC ENERGY

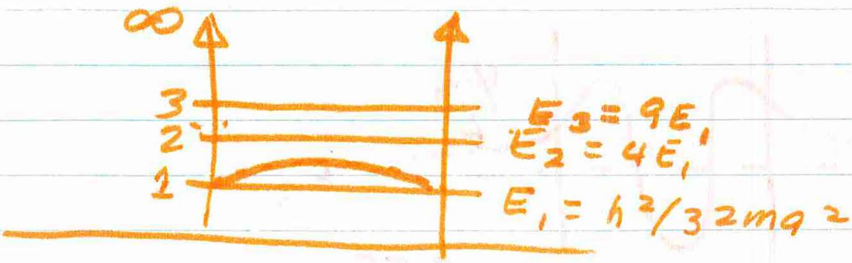
THE INFINITE WELL $(V_0 \to \infty)$



As $V_0 \to \infty$

$$ka = n\,\pi/2$$

$$\Rightarrow E_n = \frac{n^2\pi^2}{4}\,\frac{\hbar^2}{a^2}\,\frac{1}{2m} = \frac{h^2 n^2}{32\, m a^2}$$



$$\psi_1 = A_1 \cos \frac{\pi}{2a} x$$

$$\psi_2 = A_2 \sin \frac{2\pi}{2a} x = A_2 \sin \frac{\pi x}{a}$$

FROM ψ_1 . $A_1 = 1/\sqrt{a}$

ALL $A_n = 1/\sqrt{a} \quad \forall A$

$$\langle P_x \rangle = \text{MOM} = \int_{-a}^{+a} \psi_1^* \left(\frac{\hbar \partial}{j \partial x} \right) \psi_1 \, dx$$

FOR $V < V_0$, $\psi = 0$

SHR. EQUATION

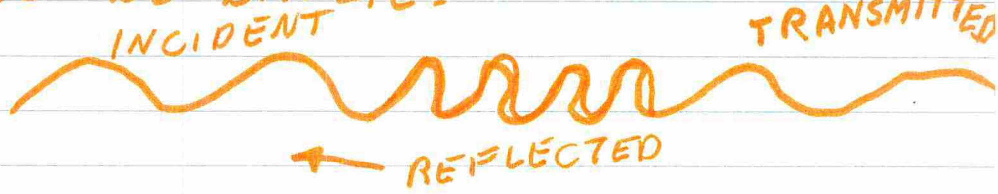
$$\frac{d^2\psi}{dx^2} + \frac{\hbar^2}{2m}(V-E)\psi = 0$$

$$p = \sqrt{2mK}$$

DE BROLIE

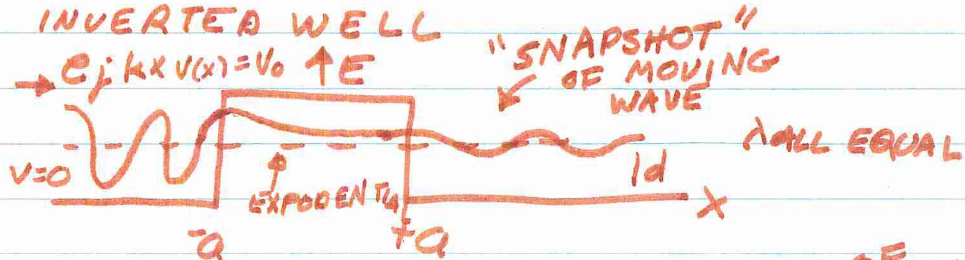
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

FOR $V > V_0$, ~~WAV~~ λ CHANGES ACCORDING TO DE BROLIE:



6-3-70

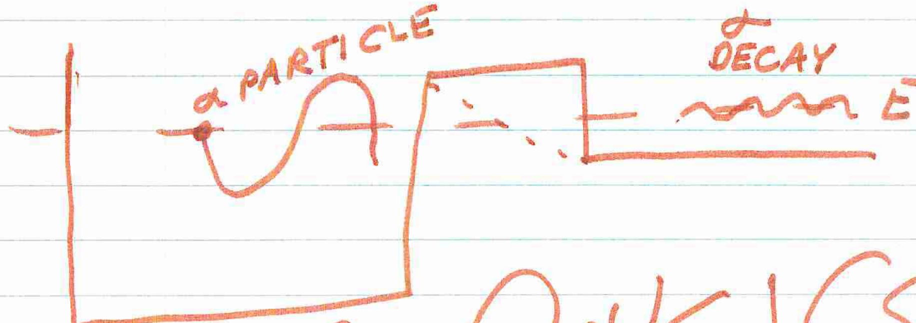
INVERTED WELL



$\int_V \psi^* \psi \, d\vec{r}$ = PROBABILITY OF FINDING PART IN V $\propto d^2$

QUANTIZATION FOR HUMP IS NO GO

<u>CALLED "TUNNELING"</u>



NO MORE PHYSICS!

EVER!

PHYSICS V - TEST 1

April 24, 1970

Name ROBERT J. MARKS

I. In the following complete the statement, choose the best response, or perform the indicated operation. (2 points each)

1. An inertial system is one in which IS MOVING AT A CONSTANT V RELATIVE TO REFERANCE (TWO FRAMES OF REFERENCE, DIFFERING IN V, IN WHICH PHYSICAL QUANTITIES CAN BE COMPARED)

2. According to Maxwell's equations, an electromagnetic wave in a vacuum propagates at speed c where c is the ratio of ELECRIC & MAGNETIC FIELDS.

16

3. The value of c in mks units is $3 \times 10^8 \frac{m}{sec}$.

4. What, where, or who is Algol? A BINARY STAR.

5. As far as classical E & M is concerned, what was the purpose of the aether? TO PROPOGATE LIGHT.

6. What was Fresnel's suggested explanation of Arago's null experimental result? THERE WAS "ETHER DRAG". THE EARTH DRAGGED THE ETHER.

7. Fresnel's explanation led to the following expression for the velocity V of light in a moving medium:

$$V = c/n + (1 - 1/n^2)\, v$$

where v is VELOCITY OF MOVING MEDIA.

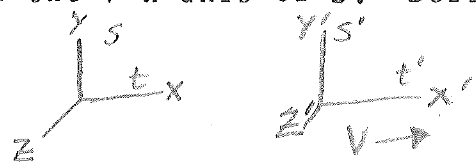
8. The classic experiment to detect the earth's motion through the aether was performed by MICHELSON ______.

9. The result of that experiment was NEGATIVE (NO SHIFT ON ROTATION).

10. Who performed the aether experiment that Einstein considered crucial? FIZEAU.

II. 1. Write down the equations of the Lorentz transformation which transform the coordinates of system S into those of system S' which is moving at a constant speed v down the + x axis of S. Define any symbols introduced.

(15 points)



$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

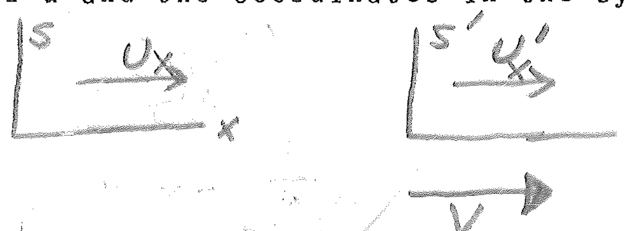
$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

2. Derive an expression for u', the x-component of the velocity of an object measured in the S'system in terms of u and the coordinates in the system S.

(15 points)



$$U_x = \frac{dx}{dt}$$

$$U_x' = \frac{dx'}{dt'} =$$

$$U_x' = \frac{dx'}{dt}\frac{dx}{dt'}\frac{dt}{dx}$$

$$= U^x$$

$$\frac{dx'}{dt} = -V$$

$$x = \gamma(x' - Vt')$$

$$\frac{dx}{dt'} = -V$$

ARG!

3. State the two basic assumptions of Einstein's theory of special relativity. (10 points)

1) THE SPEED OF LIGHT, AS MEASURED BY AN OBSERVOR, IS INDEPENDENT OF THE VELOCITY OF THE SOURCE

8

2) NO ABSOLUTE UNIFORM MOTION CAN BE DETECTED

4. Suppose you are moving with a speed of 0.75 c past a man who picks up an object and then sets it down. If you note that he held it for 9 seconds, how long does he say he held it? (15 points)

$V = .75\,c$

$$t = \frac{t'}{\sqrt{1 - V^2/c^2}} = \frac{t'}{\sqrt{1 - .562}} = \frac{t'}{\sqrt{.44}} = \frac{t'}{.663}$$

5

$$t = \frac{9}{.663} = 13.6 \text{ SECONDS}$$

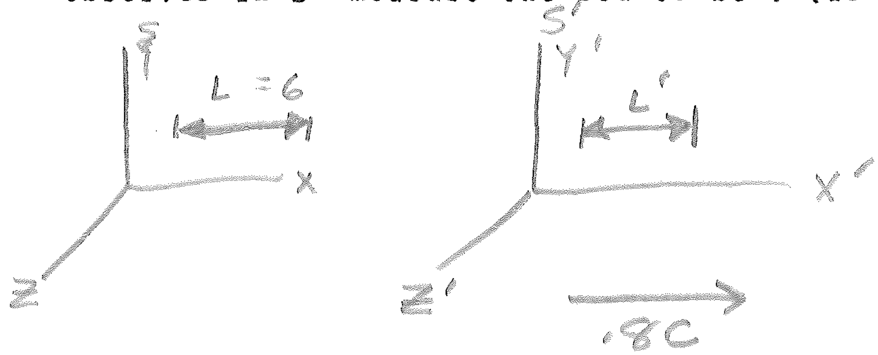
5. Does the non-relativistic formula ½ mv^2 over-estimate or under-estimate the kinetic energy of an object moving at speeds near c ? (Don't guess!)(10 points)

UNDERESTIMATED

THIS FORMULA IS DERIVED FROM THE CLASSICAL EXPRESSION F=ma, WHICH STATES IN EFFECT A MASS MAY ACHEIVE ANY VELOCITY. IN TRUTH, A MASS TAKES A MUCH LARGER AMOUNT OF ENERGY TO REACH VELOCITIE'S NEAR C, AT WHICH TIME THEY WILL HAVE A MUCH LARGER K.E. THAN mv^2

10

6. A measuring rod is at rest in system S'. If S' moves down the +x axis of S at 0.8c and an observer in S measures the rod to be 6 meters long, how long does an observer in S' measure the rod to be ? (15 points)



$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} = \frac{1}{\sqrt{1 - .64}} = \frac{1}{\sqrt{.36}} = \frac{1}{.6}$$

$$.6L = L'$$

$$L = 6/.6 = 10\,m$$

MAIL BOX # 385-2

PHYSICS V - TEST II

(60)

May 14, 1970

Name Bob Marks

I. In the following fill in the blank with the word or phrase that best completes the statement. (2 points each)

1. The Boltzmann constant is defined as R/NA (R=GAS CONSTANT; NA=AVAGADRO'S #).

2. According to the rule of Dulong and Petit, the molar specific heat of a solid is Cp = 3.8 .

3. An important assumption of our kinetic theory derivation of the gas law was that no forces act on the molecules except DURING COLLISIONS .

10

4. A classical analysis indicates the maximum kinetic energy of photoelectrons will be proportional to λ .

5. The experiment of Zartman and Ko was designed to ______ .

6. The average kinetic energy per degree of freedom for an ideal gas is $\frac{1}{2}kT$.

7. According to the Rayleigh-Jeans law, the amount of energy radiated by a black body at all wavelengths is $\propto \lambda^{-4}$.

8. The Thompson experiment was designed to measure e/m (CHARGE TO MASS RATIO) for electrons.

9. The magnitude of the electron's charge was measured in a classic experiment devised by MICKELSON .

10. A plot of stopping potential vs. light frequency for the photoelectric effect is a straight line of slope h/e (h=PLANCK'S CONST.) (e=ELECTRON CHARGE) .

II. (a) Write down the Maxwell distribution of kinetic energy for a gas with N molecules / unit volume. (5 points)

$$\phi(K) = 2N\sqrt{\frac{K}{\pi(kT)^3}}\, e^{-\frac{K}{kT}}$$

5

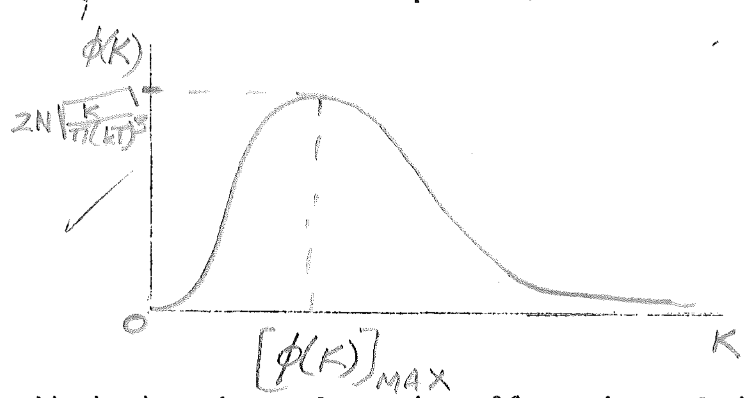
(b) Show by writing down an appropriate integral equation (complete with limits) how you might calculate the average kinetic energy of molecules that have a Maxwell distribution. (5 points)

$$\overline{K} = \int_0^\infty K\,\phi(K)\,dK \Big/ \int_0^\infty \phi(K)\,dK$$

5

20

(c) Sketch on the axes below the Maxwell kinetic energy distribution ϕ (K) vs. K. (5 points)



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III. (a) If you studied the photoelectric effect in calcium, you would find for light of the indicated frequencies the following stopping potentials:

frequency	stopping potential
$1.18 \times 10^{15}\ sec^{-1}$	1.95 volts
0.96×10^{15}	0.98
0.82×10^{15}	0.50
0.74×10^{15}	0.14

5

Plot these data carefully on the attached graph paper. Determine from your plot the

(1) minimum frequency
(2) maximum wavelength
(3) the work function in volts
(4) the slope in joule-sec. (20 points)

(b) State two results of the photoelectric effect experiment that cannot be explained by classical physics. (5 points)

1) SUPPORT OF THE EXISTANCE OF PHOTONS

2) LIGHT INTENSITY DOES NOT EFFECT THE STOPAGE VOLTAGE

3

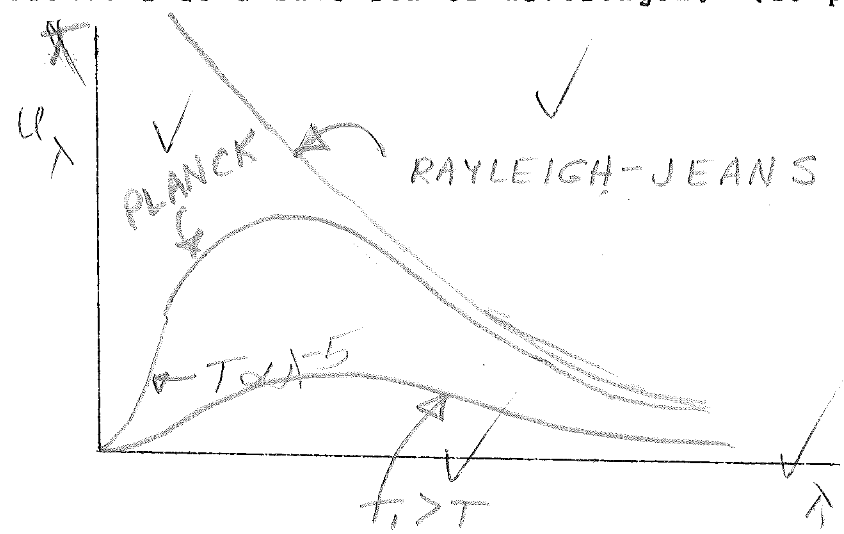
(c) Using an energy diagram such as was used in class, outline briefly Einstein's derivation of the photoelectric equation. (10 points)

0

12

IV. (a) On the axes below, label and sketch carefully the theoretical curves predicted by the Rayleigh-Jeans and Planck equations for radiation emitted by a black body at temperature T as a function of wavelength. (10 points)

9



(b) On the plot in (a) sketch also the experimental curve measured for a black body at temperture T^1 which is less than T. (10 points)

10

(c) Planck showed that the difficulty with classical theory was concerned with the calculation of the average energy per degree of freedom of the oscillators. Write down (don't derive) the classical and quantum expressions for the average energy. (10 points)

PLANK SAID

$$E = \sum_0 E_n e^{-E_n/kT}$$

WHERE $E_n = nhf$ (n IS AN INTEGER)

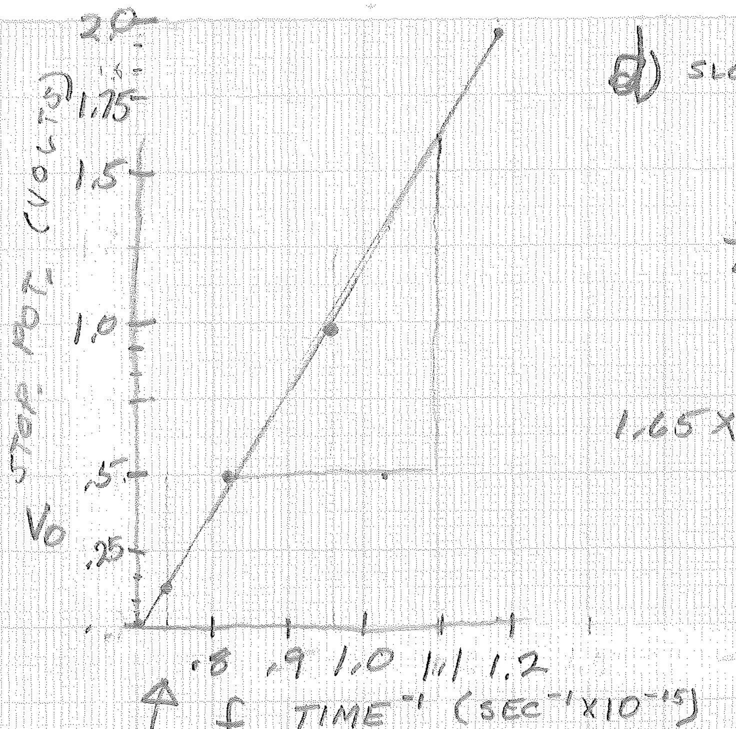
18

8

$$\therefore \bar{E} = \frac{hc/\lambda}{(e^{hc/\lambda kT} - 1)}$$

CLASSICAL SAID

$$E = \int (E_n e^{-E_n/kT}) d(E_n) = \frac{1}{2} kT$$



a) SLOPE = $\frac{4.45 \text{ V-SEC}}{2.7 \times 10^{+15}}$ ✓

$= 1.65 \times 10^{-15}$ V-SEC

$\frac{6.63 \times 10^{-34} \text{ J-SEC}}{4.14 \times 10^{-15} \text{ V-SEC}} = 1.6 \times 10^{-19} \frac{J}{V}$

$1.65 \times 10^{-15} \text{ V-SEC} \frac{J}{1.6 \times 10^{-19} V}$

$= 1.03 \times 10^{4}$ J-SEC ← WRONG!

~~b) $V = \frac{1.65 \times 10^{-15}}{T} + V_{INT}$~~

~~$0 = \frac{1.65 \times 10^{-15}}{.7 \times 10^{15}} +$~~

$eV_0 = hf - \phi \quad (\phi = hf_0)$

$V_0 = \frac{h}{e} f - \phi/e$

$f_0 = .7 \times 10^{+15}$ HZ = MINIMUM f

$V_0 = \frac{h}{e} f - \frac{hf_0}{e}$

SLOPE = h/e

Box No. 385-2

(45)

June 5, 1970

PHYSICS V - TEST III
(100 points)

Name BOB MARKS

(You may find some of the following information useful:

$e = 1.6 \times 10^{-19}$ coulombs, $h = 6.62 \times 10^{-34}$ joule-seconds, $c=3\times10^{8}$ m/sec,

electron mass = 9.11×10^{-31} kgm, one Angstrom = 10^{-10} m)

I. (a) An electron is accelerated from rest through a potential difference of 250 volts. What is the deBroglie wavelength associated with this electron? (10)

$$\lambda = \frac{h}{p} = \frac{h}{\frac{1}{2}mv^2} = \frac{6.62\times10^{-34}\ J\,sec}{\frac{1}{2}m_e v_e^2}$$

4 ?

(b) Suppose the potential difference in part (a) were changed to 25,000 volts. The correct answer would then be (circle the right one)

1. 100
2. 1/100
3. 10
4. 1/10
5. a little less than 1/10
6. a little more than 1/10

0 X

times the answer to part (a). (5)

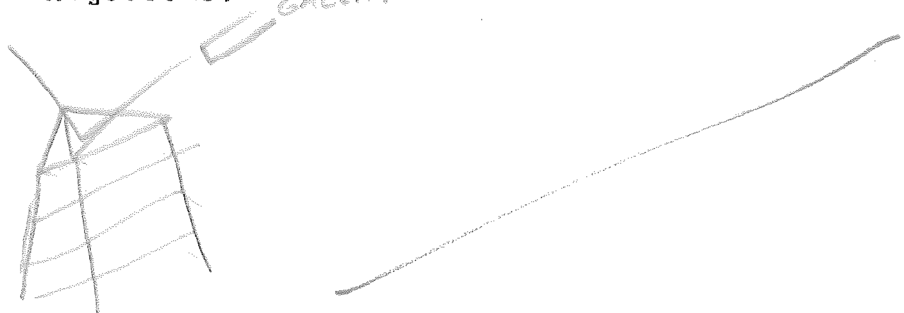
(c) Explain the reason for your choice in part (b). (10)

$V \propto v^2$

10 $V_0 \propto v_0^2$

$\frac{1}{100} = \frac{1}{(10)^2} \Rightarrow v$ WOULD INCREASE BY A FACTOR OF 10

(d) The electrons in part (a) are to be used in a Davisson and Germer experiment. At what angle would you expect the first maximum of the diffraction pattern to occur if the crystal used has a spacing between planes of 1.985 Angstroms? (10)



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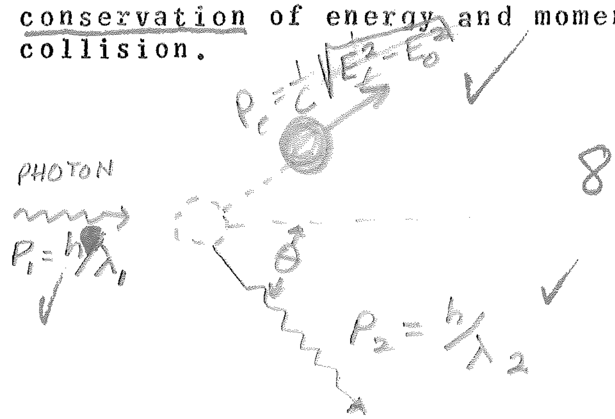
II. (a) Diagram and label carefully a collision between a high energy photon and a free electron (Compton effect) and write down the equations for conservation of energy and momentum in the collision. (15)



8

$$\lambda_1 - \lambda_2 = \frac{h}{mc}(1 - \cos\theta)$$

(b) Which of the quantities in the equation in part (a) would be measured in a Compton effect experiment? (5)

2 Movement of the change in frequency

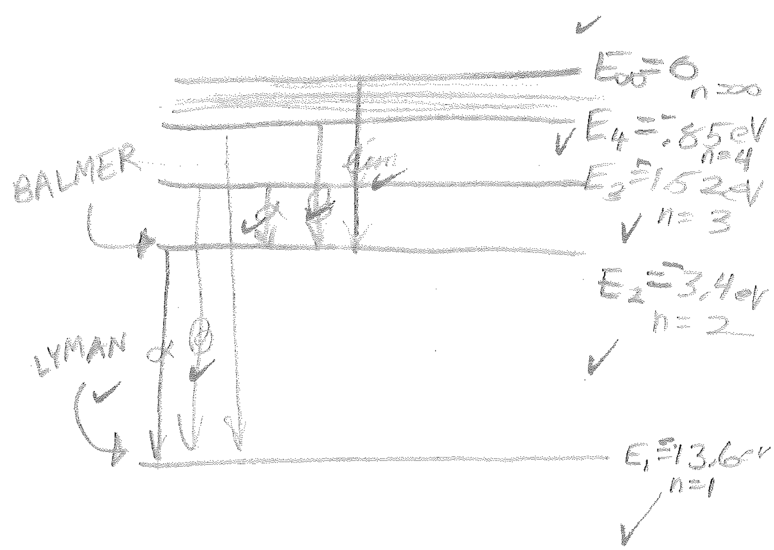
III. (a) In the space at the right sketch carefully the energy levels for hydrogen (according to Bohr's theory) for n = 1,2, 3,4, and ∞. Show n and E values(in eV). (10)

$E_n = \frac{E_1}{n^2}$

$E_{1(H)} = 13.6$ eV

✓

9



(b) On the diagram above shown by arrows the following spectral lines: Lyman α, Lyman β, Balmer α, Balmer β, and Balmer limit. (5)

5

(c) From information in the diagram compute the wavelength of the Balmer α line. (5)

$hf = E_2 - E_1$ $\quad f =$

$= 3.40 - 1.52$

$h\lambda = \frac{1.88}{f}$

1

IV. (a) Write down the five postulates of Schrödinger's wave mechanics. (20)

$f = E/h$; $\lambda = h/p$; (PARTICLES DISPLAY WAVE PROP.)

$\psi'(x)$ MUST BE CONTINUOUX

✓ $\psi(x) \to 0$ AS $x \to \infty$

(DeBroglie's postulate) (CONT.)

4

ANALOGOUS TO DERIVATION OF WAVE PROPOGATION IN AN E FIELD

(b) The Schrödinger equation is the wave mechanical equivalent of what classical statement? (5)

The differential equation something like,

$\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{c^2} \sim$

which described propagation of electromagnetic waves in an E field.

I) SPECIAL RELATIVITY

A) THE EINSTEIN POSTULATES

1) SPECIFICALLY:

a) ABSOLUTE, UNIFORM MOTION CAN'T BE DETECTED

b) THE SPEED OF LIGHT IS INDEPEN. OF THE SOURCE

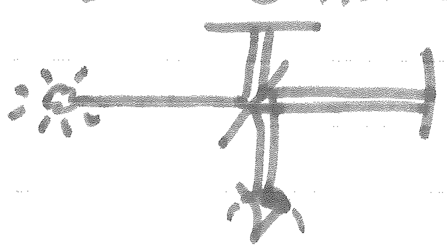
2) SUPPORTED FROM MAXWELL'S EQUATIONS

3) SUPPORTED: NO PROOF OF THE EXISTANCE OF ETHER

B) MICHELSON-MORELY EXP.

1) FIZEAU'S EXP. - NOT EXACT ENOUGH

2) MICHELSON'S INFEROMETER



a) OBSERVE INTERFERENCE PATTERN, ROTATE 90°, OBSERVE ANOTHER, THUS COMPUTING LIGHT DRAG IN ETHER

b) NO SHIFT OBSERVED UPON ROTATION

3) ABERRATION OF LIGHT

a) TELESCOPE MUST BE TILTED TO GET IMAGE OF STAR AT ANGLE FROM STAR'S TRUE POSI.

b) EARTH DOESN'T DRAG ETHER

C) KINEMATICAL CONSEQUENCES OF EINSTEIN'S POSTULATES

1) TIME DILATION

$$\Delta t = \Delta t' / \sqrt{1 - V^2/c^2}$$

2) LENGTH CONTRACTION

$$L_0 = L' / \sqrt{1 - V^2/c^2}$$

3) LIFETIMES (HALF LIFE)

$$\tau = \tau' / \sqrt{1 - V^2/c^2}$$

D) SIMULTANEITY & CLOCK SYNCHRONIZATION

1) TWO EVENTS IN A REFERENCE FRAME ARE SIMULTANEOUS IF THE LIGHT SIGNALS FROM THE EVENT REACH AN OBSERVOR HALFWAY BETWEEN THE EVENTS AT THE SAME TIME.

2) EVENTS THAT ARE SIMULTANEOUS IN ONE FRAME ARE NOT IN ANOTHER

3) TWO CLOCKS SEPARATED BY L_0 & SYNCHRONIZED IN REST FRAME ARE UNSYNCHRONIZED IN A REFERENCE FRAME MOVING WITH SPEED V. THE CHASING CLOCK IS AHEAD BY VL_0/c^2

4) PERPENDICULAR WAVES ARE THE SAME MEASURED IN ANY SYSTEM

E) THE LORENTZ TRANSFORMATION

1) $x = \gamma (x' + Vt')$; $x' = \gamma (x - Vt)$

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}}$$

2) $t = \gamma (t' + Vx'/c^2)$; $t' = \gamma (t - Vx/c^2)$

3) CLOCK SYNCHRONIZATION

a) $t_A' - t_B' = \gamma (x_2 - x_1) \frac{v}{c^2} = L_0 v / c^2$

b) $L_0 = \gamma (x_2' - x_1') = x_2 - x_1$

F) THE VELOCITY TRANSFORMATION

1) a) $u_x = (u_x' + v) / (1 + v u_x' / c^2)$

b) $u_y = u_y' / \gamma (1 + v u_x' / c^2)$

c) $u_z = u_z' / \gamma (1 + v u_x' / c^2)$

2) $|u| = (u_x^2 + u_y^2 + u_z^2)^{\frac{1}{2}}$

G) THE DOPPLER EFFECT

1) $\lambda = \left(\frac{c \pm v}{f_0} \right) \frac{\Delta t}{\Delta t_0} = \left(\frac{c \pm v}{f_0} \right) \gamma$

2) FREQUENCY

a) $f = f_0 / \gamma$

b) MOTION MAKING ANGLE θ BETW. SOURCE & OBSERVOR

$f = [1 \pm (v/c) \cos \theta] \gamma$

H) RELATIVISTIC MOMENTUM

1) $P = mu / \sqrt{1 - u^2/c^2}$

2) RELATIVISTIC MASS $m(u)$

$m / \sqrt{1 - u^2/c^2}$

I) RELATIVISTIC ENERGY

1) $K.E. = T = \gamma mc^2 - mc^2$

2) $E = T + mc^2 = \gamma mc^2 = \gamma E_0$

$E_0 = mc^2 =$ REST ENERGY

3) $E^2 = P^2 c^2 + E_0^2 \quad (\gamma = (1 - u^2/c^2)^{-\frac{1}{2}})$

4) FOR TWO COLLIDING PARTICLES OF EQUAL MASS

a) $M_2 = 2m_1 / \sqrt{1 - u^2/c^2}$

b) REST MASS INCREASES

$\Delta m = 2m_1(\gamma - 1)$

c) ORIGINAL KINETIC ENERGY IN S TRANSFORMED INTO REST ENERGY

$T = c^2 \Delta m_1$

d) LOSS OF ENERGY SAME IN S & S'

J) MASS & BINDING ENERGY

1) $1u / 1.66 \times 10^{-27} kg = 1$

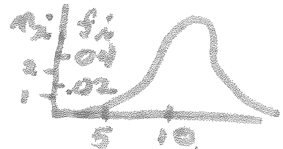
2) $1 eV / 1.602 \times 10^{-19} joule = 1$

3) $1 g \times c^2 / 5.61 \times 10^{32} eV = 1$

4) $1 u \times c^2 = 931.5 MeV$

5) THE MASS OF A NUCLEUS ISN'T THE SUM OF THE MASSES OF IT'S PARTS

II) THE KINETIC THEORY OF MATTER



A) DISTRIBUTION FUNCTION

1) $\sum_i f_i = 1 \Rightarrow$ NORMALIZATION FUNCTION

2) AVERAGE:

$$\overline{S} = \frac{1}{N} \sum_i s_i n_i = \sum_i s_i f_i$$

3) AVERAGE SQUARE:

$$\overline{S^2} = \sum_i s_i^2 f_i$$

4) STANDARD DEVIATION

a) $\sigma = \left(\sum_i (s_i - \overline{S})^2 f_i \right)^{\frac{1}{2}}$
$= \left(\overline{S^2} - \overline{S}^2 \right)^{\frac{1}{2}}$

b) 66% LIE BETWEEN $\overline{S} \pm \sigma$

B) PRESSURE OF A GAS

1) ASSUMPTIONS

a) LARGE # (N) MOLEC; MAKING ELASTIC COLLISIONS

b) MOLEC. SEPARATED BY LARGE DISTANCES COMP WITH DIA; EXERT NO FORCE ON EACH OTHER

c) NO PREFERED POSITION OR V FOR MOLEC

2) $P_x = 2nm \int_0^\infty V_x^2 f(V_x)\, dV_x = nm \overline{V_x^2}$

a) $n = N /$ VOLUME

b) $m =$ MASS OF MOLECULE

3) $\overline{V_x^2} = \overline{V_Y^2} = \overline{V_Z^2} = \frac{1}{3} \overline{V^2}$

$\therefore P = \frac{1}{3} nm\overline{v^2}$

4) $PV = \frac{2}{3} N \overline{E_K} = \frac{2}{3} U = NRT$

a) $\overline{E_K} =$ AVERAGE KINETIC ENERGY

b) $R = 1.99$ CAL/°K-MOLE $= 8.31$ J/°K-MOLE

5) $\overline{E_K} = \frac{3}{2} \frac{R}{N_A} T = \frac{3}{2} kT$

K = BOLTZMAN'S CONSTANT $\equiv R/N_A$

$= 1.38 \times 10^{-23}$ J/°K $= 8.63 \times 10^{-5}$ eV/°K

6) $V_{RMS} = (3RT/\mathcal{M})^{\frac{1}{2}}$

$\mathcal{M}$ = MASS OF A MOLE

7) $C_V = \frac{3}{2} R = 2.98$ CAL/MOLE $= \frac{dU}{dT}$

C) THE MAXWELL-BOLTZMANN DISTRIBUTION

1) $f(V_x) = \sqrt{\frac{m}{2\pi kT}}\, e^{-mV_x^2/2kT}$

2) $F(V_x, V_y, V_z) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-m(V_x^2+V_y^2+V_z^2)/2kT}$

D) OTHER DERIVATIONS (WERE DONE)

E) EQUAPARTITION THEOROM AND HEAT CAPACITIES OF GASES & SOLIDS

1) THE AVERAGE ENERGY OF $\frac{1}{2}kT$ IS ASSOCIATED WITH EACH CO-ORDINATE OR MOMENTUM COMPONENT APPEARING IN THE ENERGY AS A SQUARED TERM

$\overline{E} = \#\left(\frac{1}{2}kT\right)$

2) DULONG PETIT: MOLAR HT CAPACITIES OF MOST SOLIDS IS ABOUT

a) 6 CAL/°K-mole $\approx 3R$

b) TREAK ~~MOLEC.~~ ATOMS AS ATTATCHED BY SPRINGS (K = SPRING CONSTANT)

$E = \frac{1}{2m}(P_x^2 + P_y^2 + P_z^2) + \frac{1}{2}(K_1X^2 + K_2X^2 + K_3X^2)$

c) AT HIGH TEMP., ALL SOLIDS OBEY LAW

F) TRANSPORT PHENOMENA

1) ℓ = MEAN FREE PATH

a) $\ell = 1/n\sigma$

b) $\sigma = \pi\left(\frac{1}{2}(d_1 + d_2)\right)^2$

2) $S = \eta \frac{du}{dz}$ VISCOUS STRESS

a) η = COEFF. OF VISCOSITY

b) $S = \frac{1}{3} n \bar{v} m \ell \, du/dz$

G) BROWNIAN MOTION & THE RANDOM WALK PROBLEM

III) THE QUANTIZATION OF ELECTRICITY, LIGHT, & ENERGY

A) MEASUREMENT OF ELECTRIC CHARGE

1) FARADAY = $N_A e$

2) TOWNSEND - ESTIMATED $e = 1 \times 10^{-19}$ coul

3) THOMPSON & WILSON

4) MILLIKAN & HIS OIL DROP

a) CHARGES OCCUR IN MULTIPLES OF e

b) LITTLE OIL DROPS IN A CAPACITOR

B) MEASUREMENT OF e/m

1) ZEEMAN - ROTATITING CHARGE YIELDING ELECTRO-MAGNETIC WAVES

2) J.J. THOMPSON

a) WHEN B IS PLACED $\perp$ TO PATH, PARTICLES MOVE IN CIRCLE:

① $R = mv/qB$

② $\frac{m}{e} = B^2R^2Q/2W$
$(W = N\frac{1}{2}mv^2)$

b) J.J. THOMPSON EXPERIMENT

① $\perp$ B & E FIELDS

3) BLACKBODY RADIATION

a) CLASSICAL (RAYLEIGH-JEANS LAW)

$$f(\lambda)d\lambda = 8\pi\lambda^{-4}d\lambda$$

b) MAX PLANCK

① ASSUMED $\bar{E} = \sum_n E_n e^{-E_n/kT}$

$(E_n = nhf) \Leftrightarrow n \Rightarrow$ INTEGER

② $\therefore \bar{E} = {}^{hc/\lambda}/(e^{hc/\lambda kT} - 1)$

③ $\Rightarrow f(\lambda) = 8\pi hc\lambda^{-5}/(e^{hc/\lambda kT} - 1)$

④ h (= PLANK'S CONSTANT)

$h = 6.626 \times 10^{-34}$ J SEC $= 4.136 \times 10^{-15}$ eV SEC

C) THE PHOTOELECTRIC EFFECT

a) LENARD

① $eV_0 = \frac{1}{2}mV^2 = hf - \phi$

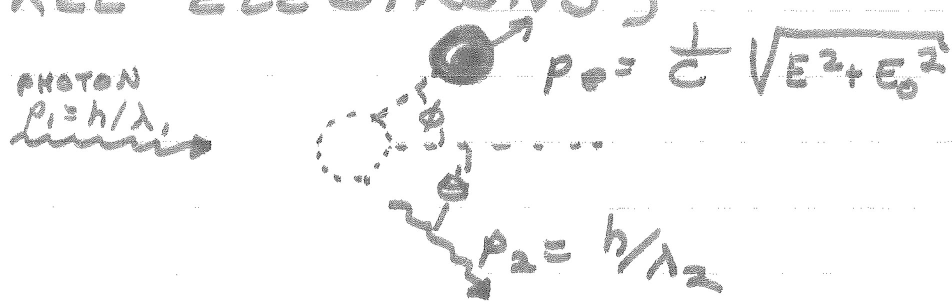
(ϕ = ENERGY NESCESARY TO REMOVE e FROM SURFACE)

② $\phi = hf_0 = hc/\lambda_t$

a) λ_t = THRESHOLD ~~FRE~~ λ

b) $hc = 1.24 \times 10^4$ eV-Å

D) X RAYS & THE COMPTON EFFECT (SCATTERING OF X RAYS BY FREE ELECTRONS)



1) $\lambda_2 - \lambda_1 = \frac{hc}{E_0}(1-\cos\theta) = \frac{h}{mc}(1-\cos\theta)$

2) h/mc = COMPTON λ

= .0243 Å FOR $\dot{e}$

IV) THE NUCLEAR ATOM

A) EMPIRICAL SPECTRA FORMULAS
(DATA COLLECTED ON EMISSION OF LIGHT BY ATOMS IN A GAS WHEN EXCITED ELECTRICALLY OR IN A FLAME)

1) JOHANN BALMER - LINES COULD BE REPRESENTED BY (FOR H)

$$\lambda = b \frac{m^2}{m^2-4} \qquad m = 3, 4, 5$$

$b) = 3645 \times 10^{-8} cm$

2) RYDBERG:

$$1/\lambda = \left(\frac{1}{\lambda}\right)_{\infty} - \frac{R}{(m+u)^2} = \text{WAVE \#} = \bar{f}$$

a) R = RYDBERG'S CONSTANT $= 1.10 \times 10^5 cm^{-1}$

b) $f = \frac{c}{\lambda} = c\bar{f}$

3) RITZ:

$$\bar{f} = R\left[\frac{1}{(m+A)^2} - \frac{1}{(n+B)^2}\right]$$

a) m & n ARE INTEGERS

b) A & B CONSTANTS

4) J.J. THOMPSON - "PLUM PUDDING" MODEL OF THE ATOM. UNABLE TO SUPPORT

B) RUTHERFORD SCATTERING (STUDENTS GEIGER & MARSDEN)

1) SHOT α PARTICLES THRU SCREEN, SOME DEFLECTED OVER 90°

2) DISPROVED THOMPSON'S ATOM MODEL

C) BOHR MODEL OF THE ATOM

1) RADIATION EMITTED BY ELECTRONS CHANGING ORBITS, RATHER THAN ROTATING

$$hf = W_{ORBIT\ 1} - W_{ORBIT\ 2}$$

2) DERIVED:

$$f = \frac{1}{2}\,\frac{kZe^2}{h}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

a) $r_n = n^2 r_0$

b) Ze = CHARGE ON NUCLEUS

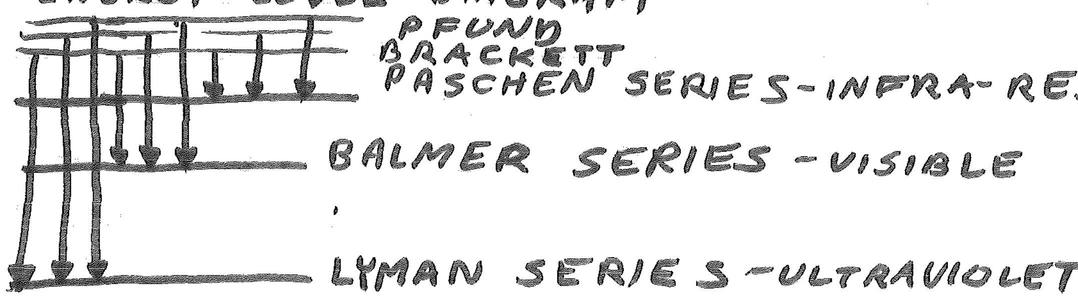
3) CORRESPONDANCE PRINCIPLE - FOR LARGE QUANTUM #'S (n), CLASSICAL & QUANTUM CALCULATIONS SHOULD BE THE SAME.

4) DERIVED:

a) $W_n = -\left(\frac{2\pi K^2 e^4 m}{h^2}\right)\frac{Z^2}{n^2} = -Z^2 \frac{E_1}{n^2}$ FOR H

$E_1 = 13.6$ eV (IONIZATION OR BINDING ENERGY)

b) ENERGY LEVEL DIAGRAM



5) IN AN ATOM

a) $E_K = \frac{p^2}{2\mu}$; $\mu = \frac{mM}{m+M}$ (M = MASS OF NUC.)

b) μ IS CALLED REDUCED MASS

c) REASON FOR VARIATION OF RYDBERG CONSTANT FROM ELEMENT TO ELEMENT

6) BOHR SAID (POSTULATED)

a) L (= ANG MOM. OF e) = $n\hbar$

b) f (OF RADIATION) = $(W_1 - W_2)/h$

D) X-RAY SPECTRA

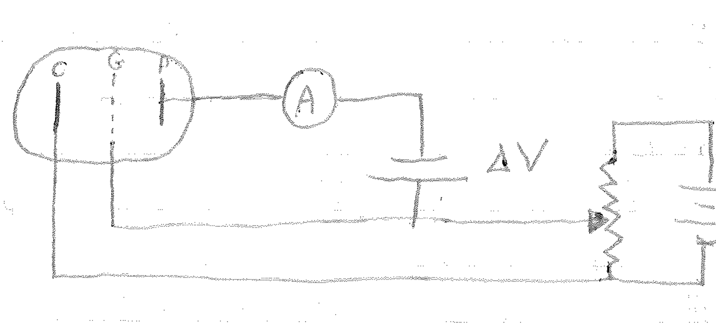
1) CRYSTAL SPECTROMETRY DEVELOPED BY THE BRAGGS

2) MOSELEY, PLOTTED $f^{\frac{1}{2}}$ VS Z (ATOMIC #)

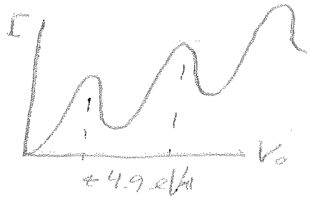
$f^{\frac{1}{2}} = A(Z-b)^2$) USING X-RAYS

(A & b CONSTANTS FOR EACH LINE)

E) THE FRANCK-HERTZ EXP.



1) ELECTRONS ACCELERATED FROM HEATED CATHODE THRU GRIDD (V_0)

2) 

CORRESPONDS TO EXCITATION OF DIFFERENT LEVELS IN Hg

F) WILSON-SOMMERFIELD QUANTIZATION RULE

$$\oint P\,dq = nh$$

1) P = COMPONENT OF MOMENTUM, SUCH AS P_x

2) q = CORRESPONDING DIRECTIONAL VALUE (AS q = x)

V) ELECTRON WAVES

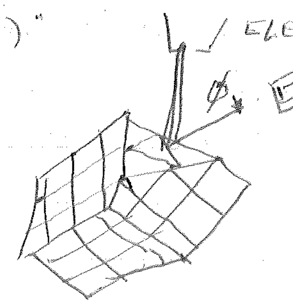
A) THE DE BROGLIE RELATIONS (ELECTRON WAVES)

1) $f = E/h$; $\lambda = \frac{h}{p}$ (p = MOMENTUM)

2) DEVELOPED BY SCHROEDINGER

B) MEASUREMENTS OF ELECTRON WAVELENGTHS

1) MADE BY DAVISSON & GERMER

2) 

MEASURED AS 1.67 Å

C) CLASSICAL WAVE EQUATIONS

1) FOR A STRING:

$$(\rho A)\frac{\partial^2 Y}{\partial t^2} = T\frac{\partial^2 Y}{\partial x^2}$$

ANSWER

$$Y(x,t) = Y_0 \cos(kx - \omega t) \quad \text{OR} \quad Y(x,t) = Y_0 e^{i(kx-\omega t)}$$

2) USEFULL PARAMETERS

a) $k\lambda = 2\pi$

b) $\omega = 2\pi f$

D) WAVE PACKETS

E) ELECTRON WAVE PACKETS

1) $\psi(x,t)$ (= WAVE PACKET)

$= \int g(k) \cos(kx - \omega t) dk$

($g(k)$ CONTAINS RANGE OF WAVE NUMBERS CENTERED ABOUT k_0)

2) V (= VELOCITY OF WAVE PACKET) $\doteq p_0/m$

F) THE PROBABILISTIC INTERPRETATION OF THE WAVE FUNCTION (RELATION OF $\psi(x,t)$ TO ELECTRON'S LOCATION)

1) $\mathcal{E}^2 \propto$ PROBABILITY OF PHOTON BEING IN UNIT ~~AREA~~ VOLUME

2) $\psi^2 \propto$ PROBABILITY OF ELECTRON BEING IN UNIT VOLUME

($\psi^2 dx \,\overline{\overline{\propto}}$ ELECTRON BEING IN INTERVAL dx)

G) THE UNCERTAINTY PRINCIPLE (HEISENBURG)

$\Delta x \, \Delta p \gtrsim \hbar \approx \Delta E \Delta t$

H) PARTICLE WAVE DUALITY

BOHR'S PRINCIPLE OF COMPLEMENTARITY - PARTICLE ASPECTS & WAVE ASPECTS COMPLEMENT EACH OTHER

I) CONSEQUENCES OF THE UNCERTAINTY PRINCIPLE

A) PARTICLE IN SMALL SPACE MUST HAVE K.E. (NOT OBSERVABLE FOR MACROSCOPIC)

B) BUNCHES ELSE

J) SUMMERY

A) ELECTRONS AND OTHER PARTICLES DISPLAY WAVE

PROPERTIES

a) $E = hf = \hbar\omega$

b) $p = h/\lambda = \hbar k$

2) IF λ IS SMALL, e & LIGHT WAVE PROPERTIES CAN BE NEGLECTED.

3) $(\text{WAVE FUNCTION})^2 \propto$ PROBABILITY

VI) THE SCHROEDINGER EQUATION IN ONE DIMENSION

A) PLAUSIBILITY ARGUMENT FOR SCHROEDINGER EQUATION (S.E.)

1) $i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi$

SOLVING YIELDS:

2) $\psi = Ae^{i(kx-\omega t)} = A\cos(kx-\omega t) + iA\sin(kx-\omega t)$

3) $P(x,t)\,dx$ (PROBABILITY) $= \psi^*\psi\,dx$

(ψ^* IS COMPLEX CONJUGATE OF ψ)

4) NORMILIZATION CONDITION

$\int_{-\infty}^{\infty}\psi^*\psi\,dx = 1$

B) THE TIME INDEPENDENT S.E.

1) LET $\psi(x,t) = f(t)\,\psi(x)$

$\Rightarrow f(t) = Ae^{-i\omega t}$

LEADS TO TIME INDEPENDENT S.E.

$-\frac{\hbar^2}{2m}\psi''(x) + V(x)\,\psi(x) = E\psi(x)$

2) PROBABILITY DISTRIBUTION

$\psi^*\psi = \psi^*(x)\,\psi(x)$

3) MOST GENERAL SOLUTION TO T.I.S.E.

$\psi(x,t) = \sum_n C_n e^{-i(E_n/\hbar)t}\,\psi_n(x)$

(E_n = POSSIBLE ENERGY OF PARTICLE)

4) $\psi(x)$ MUST MEET FOLLOWING CONDITIONS

1) SATISFY S.E.

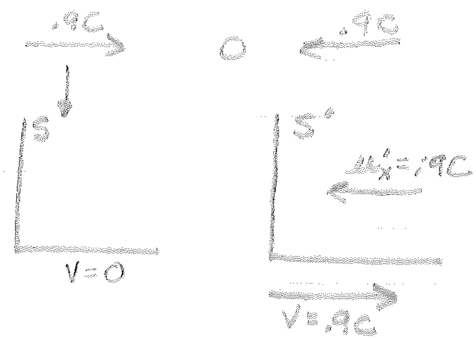
2) BE CONTINUOUS

3) $\psi'(x)$ BE CONTINUOUS

4) $\psi(x) \to 0$ AS $x \to \pm\infty$, so $\psi(x)$ CAN BE NORMALIZED

c) ENERGY QUANTIZATION FROM S.E.

3)



a) $U_x = \frac{U_x' + V}{1 + U_x' V/c^2} = \frac{1.8c}{1.81} = .994c$

b) $U_x = \frac{6\times10^4}{1 + \frac{36\times10^8}{9\times10^{16}}} = \frac{6\times10^4}{1 + 4\times10^{-8}} = \frac{6\times10^4}{1.00000004} \cong 6\times10^4 \frac{m}{sec}$

c) $\simeq 3\times10^{-5}\,\%$

4) $\lambda = \frac{(c-v)\Delta t}{f_0\,\Delta t'} = \frac{(c-v)}{f_0\sqrt{1-v/c^2}} = \frac{(c-v)\lambda'}{c\sqrt{1-v^2/c^2}}$

$\lambda^2 = \frac{(c-v)^2\lambda'^2}{c^2 - v^2} = \frac{(c-v)^2\lambda'^2}{(c-v)(c+v)} = \frac{(c-v)\lambda'^2}{c+v}$

$\frac{\lambda^2}{\lambda'^2} = \frac{c-v}{c+v}$

$c\frac{\lambda^2}{\lambda'^2} + v\frac{\lambda^2}{\lambda'^2} = c - v$

$c\left(1 - \frac{\lambda^2}{\lambda'^2}\right) = v\left(1 + \frac{\lambda^2}{\lambda'^2}\right)$

$\frac{\lambda^2}{\lambda'^2} = \frac{(1.22)^2}{(3.66)^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$c\left(\frac{8}{9}\right) = v\left(\frac{10}{9}\right)$

$v = .8c$

8) $He_3 = 3.016030$

$n = 1.008665$

4.024695

$He_4 = 4.002603$

$E_R = .022092$

$E = \frac{2.21 \times 10^{-4}\ kg}{mole} \frac{mole}{6.03 \times 10^{23}}\ 9 \times 10^{16} \frac{m^2}{sec^2}$

$= 3.30 \times 10^{-11}\ J \quad \frac{5.61 \times 10^{32}\ eV}{9 \times 10^{13}\ J} \quad \frac{1\ MeV}{10^{6}\ MeV}$

$= 2.06 \times 10^{-30}\ MeV$

$\frac{2.06 \times 10^{-30}}{21.6} \times 10^{2}\ \% = .0955 \times 10^{-28}\ \%$

$= 9.55 \times 10^{-30}\ \%$

11) $K^{\circ} \rightarrow \pi^{\circ} + \pi^{\circ}$

$M_{K^{\circ}} = 498\ MeV/c^2$

$M_{\pi^{\circ}} = 135\ MeV/c^2$

$2(135) = 270$

$\frac{498 - 270}{2} = 114\ MeV$

LABS

[illegible] theory [illegible]

[illegible] of experimental [illegible] apparatus [illegible]

[illegible] conducted [illegible]

[illegible] by [illegible]

[illegible] in tabular form [illegible]

[illegible] listing of all equations used [illegible]

include sample calculations for each equation

[illegible] discussion

[illegible] conclusions and general discussion

[illegible] will be in INK (for graphs [illegible]

[illegible] will be [illegible]) [illegible]

label all [illegible]

[illegible] will ultimately [illegible]

without name and date.

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Grading scale: A B C D F

[illegible]	[illegible]	Thursday	[illegible]	[illegible]
"	"	Tuesday	[illegible]	[illegible]
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Experimental Errors I - Errors in Measurements

The error in the calculated result of an experiment can be as important as the answer itself. It is impossible to come to any conclusion about what the experiment shows or does not show until some estimate is made of the probable magnitudes of the experimental errors and their effects on the result. Every engineer or scientist who analyzes experimental data must have some knowledge of the methods of error analysis.

The following exercise is designed to introduce you to some of the methods used in analyzing experimental errors. The exercise will be of value to you only if you work through it carefully, following the directions below, and think about it while you are doing it. You will be tested on these methods later.

Each numbered section of the exercise is called a frame. Some frames are information frames and others ask questions which you are to answer before going on. The correct answers to the questions are given below the questions and should be covered by a "mask" (piece of paper) until you have supplied a written answer to the question.

Directions:

1. Read each statement and question carefully and write your answer down (on another sheet of paper, not the exercise sheets) before looking at the printed answer.

2. Work carefully and directly through the exercise in sequence without skipping or browsing. If necessary you may refer back to earlier frames at any time.

3. If your answer turns out to be incorrect or incomplete, try to get straightened out by re-reading the pertinent information before going on to the next frame.

1. The term "error" as it is applied in experimental work does not include mistakes in arithmetic or mistakes such as recording the wrong number or reading instruments incorrectly. These mistakes may, and should, be eliminated completely by careful work. There are still, however, two kinds of errors which may be present in a series of measurements:

(a) Systematic errors-------any error that causes a measurement to yield a value which is consistently too large or too small. For example, if a voltmeter reads consistently high because of incorrect calibration, it is contributing a systematic error to the experiment.

(b) Random errors------an error which results from chance variations in the measuring device, the observation method, or the quantity being measured; there is an equal chance of these variations producing positive or negative errors. For example, there may be random error in reading a voltmeter because the observer does not always observe from the same angle. (There also may be a systematic error if the observer tends to read the meter at an angle to one side of the vertical.)

---

2. What is the principal difference between systematic and random errors?

Systematic errors produce results which are systematically too high or too low (one or the other). Random errors are due to chance variations and are just as likely to produce a value which is too high as to produce one which is too low.

---

3. In which of the following cases are the errors systematic and in which are the errors random in nature?

(a) Objects are weighed on a balance that is not first correctly "zeroed".

(b) The volume of a liquid is measured several times by different observers using an accurate graduated cylinder and noting the level of the liquid on the scale.

(a) Systematic. Readings will be systematically high or low (one or the other).

(b) Random. It will be difficult to line up the eye with the scale and liquid surface the same way every time.

4. In which of the following cases is the main type of error systematic and in which is it random?

(a) A timer is running slow.
(b) Friction is ignored in an experiment in which it is not quite negligible.
(c) A galvanometer (used for measuring small quantities of charge and currents) is being affected slightly by vibrations.
(d) Fluctuations in the line voltage of an electronic measuring device.
(e) The use of weights which are light due to part of their mass being worn away over the years.
(f) Bias of an observer which causes him to always read a scale from a point to one side of the vertical.

(a) Systematic
(b) S
(c) Random
(d) R
(e) S
(f) S

---

5. Often the largest source of error in an experiment is a systematic error. The experimenter should always look for sources of systematic error in his experiment, and try to eliminate them or correct for them as much as possible. For example, one can guard against observer bias in reading a scale by having several different observers make the same reading without knowledge of the others' results, or by constructing a viewing arrangement which makes bias impossible. He can make sure that all his instruments are properly zeroed before using them. He can calibrate his instruments and measuring devices by comparing readings taken with them with readings taken with another instrument which is known to give results which are more likely to be accurate. Systematic errors are sometimes difficult to prevent, but as you become more experienced with correct experimental procedures you will find these errors easier to detect and correct.

---

6. How might you go about eliminating systematic error from an experiment in which you suspect a timer may not be keeping accurate time, after the experiment is done?

There are probably a number of correct ways. One way would be to calibrate the timer against a timing device that is more likely to be accurate and make the appropriate corrections to all time measurements in the experiment.

---

7. Even when systematic error is not eliminated from an experiment it is often possible to estimate, at least roughly, how large it might be. When you use a resistance box with resistors which are supposed to be accurate within 1%, for example, you can assume that if the box has not been mistreated, the resistance valves are within 1% of the dial settings. A voltmeter which has specifications which state a 5% accuracy has been calibrated at the factory to have a systematic error of less than 5% at full scale reading.

8. Suppose you were doing an experiment using a 1% resistance box and a 5% voltmeter and were somewhat dissatisfied with the results. Should you go looking for a better resistance box or a better voltmeter?

A more accurate voltmeter, assuming that the voltmeter and resistance box each contribute error in about the same way to the experimental result (more about this later.)

9. Random errors cannot be eliminated from an experiment. One simple way of finding out how large they are, however, is to repeat each measurement several times, trying not to let the result of any measurement be influenced by the results of previous measurements. The variations which show up in repeated measurements of a single quantity are a measure of the uncertainty involved in measuring that quantity, assuming all systematic errors are negligible (i.e., systematic errors which are not negligible compared to this uncertainty have been eliminated). If repeated measurements of a quantity all give the same value, it does not mean that there is no random error or uncertainty in the measurements, but only that the measurements were not carried to a sufficient number of decimal places to observe random deviations. The experimenter should always estimate fractions of divisions down to the point where the last significant figure contains some uncertainty, if he wants to get maximum precision in his experiment.

10. Suppose a measurement of a particular quantity is repeated 5 times and each time the result is $x = 10.0$ units. If the measurements are made with a device which may have a systematic error of 5% and you wish to increase the accuracy of your measurement, should you try to estimate another decimal place in the readings or calibrate your device using a method having a smaller systematic error?

Since the systematic error of 5% gives an uncertainty of ±0.5 in the measurements there is not much point in determining the (much smaller) random error uncertainty by estimating another decimal place.

---

11. Suppose in the preceding measurement that the measuring device is known to have a systematic error of less than .01%. What would you do to get the maximum precision from your measurements?

Estimate another decimal place or two in the readings, whatever is necessary to start showing random deviations.

---

12. In the following frames let's suppose that systematic errors are negligible compared to random deviations (i.e., corrections have been made to eliminate most of the systematic error). If this is the case then the best way to determine a quantity accurately is to repeat the measurement several times and take an average. Individual trials will deviate from this average (deviation d_i of value x_i from average $\bar{x}$ is $d_i = x_i - \bar{x}$), but the average is the best value obtainable from these measurements. One way of indicating the amount of uncertainty present in the individual measurements is to calculate the average of the absolute value of the deviations.

$$\text{avg. deviation a.d.} = \frac{|d_1| + |d_2| + |d_3| + \ldots\ldots + |d_n|}{n}$$

where n is the number of individual measurements in the set.

---

13. Suppose a certain measurement is made 5 times with the result

$x_1 = 10.90$ units
$x_2 = 10.35$
$x_3 = 10.30$
$x_4 = 10.10$ units
$x_5 = 10.55$

what is the average and the average deviation of these measurements?

$$x = \frac{10.90 + 10.35 + 10.30 + 10.10 + 10.55}{5} = 10.44$$

$$\text{a.d.} = \frac{0.46 + 0.09 + 0.14 + 0.34 + 0.11}{5} = 0.23$$

---

14. The average deviation in a set of repeated measurements of a quantity is one way of specifying the uncertainty present in one measurement. Another way is to determine the standard deviation:

$$\sigma = \left[\frac{d_1{}^2 + d_2{}^2 + d_3{}^2 + \ldots\ldots\ldots d_n{}^2}{n - 1}\right]^{\frac{1}{2}}$$

where n = no. of deviations d_i

= no. of individual measurements in the set

15. Determine the standard deviation in the measurements of frame 13.

$$\sigma = \frac{(0.46)^2 + (0.09)^2 + (0.14)^2 + (0.34)^2 + (0.11)^2}{5 - 1}^{\frac{1}{2}} = 0.30$$

16. If in a large number of trials, the values are distributed about the average value in a way which we describe as a "normal" or "Gaussian" distribution (i.e., density of values drops off symmetrically on either side of the average—— a bell shaped density curve), then 68% of the values are between x +σ and x −σ, and 95% between x + 2σ and x − 2σ. In other words, a single measurement has a 68% chance of being between $\bar{x}$ +σ and $\bar{x}$ −σ, and a 95% chance of being between $\bar{x}$ + 2σ and $\bar{x}$ − 2σ. Thus the standard deviation σ is a measure of the uncertainty present in a single measurement. The smaller the standard deviation , the narrower the limits between which a single measurement is likely to appear.

17. In the situation of frame 13, suppose you are about to make another measurement. Between what limits can you predict that it will appear with a 68% probability? Between what limits can you predict that it will have a 95% chance of appearing? (Note: The limits between which it is absolutely certain, that is, 100% certain, to appear are probably + ∞ and −∞ or o and ∞depending on the nature of the measurement.)

68% chance of appearing between 10.14 and 10.74
95% chance of appearing between 9.84 and 11.04

18. After having made several measurements of a quantity we are not generally as interested in the uncertainty in a single measurement as we are the uncertainty in the average of the set. One way of determining the uncertainty in the average would be to take a large number of data sets and look at the distribution of the averages. These averages are more reliable (i.e. less uncertain) than the individual measurements of a set and therefore won't spread out as far. The standard deviation in the mean (average) σ_m is a measure of the uncertainty in the average and can be determined from the standard deviation σ (uncertainty in a single measurement) by the equation:

$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

where σ_m = the standard deviation of the averages of many sets of data.

n = the number of measurements or trials in one set.

σ = the standard deviation of the individual trials in one set.

19. Determine the standard deviation in the mean in the situation of frame 13.

$$\sigma_m = \frac{0.30}{\sqrt{5}} = 0.13$$

---

20. The result of repeated measurements of a single quantity is generally stated

$$x = \bar{x} \pm \sigma_m$$

assuming systematic errors are negligible. Using the data of frame 13, state the result of these measurements in this form.

$$x = 10.44 \pm 0.13$$

21. If systematic error is likely to be larger than random error, then it isn't very useful to determine σ_m which is a measure of the uncertainty in the mean due to random error only. If systematic error is dominant (or at least not negligible) in the measurement the result of the measurement is stated $x = \bar{x}$ ± estimated uncertainty.

22. Assuming that the measurements of frame 13 were made using an instrument that could only be read with 5% accuracy, state the results of these measurements in the form $x = \bar{x} \pm$ ________.

5% of 10.44 is about 0.52
Estimated uncertainty (random and systematic) = 0.6
$x = 10.4 \pm 0.6$

(Note that one significant figure has been dropped in both numbers because the uncertainty is only estimated and it would be a bit optimistic to carry another place.)

TEST: (Answer the following):

1. Explain the difference between random error and systematic error.

2. Give two examples of each type of error.

3. How might one decrease the amount of systematic error in an experiment? Explain using a specific example.

4. Suppose the possible systematic error in a voltmeter reading is estimated at 5 volts and several voltage readings are taken as follows:

V	100	101	101	99	102	100	volts
Trial	1	2	3	4	5	6	

State the result of these measurements along with its uncertainty.

5. Suppose the above measurements were made using a highly accurate potentiometer with systematic error of less than 0.1 volt. State the result of these measurements along with its uncertainty.

EXERCISE

Make a series of measurements of some fundamental quantity (i.e., mass, length, or time) using any measuring devices at hand. Determine the random error standard deviation in the mean σ_m, and estimate the uncertainty due to systematic error. State the result of your measurements along with the uncertainty in the result.

Experimental Errors II - Errors in Calculated Results

The following exercise is designed to introduce you to some methods of determining the uncertainty present in a numerical result which has been calculated from experimental data. Experimental errors are always present in measurements of any kind, and these errors contribute to errors in the results obtained when calculations are made using the measured quantities. The exercise will be of value to you only if you work through it carefully, following the directions below, and think about it while you are doing it. You will be tested on these methods later.

Directions:

1. The correct answers to the questions are given below the questions and should be covered by a "mask" (piece of paper) until you have supplied a written answer to the question. Read each statement and question carefully and write your answer down (on another sheet of paper, not the exercise sheets) before looking at the printed answer.

2. Work carefully and directly through the exercise in sequence without skipping or browsing. If necessary you may refer back to earlier frames at any time.

3. If your answer turns out to be incorrect or incomplete, try to get straightened out by re-reading the pertinent information before going on to the next frame.

PRETEST:

Given $\sigma = \left[\frac{d_1^2 + d_2^2 + d_3^2 + \ldots\ldots + d_n^2}{n - 1}\right]^{\frac{1}{2}}$

$\sigma_m = \frac{\sigma}{\sqrt{n}}$

answer the following questions.

1. Which one of these two quantities is a measure of the uncertainty, due to random error, in the average of a set of n measurements?

σ_m is the standard deviation in the mean (average)

---

2. Suppose the possible systematic error in a voltmeter reading is estimated at 5 volts and several voltage readings are taken as follows:

V	88	90	89	90	volts
Trial	1	2	3	4	

State the result of these measurements along with its uncertainty.

Average $\bar{V}$ = 89.25 volts
σ = 0.96
σ_m = 0.48

The systematic error of 5 volts is nearly 10 times as great which means that the calculation of the random error standard deviation σ_m is meaningless.

Answer: V = (89 ± 5) volts

---

3. Suppose the measurements of frame 2 were made with a very accurate potentiometer which is supposed to have less than .01 volt systematic error. State the result of the measurement along with its uncertainty.

V = (89.25 ± 0.48) volts (systematic error negligible)

IF YOU HAVE TROUBLE UNDERSTANDING THE PRECEDING ANSWERS, REVIEW "EXPERIMENTAL ERRORS I" OR ASK YOUR INSTRUCTOR ABOUT IT BEFORE GOING ON.

---

4. In the following frames x_1, x_2, x_3......... are the results obtained in measuring several different quantities experimentally. The uncertainties in these measurements are represented by Δx_1, Δx_2, Δx_3.......... We are interested in knowing how to calculate the uncertainty in the result of adding, subtracting, multiplying, dividing, etcetera, two or more quantities whose individual uncertainties are known.

<u>Rule 1:</u> The uncertainty in the result of <u>addition or subtraction</u> is the square root of the sum of the squares of the uncertainties of the separate terms.

$$\Delta x = \sqrt{(\Delta x_1)^2 + (\Delta x_2)^2 + \ldots\ldots\ldots}$$

5. Given
$x_1 = 7.20 \pm 0.30$
$x_2 = 2.30 \pm 0.20$
$x_3 = 5.10 \pm 0.05$

determine $x = x_1 - x_2 + x_3$ (value and uncertainty).

$$x = (7.20 - 2.30 + 5.10) \pm \left[(0.30)^2 + (0.20)^2 + (0.05)^2\right]^{\frac{1}{2}}$$

$$x = 10.00 \pm 0.36$$

(Note that uncertainties don't add directly as would known errors, since there is a possibility of a positive error in x_1 being partially cancelled by a negative error in x_3 and so forth.)

6. <u>Rule 2:</u> The <u>percentage</u> uncertainty in the result of multiplication or division is the square root of the sum of the squares of the percentage uncertainties of the factors. In other words in the multiplication or division of x_1, x_2, x_3.......,

$$\frac{\Delta x}{x} = \sqrt{\left(\frac{\Delta x_1}{x_1}\right)^2 + \left(\frac{\Delta x_2}{x_2}\right)^2 + \left(\frac{\Delta x_3}{x_3}\right)^2 + \ldots\ldots\ldots}$$

7. Given x_1, x_2, x_3 as in frame 5, find $x = \frac{(x_1)(x_2)}{x_3}$ (value and uncertainty).

$$x = \frac{(7.20)(2.30)}{5.10} = 3.25$$

$$\frac{\Delta x}{x} = \sqrt{\left(\frac{0.30}{7.20}\right)^2 + \left(\frac{0.20}{2.30}\right)^2 + \left(\frac{0.05}{5.10}\right)^2} = 0.097$$

$$\Delta x = (.097)(3.25) = 0.32$$

$$x = 3.25 \pm 0.32$$

---

8. In a calculation that includes both addition (or subtraction) and multiplication (or division) the calculation for uncertainty x is broken into parts. For example in determining Δx for the calculation $x = (x_1 + x_2)x_3$ rule 1 would be used to determine an uncertainty for the sum of x_1 and x_2, and then rule 2 applied to this uncertainty and Δx_3 to determine Δx for the product.

---

9. Using the values for x_1, x_2, and x_3 given in frame 5, determine the value and uncertainty in the result of the calculation

$$x = \frac{x_1}{x_2} + x_3$$

$$x = \frac{7.20}{2.30} + 5.10 = 8.23$$

$$\frac{\Delta x_{12}}{x_{12}} = \sqrt{\left(\frac{0.30}{7.20}\right)^2 + \left(\frac{0.20}{2.30}\right)^2} = .096$$

$$\Delta x_{12} = (.096)(3.13) = 0.30$$

$$\Delta x = \sqrt{(0.30)^2 + (0.05)^2} = 0.30$$

$$x = 8.23 \pm 0.30$$

---

10. One advantage of doing a calculation such as the above is that it shows which of the measured quantities, x_1, x_2, or x_3 is contributing most of the uncertainty to the result of the calculation. This in turn tells the experimenter which measurement to concentrate his attention on if he wants to improve the accuracy of his result.

(a) Does the quotient $\frac{x_1}{x_2}$ or the quantity x_3 contribute the most uncertainty to x? (Look at the calculation for Δx.)

(b) Does x_1 or x_2 contribute the most uncertainty to $\frac{x_1}{x_2}$? (look at the calculation for $\frac{\Delta x_{12}}{x_{12}}$.)

(c) Which of the quantities x_1, x_2, and x_3 would you concentrate on if you wanted to improve the accuracy of x?

(a) $\frac{x_1}{x_2}$ (b) x_2

(c) x_2 first

11. Rule 3: The percentage uncertainty in the result of raising a quantity to the nth power is n times the percentage uncertainty in the quantity. In other words, if $x = x_1^n$,

$$\frac{\Delta x}{x} = n \frac{\Delta x_1}{x_1}.$$

12. If x_1 is as given in frame 5, find the value and uncertainty in:

(a) x_1^3

(b) $x_1^{\frac{1}{2}}$

(a) $\frac{\Delta x}{x} = 3\left(\frac{0.30}{7.20}\right) = 0.125$

$x = (7.20)^3 = 373$

$\Delta x = (0.125)(373) = 47$

$x = 373 \pm 47$

(b) $\frac{\Delta x}{x} = \frac{1}{2}\left(\frac{0.30}{7.20}\right) = 0.0209$

$x = (7.20)^{\frac{1}{2}} = 2.68$

$\Delta x = (0.0209)(2.68) = 0.06$

$x = 2.68 \pm 0.06$

---

13. Although the preceding three rules apply to a great many calculations, there is a more general rule from which they are derived, and which applies to other calculations as well.

<u>Rule 4:</u> If x is a function of x_1, x_2, x_3.........., then the uncertainty in x is

$$\Delta x = \sqrt{\left(\frac{\partial x}{\partial x_1}\right)^2 (\Delta x_1)^2 + \left(\frac{\partial x}{\partial x_2}\right)^2 (\Delta x_2)^2 + \ldots\ldots\ldots\ldots}$$

---

14. Suppose a certain angle θ is measured as $\theta = (1.00 \pm 0.05)$ radians. What is the value and uncertainty of sin θ?

$x = \sin\theta = 0.841$

$x_1 = \theta$

$\Delta x = \sqrt{(\cos\theta)^2 (\Delta\theta)^2} = (.0540)(0.05) = 0.027$

$x = \sin\theta = 0.841 \pm 0.027$

TEST

1. An object travels a distance $L = (10.6 \pm 0.4)$ meters in a time $t = (4.3 \pm 0.2)$ sec. What is the average velocity (value and uncertainty)?

2. Two experimental quantities $x_1 = 8.23 \pm 0.56$ and $x_2 = 6.54 \pm 0.42$ are to be added. What is the value and uncertainty of $x = x_1 + x_2$?

3. What is the value and uncertainty of x_2^2 in question 2?

4. Given $\theta = (0.25 \pm 0.07)$ radians, find the value and uncertainty of $\cos\theta$.

AN EXPERIMENTAL STUDY OF σ .

Consider an experiment in which a quantity x is measured r times. Let each measured value of x be denoted by x_j where $j=1, \ldots, r$. We can compute the standard deviation of the measured values of x which we will call $\sigma(x)$. We can also compute the standard deviation of the mean value $\bar{x}$ of x which we will call $\sigma_m(\bar{x})$. This latter computation is simply $\sigma_m(\bar{x}) = \frac{\sigma(x)}{\sqrt{r}}$ (so we are told). We wish to investigate here whether or not $\sigma_m(\bar{x})$ is in fact smaller than $\sigma(x)$ by the factor $\frac{1}{\sqrt{r}}$. In order to do so we must obtain $\sigma_m(\bar{x})$ by some other presumably more direct method.

Suppose we perform the whole experiment s times, each time obtaining a value $\bar{x}$. Let each value of $\bar{x}$ be denoted by $\bar{x}_k$ where $k=1, \ldots, s$. We can compute the mean value of $\bar{x}$ which we will call $\bar{\bar{x}}$. Here $\bar{\bar{x}} = \left[\frac{\sum_{k=1}^{s} \bar{x}_k}{s}\right]$. We can also compute the standard deviation of the values of $\bar{x}$ which we call $\sigma(\bar{x})$. Here $\sigma(\bar{x}) = \frac{\left[\sum_{k=1}^{s}(\bar{\bar{x}} - \bar{x}_k)^2\right]^{1/2}}{\sqrt{s-1}}$.

<u>Obviously</u> $\sigma(\bar{x})$ is a direct determination of $\sigma_m(\bar{x})$.

Therefore by doing the experiment s times we can directly measure $\sigma(\bar{x})$ and test whether or not $\frac{\sigma(\bar{x})}{\sigma(x)}$ is approximately $\frac{1}{\sqrt{r}}$.

To carry this study of uncertainty one final step— what is the standard deviation of the experimentally determined quantity $\frac{\sigma(\bar{x})}{\sigma(x)}$? (!) With the data obtained at this point we can at least calculate directly the standard deviation of the quantity $\sigma(x)$ (but not of $\sigma(\bar{x})$). Call this $\sigma[\sigma(x)]$.

To generate data construct a simple pendulum of length ~1m from string and a small weight. Using your wrist watch or a stop watch, measure the time for the pendulum to execute ten full swings. Let this measured quantity be x_j (t_j if you wish). Let $r = 7$ and $s = 5$.

Give some thought and planning to the organization of data tables before you begin collecting data.

GRAPHICAL ANALYSIS

Often one of the aims of an experimental investigation is the determination, from measurements made in the laboratory, of how one of two interdependent quantities, y, depends on the other, x. Graphical methods provide us with a very useful tool in this type of analysis.

I. Plotting Graphs

Suppose one is interested, for example, in finding in a particular experiment a mathemical relationship which expresses the velocity of a moving object v as a function of the time t. In this case velocity is the "dependent variable" whose dependency on the "independent variable" time is to be established from the following data.

Time (sec)	Velocity (magnitude) (cm/sec)
1.00	1.9
2.00	1.9
3.00	3.0
4.00	3.9
5.00	6.5
6.00	11.0

Suppose that in this experiment the time measurements are very precise and their errors can be ignored while the velocity measurements are estimated to have a standard deviation (see instruction sheet on "Measurement, Probability, and Experimental Errors) of about $\pm$ 0.30 cm/sec. The steps to be followed in constructing a graph which illustrates the dependence of velocity v on time t (or any quantity y on another quantity x) are summarized below.

(a) The dependent variable (quantity whose dependency on the other is to be determined) is plotted vertically (velocity versus time rather than vice versa).

(b) Scales should be chosen which are easy to plot and easy to read and which make the graph large enough to be read easily and accurately (occupying a full page if possible).

(c) Scales usually start at zero but sometimes this would cause the data to be crowded into one part of the graph. In such a case it is a good idea to suppress the zero (start the scale at some value other than zero or show a break in the scale). However, <u>it should be made obvious</u> to someone looking at the graph <u>that the zero has been suppressed</u>.

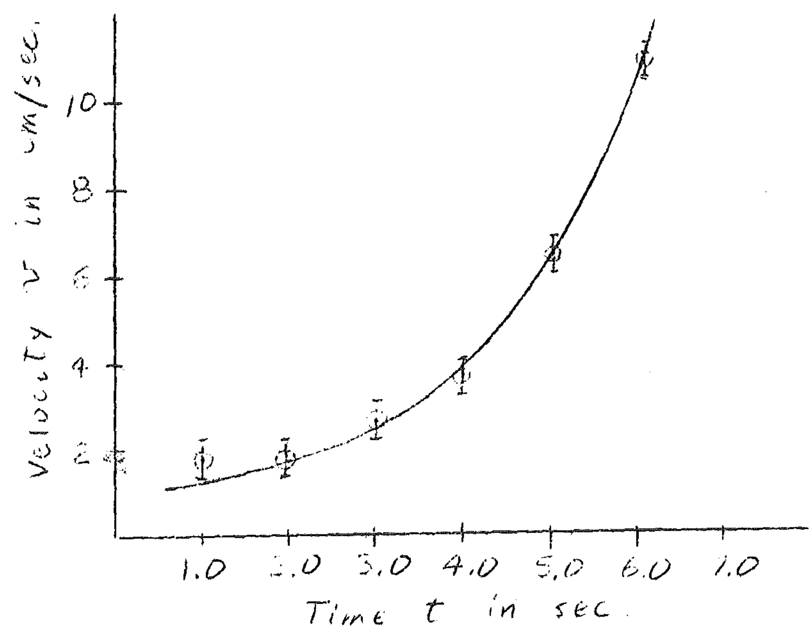
(d) The graph should have a title and each of the axes should show the quantity plotted along that axis and the numerical scale and units for that quantity.

(e) The experimental points are marked clearly on the graph by drawing a small circle around each of them and drawing an "error line" (in the above example extending 0.30 cm/sec above and below the data point) at each point.

(f) Draw the simplest possible smooth line or curve (i.e. the simplest curve is a straight line, the next is a curve whose curvature is always in the same direction and doesn't change magnitude suddenly, etc) among the points, with no more details of shape and curvature than is justified by the size of the estimated errors. If the magnitude of the standard deviations are estimated correctly and the line is drawn correctly the curve should cut about two thirds of the error lines (very roughly).

When these steps are applied to the example of the moving object given above, a graph results such as that shown in the following figure.

<u>Velocity v versus Time t</u>



This sample graph is actually too small for accuracy

II. Determination of a Mathematical Relationship

If a graph of dependent variable y versus independent variable x turns out to be a straight line, the dependence of y on x is expressed by the equation

$$y = ax + b \tag{1}$$

The slope a and y intercept b of the line can be taken directly from the graph (see part III) thus establishing the relationship between quantity y and quantity x in this experiment.

If the graph of y versus x is curved, however, as it is in the case of the velocity of an object versus the time in part I, the quantities must be related by some other equation. For example, one might guess that y is related to x according to an equation of the type

$$y = ax^n + b \tag{2}$$

where n might be an integer $-1, \pm 2, \pm 3, \pm 4,$ ....or a fraction $\pm 1/2, \pm 1/3, \pm 1/4,$ ....To decide which values of n are truly possibilities one should study the graph of y versus x and equation (2). In the case of the velocity versus time graph of part I, for example, negative values of n should be immediately discounted since equation (2) would predict a decrease in y for increasing x. Fractional values of n are just as unlikely since as x increases, the graph shows y increasing faster and faster (perhaps indicating $n = +2$ or $+3$, etc.).

To see if the velocity - time ($y = v$, $x = t$) data for the moving object example of part I fits equation (2) with $n = +2$ one could graph $Y = v$ versus $X = t^2$ from the experimental values of v and the corresponding values of t^2. If the graph of Y versus X from the data is a straight line, the experimental results fit a relationship

$$Y = a X + b$$

or $$v = a t^2 + b \qquad \text{(equation 2 with } n = +2\text{)}$$

where a and b are the slope and intercept of the line. If such a graph was not straight, but was straighter than a graph of v versus t, then one might try a graph of $Y = v$ versus $X = t^3$ and so on until a straight line was found. The same general procedure could be followed in cases where n is thought to be a fraction or have a negative value. If the data are to be represented by the equation

$$y = ax^{-1/3} + b \tag{3}$$

then a graph of y versus $x^{-1/3}$ should yield a straight line.

Another type of relationship between quantities which appears often is

$$y = Ae^{ax} \qquad (4)$$

where A and a are positive or negative constants. If equation (4) accurately represents the data, then

$$\ln y = ax + \ln A$$

or $$Y = ax + b$$

making the substitutions $Y = \text{Ln } y$ and $b = \ln A$. Therefore if $Y = \ln y$ is plotted vertically against x horizontally, a straight line of slope a and intercept $b = \ln A$ should result. The values of a and A can be determined from this line.

III. Determination of Slope and Intercept

The slope and intercept of a straight line are found as follows: First the x and y coordinates of two widely separated points on the line are determined (note that the points must be widely separated for accuracy and the points are points on the line, not data points). The slope of the line is defined

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

and should have the same value (for a straight line) regardless of what two points are chosen. The y intercept is obtained by extending the line back to $x = o$ and noting the value of y at this point on the line (this is the intercept b).

A more reliable determination of slope, a, and y intercept, b, results when one computes the slope and intercept of the straight line which minimizes the sum of the squares of the deviations of the data points from the line (see instruction sheet on "Method of Least Squares").

References:

1. Kruglak and Moore, "Basic Mathematics for the Physical Sciences", chapter 7.

2. G. Wootan, Inc., "Graphs"

3. Ford, "Basic Physics", section 7.6

The Oscilloscope

Introduction

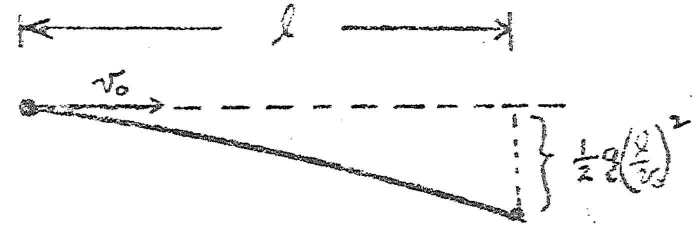
We know from our studies of the motion of objects in a uniform gravitational field, that when a mass m is thrown horizontally with some initial velocity v_o it will follow a parabolic path (Fig. 1-a) and will fall a distance

$$\frac{1}{2}\, g\left(\frac{\ell}{v_o}\right)^2$$ in the time it takes to

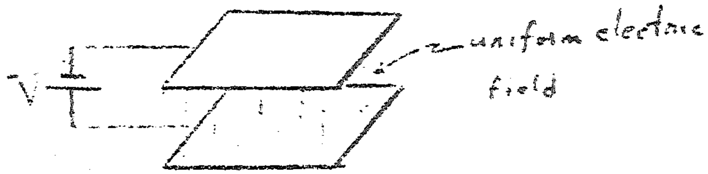
travel a distance ℓ horizontally. One can produce a uniform electric field, by connecting a battery between two parallel metal plates as in Fig. 1-b, and c. If an electron is in the region between the two plates it is subject to a constant electric force, just as mass m in a uniform gravitational field is subject to a constant gravitational force. As a consequence of this, if the electron enters the field with a velocity v_o as indicated in Fig. 1-c, it will "fall" a distance

$$y_1 = \frac{1}{2}\,\frac{Ve}{md}\left(\frac{\ell}{v_o}\right)^2$$ in the time it takes

the electron to travel a distance ℓ horizontally. Here e is the charge on the electron, m is the mass of the electron, d is the distance between the plates, and V is the difference of potential between the plates. If e, m, d, l and v_o are all held constant, then y_1 is directly proportional to V. When the electron leaves the electric field between the plates, its path becomes a



(a)



(b)

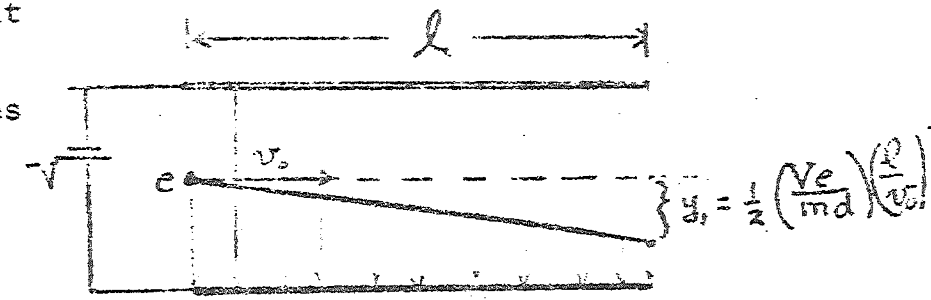


Fig 1 (c)

straight line until it strikes some obstruction such as the screen in Fig. 2. It is relatively easy to show that

$$Y = \text{const } V$$

provided e, m, d, l, v_o and D are held constant. Here Y and D are the distances shown in Fig. 2.

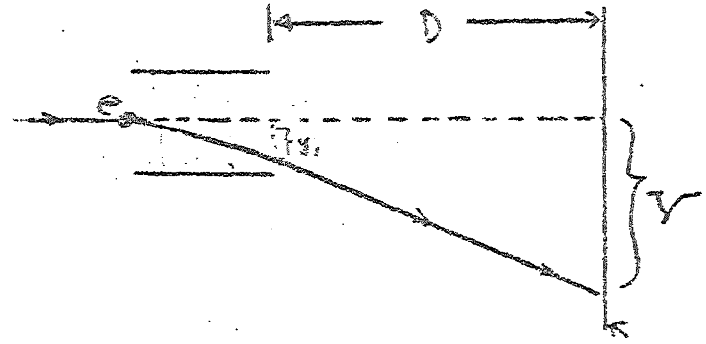


Fig 2

In an oscilloscope, electrons are accelerated until they attain some velocity v_o and then pass in turn through two sets of plates as indicated in Fig. 3. If there is any potential difference between the first pair of plates, the electrons will be deflected vertically an amount proportional to this potential difference; if there is any potential difference between the second pair the electrons will be deflected horizontally an amount proportional to this potential difference.

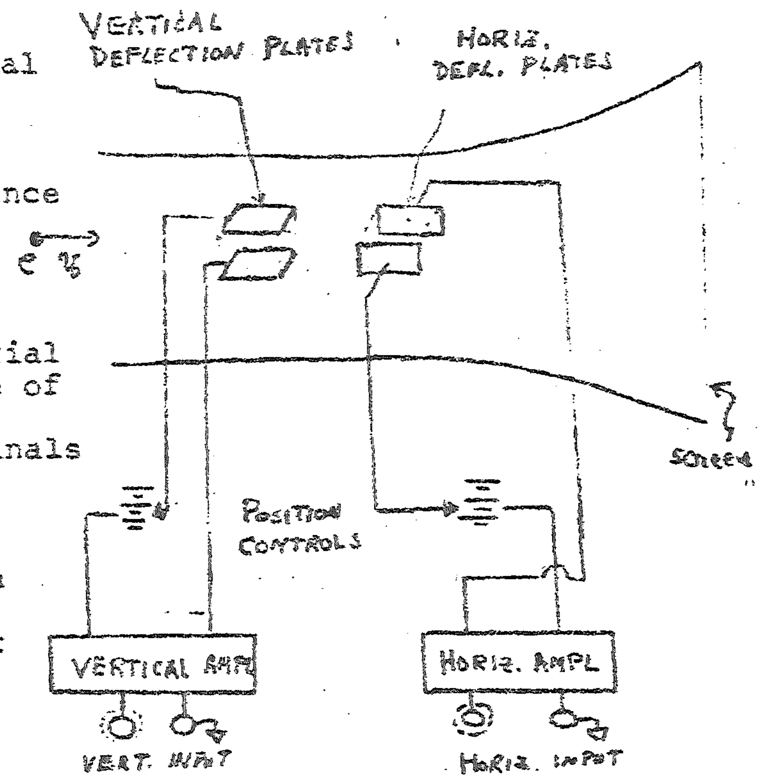


Fig. 3 shows schematically some of the essential features of the oscilloscope. Any difference of potential applied to the input terminals of either the horizontal or vertical input terminals is first multiplied by some constant factor (determined by the setting of the amplifier controls) and then applied to the deflection plates. The vertical and horizontal position controls allow a potential difference to be applied to the deflection plates even without an input, and thus permit positioning of the electron beam.

Procedure

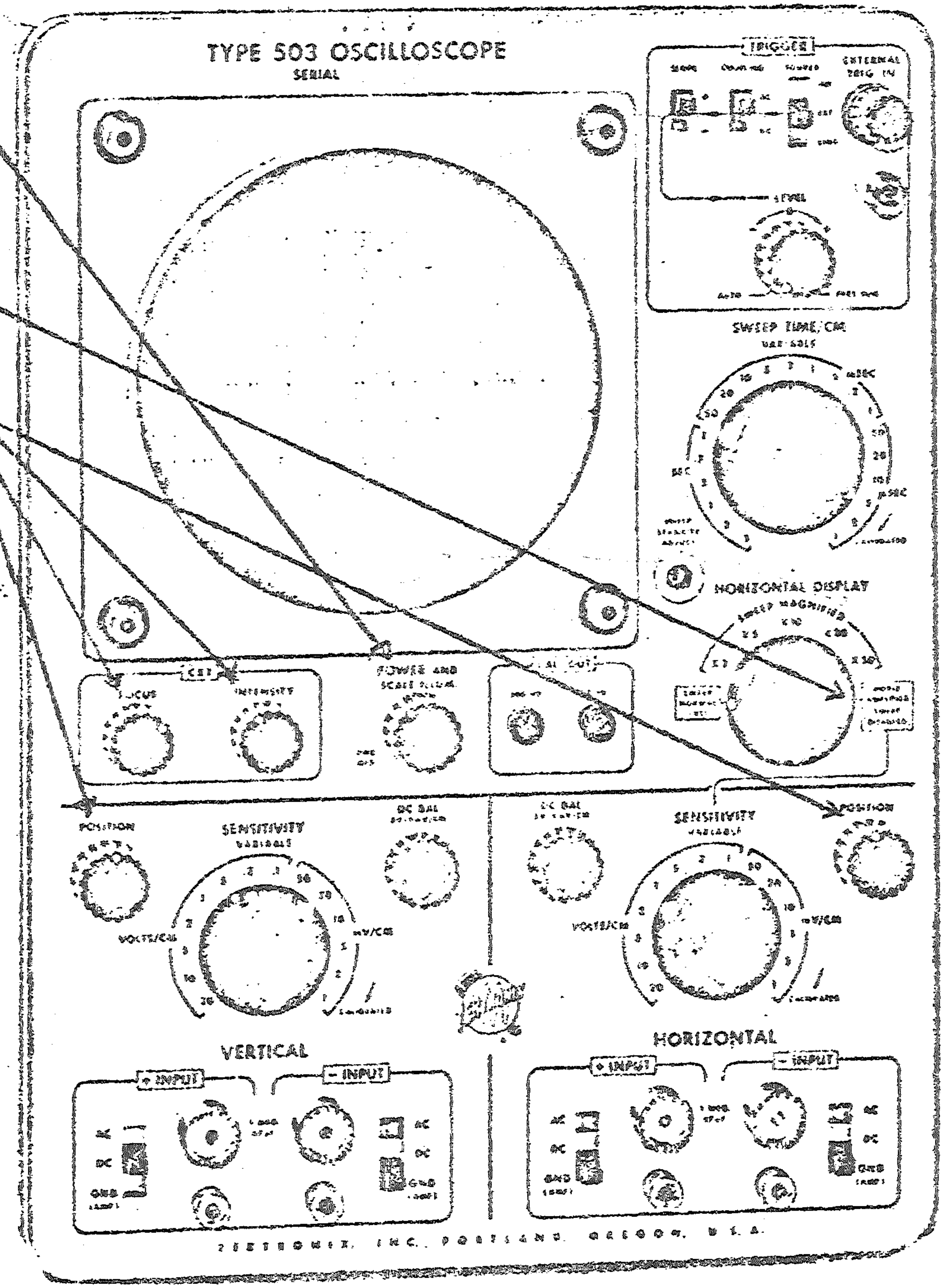
1. Plug in the oscilloscope and rotate the power switch clockwise to turn on the oscilloscope. Note the effect that this rotation has on the amount of scale illumination.

2. Set the horizontal DISPLAY switch to the Horizontal Amplifer Sweep Disabled position.

3. Set the INTENSITY, FOCUS, and both POSITION KNOBS so that the white dot of each knob is at its highest point

4. As soon as the scope has warmed up a small spot should appear near the center of the screen. Note the effect of turning the intensity knob clockwise and counterclockwise. Set the intensity control so that the spot is barely visible.

5. Note the effect of the position controls. Use them to set the spot to the center of the screen.

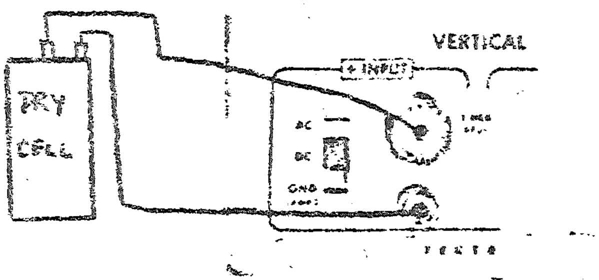


6. Set both sensitivity switches to the 1 volt/cm position.

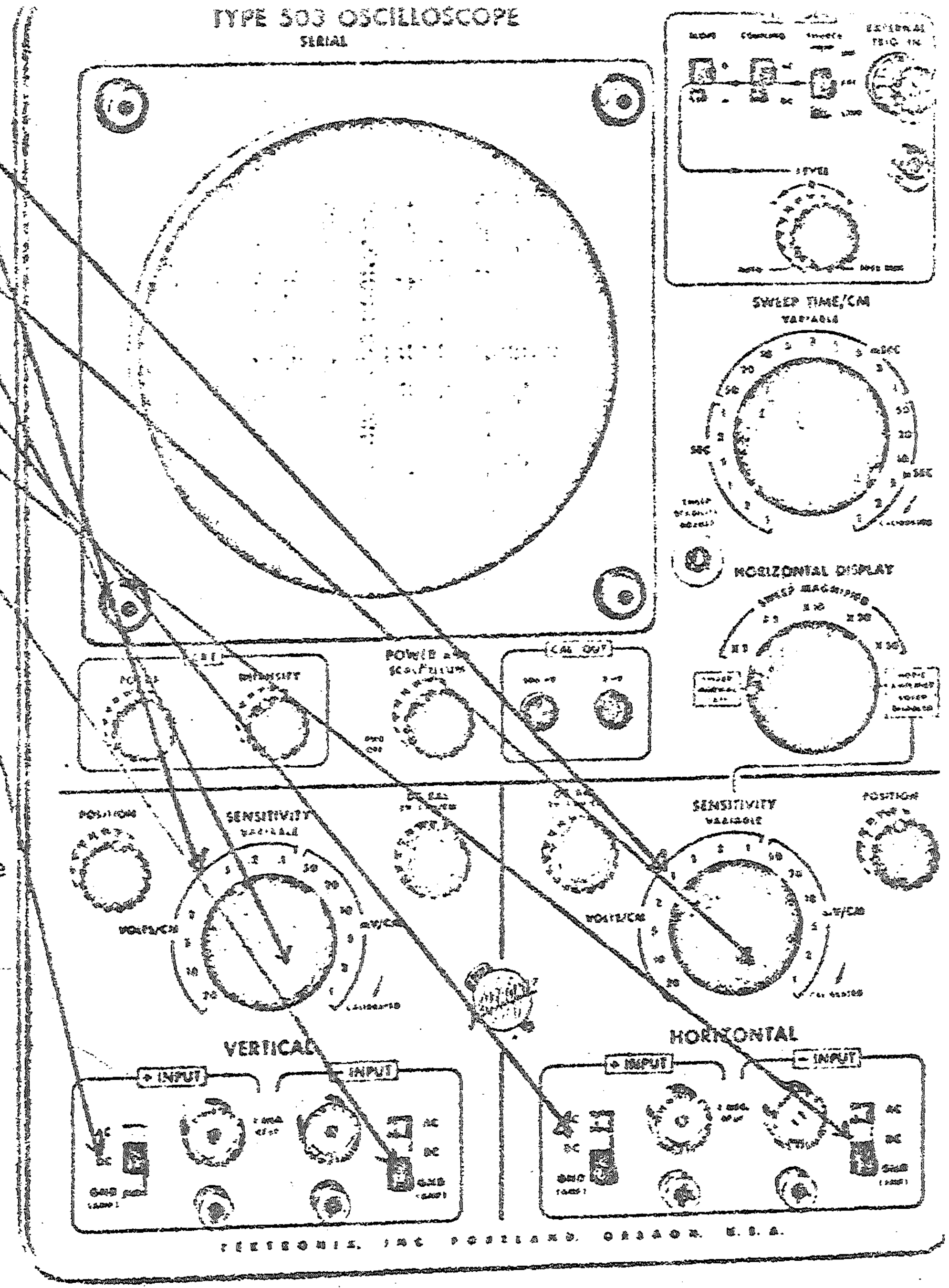
7. Rotate both red sensitivity controls to their maximum clockwise position

8. Set both horizontal and vertical + input switches to DC and set both - input switches to GND.

9. Recenter the spot if necessary. Connect a dry cell to the + vertical input terminals as indicated below:



Note that the spot moves up approximately 1½ cm. The dry cell maintains a difference of potential of approximately 1.5 volts between its two terminals, and the sensitivity controls are set at 1 volt/cm, so that 1½ volts produce a deflection of 1.5 cm.



10. Reverse the connections at either the dry cell or the input terminals and note that the spot moves down 1½ cm. Whenever the top (large) input terminal is higher in potential than the lower (small) input terminal, the spot is deflected upward and vice versa.

11. Repeat steps 9 and 10 except use the horizontal + input terminals. Note that when the top (large) input terminal is higher in potential than the lower terminal the spot moves to the right and vice versa.

12. Set the vertical sensitivity switch to 0.5 volts/cm and re-center the spot if necessary. Connect the 1.5 volt dry cell to the vertical input. Note that the spot is deflected approximately 3 cm.

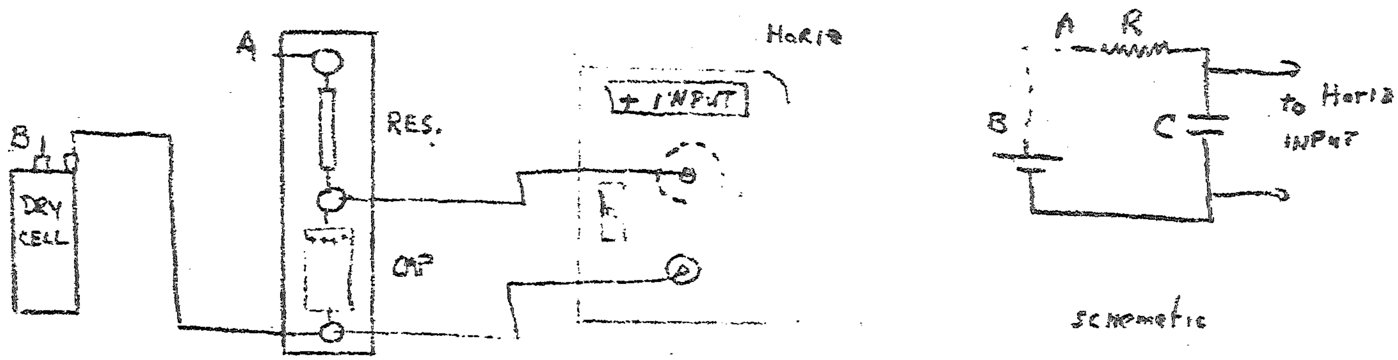
13. Repeat step 12 with different settings of the sensitivity switch, until you are confident you understand the function of this switch. Repeat, using the horizontal + input terminals, and different settings of the horizontal sensitivity switch.

14. Set the vertical sensitivity switch to 1 volt/cm and rotate the red sensitivity control so that its white dot is approximately vertical Re-center the spot if necessary. Connect the 1½ volt battery to the + vertical input terminals and note that the spot is deflected only about 1 volt, instead of 1.5 cm. Remove the battery, rotate the red sensitivity control knob to its extreme counterclockwise position, re-center the spot, and re-connect the battery. Note that the spot is deflected only about 0.5 cm.

15. Repeat step 13 with different settings of the red sensitivity control knob.until you are confident that you understand the function of this control. Repeat, using the horizontal + input terminals and the horizontal sensitivity control knob.

16. Use your oscilloscope to measure the potential difference between the terminals of the black box provided. Position the spot and choose the sensitivity setting so that the deflection produced by the unknown voltage is as large as possible, but still on scale. Remember that it is only when the red knob sensitivity control is in its extreme clockwise position that the sensitivity is actually that marked on the sensitivity switch.

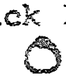
17. Wire up the circuit shown below.



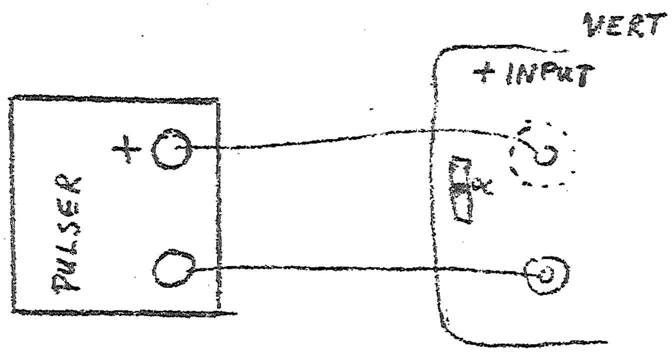
Set the horizontal sensitivity switch at 0.1 volt/cm and position the spot so that it is at (-4,0). Connect points A and B and note that the spot begins to move to the right. Note that it moves with approximately constant velocity over about the first three centimeters and then slows down as it moves along the rest of its path. Take another lead and short out the capacitor (i.e., connect the lead across the capacitor). Note that the spot jumps back approximately to (-4.0). Remove the short and the spot again sweeps to the right. Short the capacitor, set the horizontal sensitivity switch to 20 mv/cm and adjust the position of the spot to (-5.0). Now remove the short and note that the spot moves across the entire screen at very nearly constant velocity. When it reaches the right end of screen, short the capacitor momentarily and note that the spot jumps back to (-5.0) and then begins moving to the right at constant speed. A circuit which produces this type of horizontal motion of the spot is called a "sweep" circuit. All oscilloscopes have a built-in sweep circuit which may be used to produce this type of horizontal motion of the spot. Disconnect your circuit. Set the black LEVEL control switch to the AUTO. position.

18. Set the HORIZONTAL DISPLAY switch to the [SWEEP NORMAL (x1)] position. Set the SWEEP TIME/CM switch to 1 sec., and set the red sweep time/cm control knob to its extreme clockwise position. Note that the spot moves horizontally to the right at a constant speed, then jumps quickly to the left edge of the screen and begins moving to the right again at constant speed. If necessary, adjust the position control so that the spot starts from (-5.0). Use your clock to determine how long it takes the spot to cover 10 cm. It should be close to 10 secs., since the switch is set at 1 sec, which means the spot should take 1 sec. to traverse each centimeter. Set the SWEEP TIME/cm switch to .5 secs. and use your clock to determine how much time it takes for the spot to move 10 cm. It should be very nearly 5 secs., since with the switch at .5, it should take .5 sec. for the spot to travel each cm. Continue to experiment with different settings of this switch until you are confident that you understand the function of this switch. Note that when this switch is set at, say, 2 m sec (2 millisec = 2×10^{-3} secs.) one can no longer observe the motion of the spot. All one observes is the path of the beam as it moves to the right. With this setting, the spot moves to the right such that it takes .002 seconds to traverse each centimeter.

19. Set the SWEEP TIME/cm switch to 1 sec., and set the red sweep time/cm control knob approximately vertically   Measure the time for the spot to move 10 cm. (Note that it now requires more than 1 sec. for the spot to cover 1 cm.) Repeat with this red control knob set at its extreme counterclockwise position. Continue to experiment with different positions of this control until you are confident that you understand its function. Remember that it is only when this control is at its extreme clockwise position that the sweep speeds are those indicated by the switch position.

20. There are three small black two-position switches near the top right hand edge of the oscilloscope marked SLOPE, COUPLING AND SOURCE. Set the first of these to its + position, the second to DC, and the third to INT. Set the black level control knob so that the white dot is approximately vertical  . Set the Vertical Sensitivity

switch to .5 volt/cm. Set the SWEEP TIME/cm switch to 2 m sec. Connect the small box marked PULSER to the vertical input terminals, as indicated below.



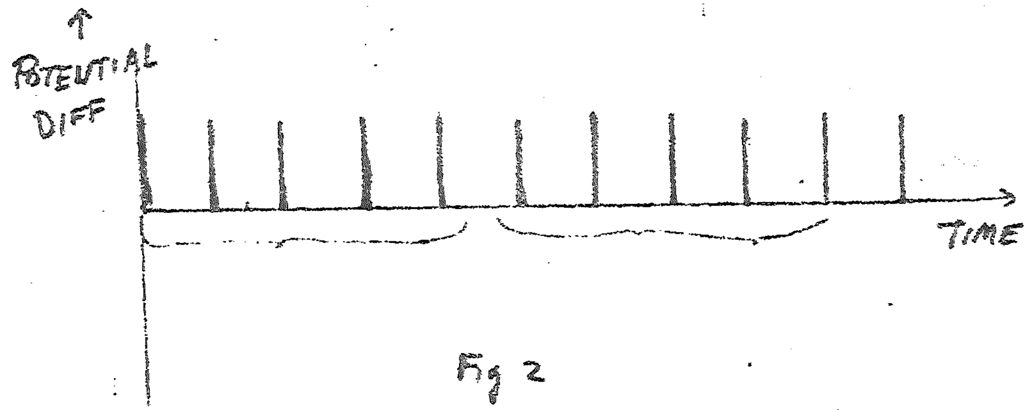


Fig 2

The pulser is a device which produces a potential difference between its two terminals which varies with time, as indicated in Fig. 2. Part of this pattern should be observed on the screen of the oscilloscope. It may be necessary to turn up the intensity (brightness) control, and/or to rotate the level control knob slightly clockwise from its vertical position. The function of the level control is to start the spot moving horizontally exactly at the time when the pulser is emitting a pulse. When adjusted correctly, the pattern appears to be stationary. While the sweep circuit is moving the spot from left to right at constant speed, the voltage from the pulser is deflecting it vertically, so one obtains a plot of the pulser voltage as a function of time. The first time the spot moves horizontally across the screen, one gets a plot of the first five pulses emitted by the pulser; the next time the spot moves across the screen, one gets a plot of the next five pulses emitted by the pulser, etc. Since each group of five pulses emitted look exactly the same as every other group of five pulses, the pattern appears to be stationary. Note that if the level control knob is rotated too far clockwise or counterclockwise from its vertical position, the pattern will disappear. Note the appearance of the pattern when the SWEEP TIME/ cm switch is at 1 m sec., .5 m sec., .1 m sec., 50 μ sec., 20 μ sec., and 10 μ sec.

21. Determine the time interval between the pulses emitted by the pulses. Use any convenient setting of the SWEEP TIME/cm switch. Make your measurement as precise as possible by using as much of the horizontal scale as possible.

Rev 5/22/72

Simple Pendulum

Object To compare the behavior of a real pendulum with the predictions based on a model.

Discussion

The real pendulum consists of a metal cylinder hung by a string from a fixed support and free to swing in a calibrated arc. A lamp, lens, photo-cell, timer, and oscilloscope make it possible to determine both the velocity of the cylinder at the bottom of its swing, and the time for one half a cycle.

The model consists of a particle of mass m, hung from a fixed point by a string of length L, free to swing in a vertical plane, and free of all dissipative (frictional) forces. If the particle is released from rest from some point such as P (Fig. 1) then it is easy to show the velocity it will have when it reaches 0 is given by

$$v_0 = \sqrt{2g\,L\,(1 - \cos A)} \qquad (1)$$

To calculate the time for the particle to go from 0 to Q is more difficult. We could get a first approximation to this time by using one half of v_0 as the *average* velocity over the path 0 Q, that is

$$t_{0\to Q} = \frac{\text{arc } \widehat{0Q}}{\left(\frac{v_0}{2}\right)} = \frac{LA}{\frac{v_0}{2}}$$

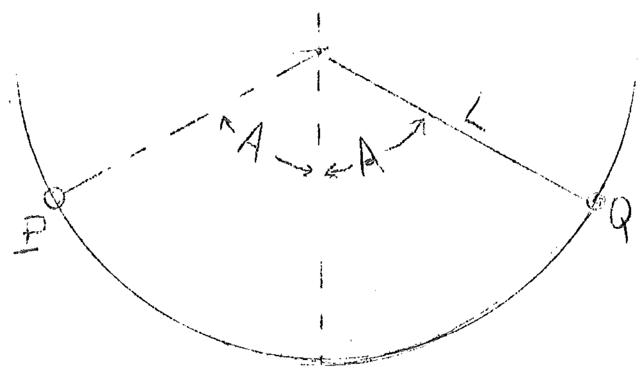


Fig 1

A better approximation would be to first divide the path $\widehat{0Q}$ into a number N of *equal* pieces, labeled

$\Delta s_0,\ \Delta s_1,\ \Delta s_2,\ \ldots,\ \Delta s_{N-1}$

as indicated in Fig. 2. As an approximation we could say that the time t_0 it takes to traverse the distance Δs_0 would be

$$\Delta t_0 = \frac{\Delta s_0}{(v_0 + v_1)/2}$$

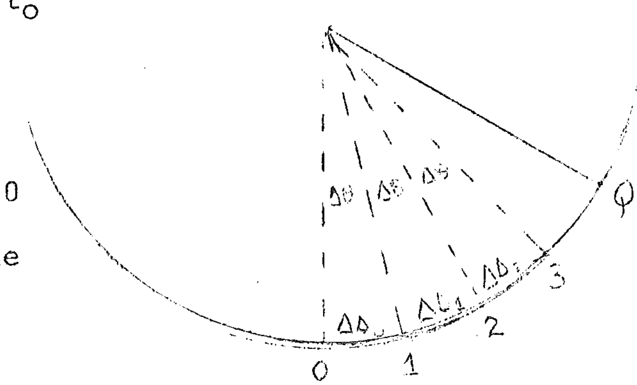


Fig 2

where v_0 is the velocity at point 0 and v_1 is the velocity at point 1. Similarly, the time to traverse the ith segment Δs_i would be

$$\Delta t_i = \frac{\Delta s_i}{(v_i + v_{i+1})/2}$$

The total time to go from 0 to Q is then

$$t_{0\to Q} = \sum_{i=0}^{N-1} \Delta t_i = \sum_{i=0}^{N-1} \frac{\Delta s_i}{(v_i + v_{i+1})/2}$$

This approximation improves as N is made larger, and one can get as close to the correct answer as one desires by making N sufficiently large.

Using the principle of conservation of total mechanical energy one can easily show that

$$v_i = \sqrt{2gL\left[\cos(i\Delta\theta) - \cos A\right]} \qquad (2)$$

Since all the Δs are the same size and equal to $L\,\Delta\theta = L\,\frac{A}{N}$ the time to go from 0 to Q may be written

$$t = \sum_{i=0}^{N-1} \frac{L\,\Delta\theta}{\dfrac{\sqrt{2gL\left[\cos(i\Delta\theta) - \cos A\right]} + \sqrt{2gL\left\{\cos([i+1]\Delta\theta) - \cos A\right\}}}{2}}$$

$$= \sqrt{\frac{2L}{g}}\;\frac{A}{N}\sum_{i=0}^{N-1}\frac{}{\sqrt{\cos(iA/N) - \cos A} + \sqrt{\cos([i+1]\,A/N) - \cos A}}$$

$\uparrow$ (3)

Method

The electronic timer or clock is controlled by a light beam and photo cell. The timing system operates in two modes determined by the position of the "Hold Mode" switch. When this switch is set in the "Gate" position, the clock will run only during the time the light beam is blocked. When this switch is in the "Pulse" position, the clock will start when the beam is first interrupted and will stop the next time the beam is interrupted. If the system is in the "Pulse" mode, and the cylinder is released from a point such as P, its first passage through 0 will start the clock, and its next passage through 0 will stop the clock. The clock reading will then be the time the bob takes to go from 0 to Q and back or twice $t_{0\to Q}$.

The photo cell is a resistive element whose resistance increases as the intensity of the light falling on it decreases. Consequently, if it is placed in a circuit such as shown in Fig. 3a then as the pendulum starts through the light beam, the voltage across R will begin decreasing. This decrease is used to initiate the scope sweep, so that one observes on the scope screen the manner in which the voltage across R varies as the pendulum passes through the light beam. The pattern observed on the screen is shown in Fig. 3b. The positions of the cylinder relative to the light beam that corresponds to points S and T are shown in Fig. 4. Thus, the time,t,

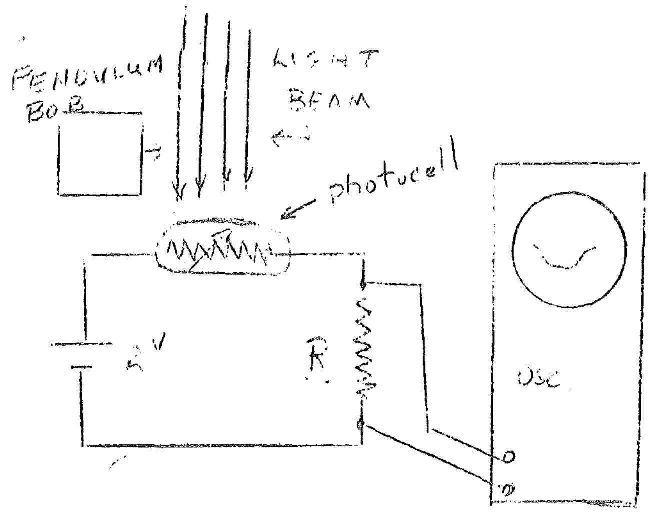
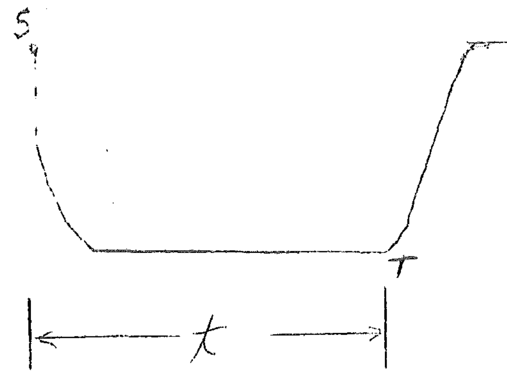


Fig 3 a

Fig 3b



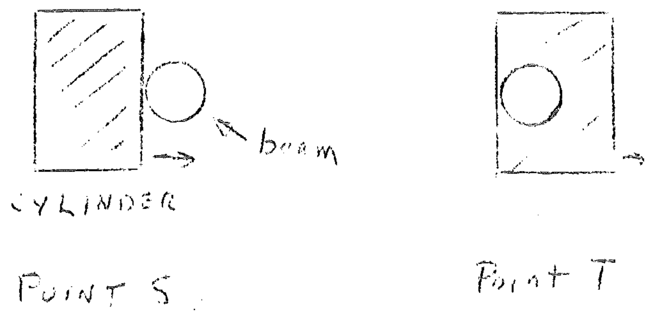


Fig 4

indicated in Fig. 3b is the time for the cylinder to travel a distance equal to the diameter of the cylinder. Dividing the diameter of the cylinder by t gives v_o.

Procedure

1. Wire up the photocell and timer as indicated in Fig.5

2. Read steps 3 through 6 and prepare (in your notebook) a suitable table for recording the data.

3. Set the "Hold Mode" switch to the "Pulse" position.

4. Use the electromagnet to position the cylinder at the point corresponding to P with A = 70°. Press the "Reset" button to set the clock to zero. Release the cylinder, allow it to swing through point 0 to Q and back through 0. Catch the pendulum after it passes through 0 on its way to P. Read and record the clock reading. The clock reading is in millisecs (10^{-3} seconds).

5. Repeat steps 4 four more times.

6. Repeat steps 4 and 5 for A = 60, 50, 40, 30, 20, 10.

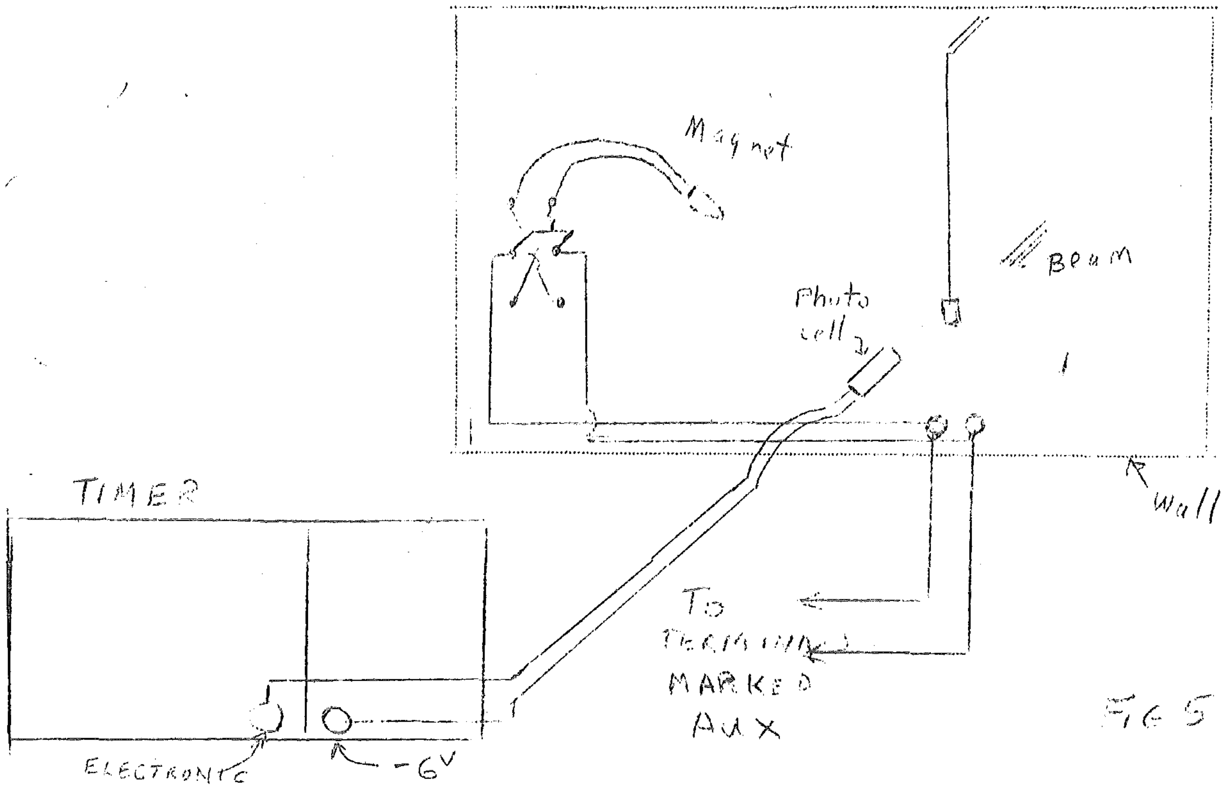


FIG 5

7. Wire up the photocell as indicated in Fig.6 . Set the three switches near the top right hand corner of the oscilloscope to -, DC, and INT. Set the "Level" control to 0, the Sweep time/Cm switch to 1 millisec/cm., and the vertical gain switch to 0.5 volts/cm. (THE sweep time/cm control knob must be in cal. position)

8. Start the pendulum swinging through about a 60° arc and note if the passage of the bob through the light beam produces a trace like that in Fig.3b. If not, adjust the "Level" control and/or the vertical gain until such a trace does occur.

9. Use the electromagnet to position the cylinder at the point corresponding to P with A = 70°. Release the cylinder and note the positions of points T and S of the scope pattern produced by the interruption of the light beam.# Record the sweep time/cm. setting.

10. Repeat step 9 several more times.

11. Repeat steps 9 and 10 for A = 60°, 50°, 40°, 30°, 20°, 10°. For each setting, use the most appropriate sweep time/cm setting.

Analysis

1. Determine the average and average deviation of each set of five time readings taken for each fixed value of A. Calculate the average and average deviation of your measurements of the length of the pendulum and the diameter of the cylinder. Use the average deviation as a measure of the uncertainty in the measurement of that quantity.

THE oscilloscope trace always starts from the same point if the position controls are not disturbed. You can determine point S by noting where the sweep starts when the Level control is at the Auto position.

2. For each fixed value of A determine v_o from your measurements of the time required for the pendulum to move through a distance equal to the diameter of the cylinder. Calculate the uncertainty in each of these values. (See Experimental Errors II, Section 6 and 7).

3. Calculate v_o for each value of A, using equation (1). Make a table comparing these calculated v_o's with the experimentally determined v_o's.

4. Write a FORTRAN program to calculate $t_{0\to Q}$ from equation (3) for the values of A used in the experiment. An N value somewhere between 50 and 100 should give adequate precision. Use the average value of your measurement of the length of the pendulum for L and 9.80 m/sec for g. Have this program processed by the computing center. (Each group is to write their own program, not merely borrow one from another group.)

5. Make a table comparing the calculated $t_{0\to Q}$ with the experimentally measured values.

6. Use the principle of conservation of total mechanical energy to calculate the velocity of the particle at point P, assuming it was released from rest at point Q (See Fig.7). From this expression derive equation (2).

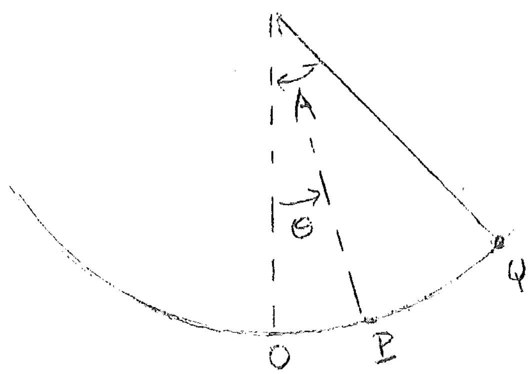


Fig 7

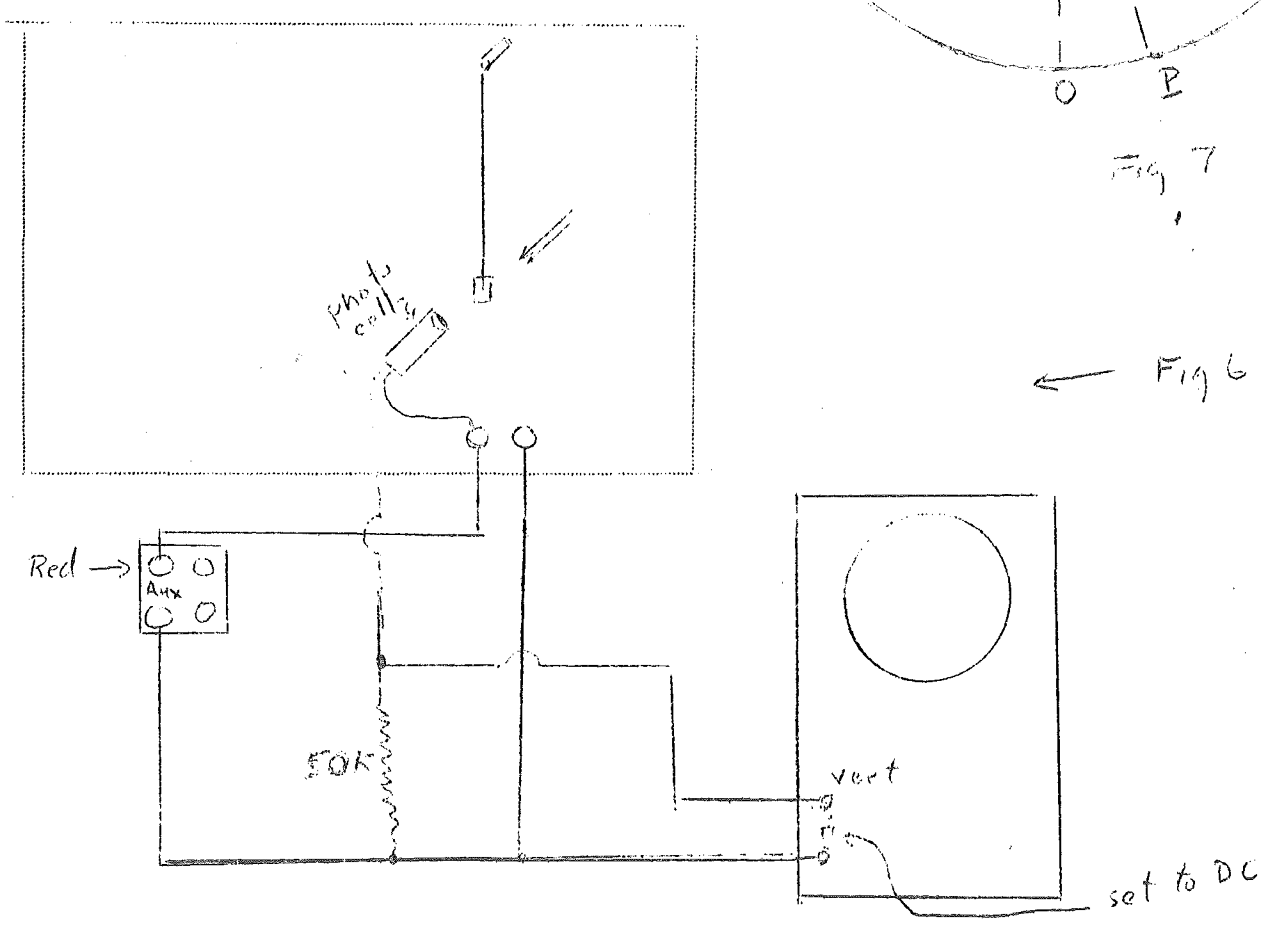


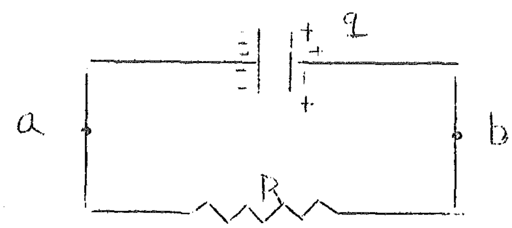
Fig 6

Measurement of Large Resistance

Ref.: Halliday, Resnick, Sect. 32-8

A. Capacitor Discharge

If a capacitor is charged so that charge q_o is on each plate and then allowed to discharge through a resistor as shown below, the potential difference across the capacitor must equal that across the resistor (they are across the same two points in the circuit ab) at all times during the discharge.



q = charge on plates at any time t
i = current at the same time

$$V_{ab} = \frac{q}{C} = i\ R \qquad i = \frac{dq}{-dt}$$

which yields the equation

$$\frac{q}{C} = -R\ \frac{dq}{dt} \qquad \text{or} \qquad \frac{dq}{dt} = -\frac{1}{RC}\ q \qquad (1)$$

Equation (1) shows that at any instant the capacitor will be discharging at a rate that is directly proportional to the charge still on its plates, and inversely proportional to the constant (RC). The fact that the discharge rate is determined by the product RC may be used to measure the resistance R indirectly by making measurements on the rate of discharge of a known capacitor C through the resistance element. From equation (1), one can show that the charge q still on the plates of a capacitor which began with charge q_o and discharged for a time t, is

$$q = q_o\ e^{-\ t/RC} \qquad (2)$$

(Show that equation 2 is a solution of the differential equation 1, and show that the time constant t = RC is the time for the charge to fall to fraction 1/e of its initial value q_o.)

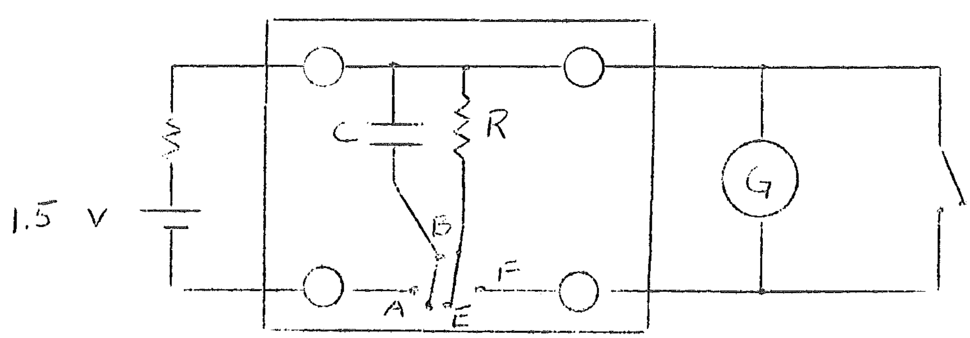
B. The Ballistic Galvanometer

The moving element of this type of galvanometer consists of a rectangular coil which is suspended between the poles of a magnet by a fine wire. When a charge q passes through the coil, the forces exerted by the magnetic field on the moving charges produce a turning moment or torque on the coil. The torque gives the coil an angular momentum, but the coil has a relatively large moment of inertia so that very little actual motion occurs in the time that it takes for charge q to pass through the coil. The coil continues to rotate however, twisting the suspension wire. The twisted suspension wire now exerts a restoring torque which decreases the angular momentum of the rotation, brings the rotation to a stop at some angle θ, and increases the angular momentum in the opposite direction. The result would be oscillatory motion of angular amplitude θ as long as no mechanical energy was lost from the system (it is almost frictionless). It is not difficult to show that the angle θ is proportional to the charge q which passed through the galvanometer coil. (Supplementary note.)

If one wishes to damp the oscillations of the galvanometer it is useful to recall that a coil rotating in a magnetic field generates an induced emf. (Chapter 35, Hall. & Res.) Short circuiting the galvanometer terminals completes an external circuit so that this emf can cause a current flow in the low resistance short-circuit. Thus the mechanical energy of the rotating coil is converted to electrical energy as in a generator, and this electrical energy is in turn dissipated as heat by the circuit resistance. The loss of mechanical energy by the system results in a quick damping of the oscillation.

Experiment

The capacitor in the circuit below may be charged by connecting A to B. It can then be discharged through resistance R for a measured amount of time t by connecting B to E for this time interval. The charge left on the plates at the end of this time interval may be determined by discharging the remaining charge through the ballistic galvanometer (connect B to F), which then deflects an amount D which is proportional to the charge q that has passed through it.



One then gets a feeling for the way the capacitor charge decays as a function of time by plotting graphs showing the galvanometer deflection D as a function of discharge time t and the natural logarithm of the galvanometer deflection as a function of time. The resistance R may be determined from the slope of the latter graph (first prove that $\ln D = \ln D_o - \frac{1}{RC}t$).

One thing that should be considered in this experiment is the fact that since R is large there may be other paths of comparable resistance through which the capacitor can discharge (even when B and D are not connected for example). Investigate this possibility before making any claims as to the accuracy of your result for the value of R.

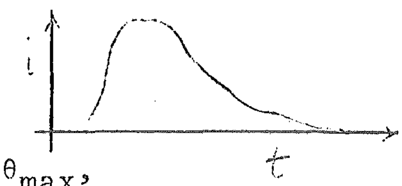
Note: The capacitors have the following values for the different circuit boards:

No. 1 C = 0.88 μF
No. 2 C = 0.90
No. 3 C = 0.88
No. 4 C = 0.88
No. 5 C = 0.91

Supplementary note
(ref.-Fund. of Elec. and Mag., A.F. Kip, McGraw-Hill, 1962)

How It Works

When a short pulse of electric current $i = i(t)$ passes through the coil of a ballistic galvanometer



question 1 - What happens?

answer 1 - It deflects through an angle θ_{max}, corresponding to a deflection (scale reading) $D \propto \theta_{max}$, and there exists a simple relation between D and $Q = \int i(t)\,dt$, the total charge passing through the coil.

question 2 - WHY? (and what is the relation)?

answer 2 - The parts of a galvanometer coil which are in a magnetic field are in a practically uniform radial field, as in Fig. 33-9 in Hall. and Res. (Beware - example 3 in the text discusses a galvanometer as it is often used to measure (steady) currents, not as we use a (ballistic) galvanometer in this lab exercise to measure charge in a current pulse. The mechanism of the instruments are basically the same, however.)

The magnetic torque on a galvanometer coil, when its plane is parallel to the magnetic field, as Fig. 33-9 (by eqn. 33-7)

$$\tau = N\, i\, A\, B.$$

When, instead of a steady current i, we supply a short pulse of current such as shown above, the angular impulse given to the coil is $\int \tau\, dt$, where the integration is over an interval covering the entire time during which current flows.

Integrating, Impulse $= \int \tau dt = N\,B\,A \int i\,dt = N\,B\,A\,Q.$

This is the special feature of the ballistic galvanometer. It receives an angular impulse that depends only on the total charge pulse flowing through it and not on the way the charge flow varies with time. The pulse is short compared with the natural period of the galvanometer coil. After the impulse ends the motion of the coil is 'ballistic' - under action of the suspension, only. From mechanics we have that this angular impulse gives the coil an angular momentum $I\,\omega_o$, where I is its moment of inertia and ω_o is the angular velocity just after the pulse has passed. Thus $I\omega_o = N\,B\,A\,Q$. The initial angular momentum $I\omega_o$ corresponds to an initial kinetic energy $\frac{1}{2}I\omega_o^2$. As the coil turns against the restoring torque of the galvanometer coil suspension, this kinetic energy is converted into potential energy. At the maximum deflection θ_{max}, Supplementary Note (Continued)

conversion is complete and we may write

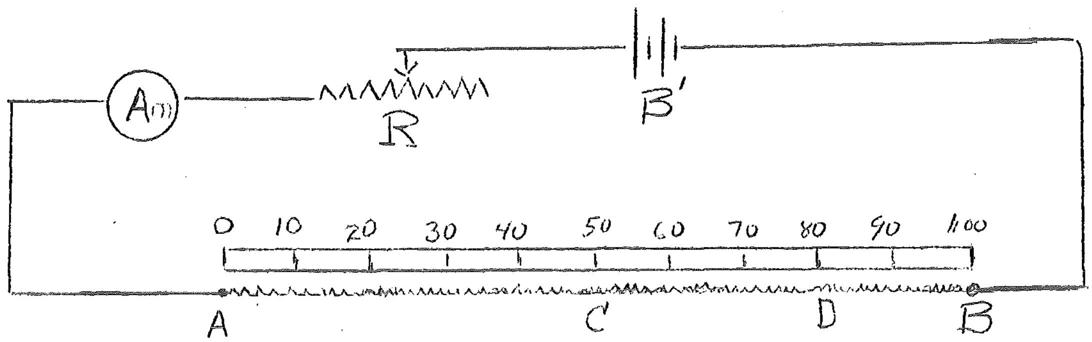
$$KE = \tfrac{1}{2}I\omega_o^2 \qquad PE = \tfrac{1}{2}k\theta_{max}^2$$

where k is the torque constant of the suspension (text, see torsional pendulum). Solving for ω_o and substituting in $KE = PE$ gives $Q = \left((Ik)^{\frac{1}{2}}/NBA\right)\cdot\theta_{max}$, or $Q \propto D$.

Tell in your report how you might verify this last relation experimentally.

The Potentiometer

The potentiometer is a widely used device for measuring D.C. voltages (potential differences). It is not as convenient to use as a voltmeter, but it has the advantage that it is more precise, and it doesn't affect the circuit to which it is connected. The principle of operation of a potentiometer can be understood from the following circuit. A 1-meter long uniform



wire, AB, is stretched along a meter stick and a known current is arranged to flow through this wire by means of a battery, B', a variable resistance R, and an ammeter. Let us suppose, for simplicity, that the 1-meter wire has a total resistance of exactly 1 ohm and the current in the wire is exactly 1 ampere. Then, according to Ohms law, the difference of potential between points A and B would be exactly 1 volt, the difference of potential between A and C exactly 0.5 volt, the difference in potential between A and D exactly 0.8 volt, and so on. Since (conventional) current always flows in a resistance from a higher to a lower potential, point A is 1 volt higher in potential than point B, is 0.5 volt higher in potential than point C, etc.; in fact, A is higher in potential than any other point of the uniform wire. Now suppose we have a battery whose emf is exactly 0.5 volt. Now the positive pole of a battery is always higher in potential than the negative pole by an amount equal to the emf of the battery, so for our battery of emf equal to 0.5 volt, the positive pole is 0.5 volts higher in potential than the negative pole. Suppose we connect the positive pole to point A, as in Fig. 2. After we make this connection, point A and

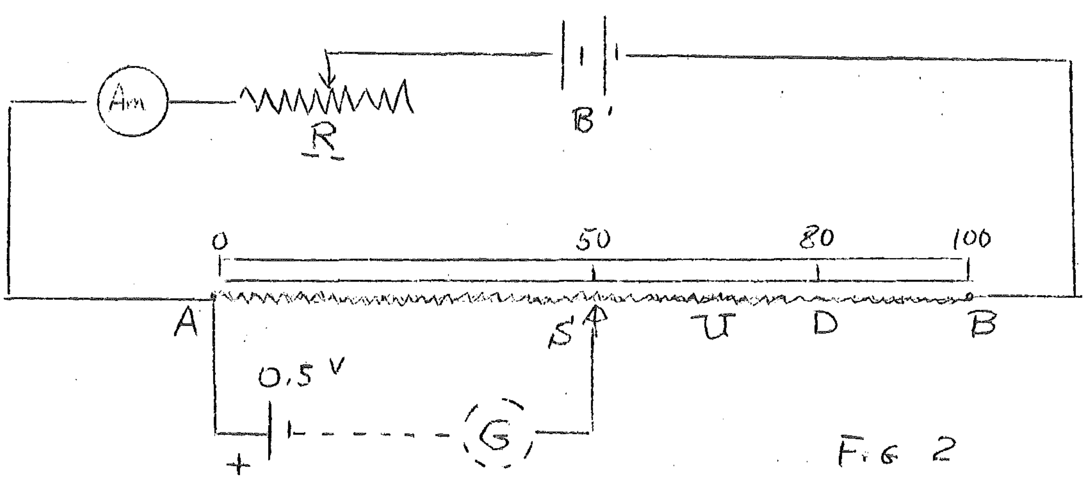


Fig 2

the positive pole of the battery are at the same potential since no current is flowing in the lead connecting A and the positive pole. The negative pole is always 0.5 volts lower in potential than the positive pole, point C is 0.5 volts lower in potential than point A, hence point C and the negative pole of the battery are at the same potential. If we connect the negative pole to point C via a sensitive ammeter, G, as indicated by the dotted lines, no current will flow, since current only flows when there is a difference of potential. If the emf of the battery had been 0.8 volts, we could have connected the negative pole to point D and no current would flow in the galvanometer. If we have a battery whose emf is unknown we can move the slider S along the wire until we reach some point, U, where the galvanometer reads zero. Then we know immediately that the difference of potential between the battery terminals is exactly that between A and U, which can be read directly from the meter stick. This is the principle of operation of a potentiometer.

It should be apparent that with the above arrangement, one could not measure emfs greater than 1 volt, since the difference of potential between the two ends of the wire is only 1 volt. To measure emfs between 0 and 2 volts, we could decrease R until the current in the wire were exactly 2 amps. Now the difference of potential between A and B would be 2 volts, that between A and C, 1.0 volt, and that between A and D, 1.6 volts. We could proceed exactly as before to determine the unknown emf. It should be clear that by properly choosing the current that is to flow in the uniform wire, one can measure any size emfs.

It turns out that the current in the uniform wire can be adjusted to the desired value more precisely by means of what is called a standard cell (battery) than by using an ammeter. Suppose we have a battery whose emf is known to be exactly 0.500 volts, and suppose we wish to adjust the current in the wire to be exactly 2 amperes. We know that if the current were 2 amperes, the difference of potential between A and the 25 cm point on the wire would be exactly 0.5 volts. We connect the positive pole of our battery to point A, and the negative pole via a sensitive ammeter

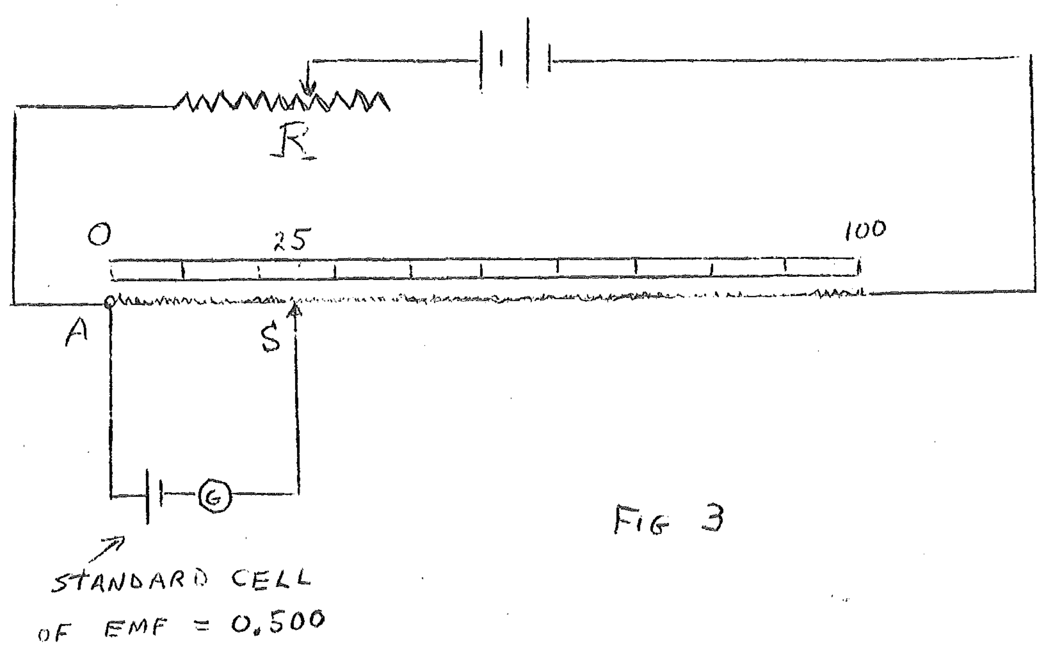


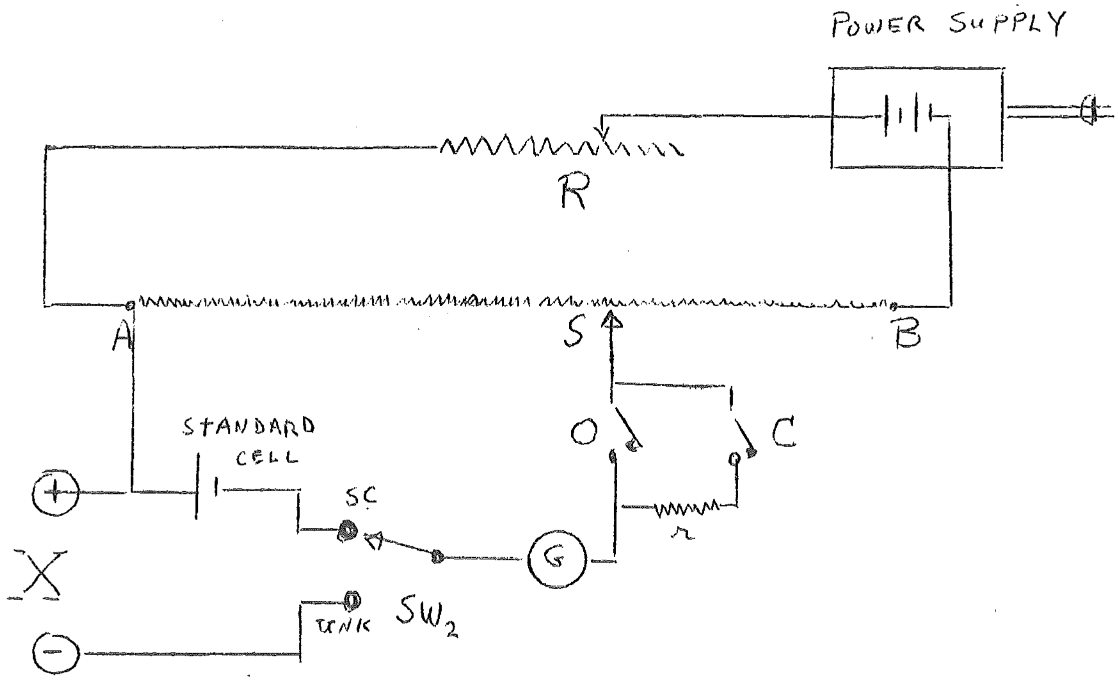
FIG 3

(galvanometer), G, to the 25 cm point of the wire as in Fig. 3. Now we adjust R until the galvanometer reads zero. When this is the case, the potential difference between A and the 25 cm point must be exactly the same as the difference of potential between the poles of the battery, namely 0.500 volts; hence, the current in the wire must be exactly 2 amperes. Suppose the emf of the standard cell were 1.35 volts. To what point of the wire should the sliding contact S be connected if it is desired to adjust the current in the wire to 2 amperes?

The potentiometer you will use in this experiment differs from the one described above in that the uniform wire, AB, is wrapped as a helix around a cylinder which is then enclosed in a plastic case. The position at which the slider S makes contact with the wire is indicated by a dial mounted on the top of the cylinder. When the dial reads zero, the slider is at the end of the wire corresponding to point A, and when the dial reads 1000, it is at the end corresponding to point B, when it reads 500 it is at a point corresponding to point C, and so on. The circuit diagram of the entire set up is shown in Fig. 4. The battery B′ is actually a regulated power supply which must be plugged in to an AC outlet to function. The variable resistance R is enclosed in a green plastic cylinder and its resistance is varied by turning the knob at one end of the cylinder. The standard cell is a flat cylindrically shaped mercury cell having an emf approximately equal to 1.35 volts. (The exact value is written on the cell). The switch SW_2 simply makes the process of calibrating of the potentiometer (i.e., adjusting the current to the proper value) more convenient. The swtiches marked C and 0 are provided to prevent a large current being drawn from the standard cell. <u>Switch C is used to obtain a preliminary balance</u>, and then <u>swtich 0 is used for the final balance. The proper technique is to tap the switches rather than hold them closed for a long period of time</u>.

<u>Procedure</u>

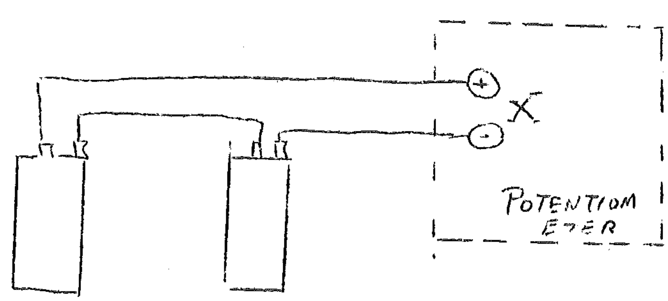
1. The resistance of the uniform wire is 100 ohms. If the current in this wire is adjusted to be .020 amperes, then the potential difference between end A and B would be 2 volts. If this <u>were</u> actually the case, then there would be some point of the wire which would differ in potential from A by exactly the standard cell voltage. Calculate the dial reading that would correspond to this point and set the dial to that value (e.g., if your standard cell is 1.350 volts, the dial should be set to 675). Rotate switch SW_2 to the S.C. side and adjust the resistance R until the galvanometer indicates zero current when switch 0 is closed momentarily. <u>Always use switch C to obtain a preliminary balance before using switch 0.</u> Your potentiometer is now calibrated, and there is a difference of potential of 2 volts between the two ends of the wire.



2. Rotate SW_2 to the UNK position. Connect a dry cell to the terminals marked X, being careful to connect the positive (center) terminal of the battery to the terminal marked +. Now move the slider until a point of the wire is reached such that the galvanometer doesn't deflect when 0 is held down momentarily. (Again, use the switch marked C to obtain a preliminary balance). Note the reading on the dial when a balance is obtained, and from this calculate the emf of the dry cell. It should be somewhere in the neighborhood of 1.5 volts.

3. Reverse the connections at the dry cell so that the positive terminal of the battery is connected to the negative terminal of the potentiometer. Using only the C switch, note that it is impossible to find a position of the slider which will produce zero galvanometer deflection. Is the galvanometer deflection smaller when the slider is at the zero end of the scale (point A) or when the slider is at the 1000 div mark (point B).

4. Connect two dry cells in series and then connect the combination to the potentiometer terminals, as indicated below. This combination has an emf of about 3 volts



Using only the C switch, note that it is impossible to find a position of the slider which will produce zero galvanometer deflection. Is the galvanometer deflection smaller when the slider is at the zero end of the scale (point A) or when the slider is at the 1000 div mark (point B).

5. There are essentially only two conditions in which it is impossible to get a balance on a potentiometer; either the point of higher potential of the unknown emf is not connected to the + terminal (point A) of the potentiometer or the unknown emf is too large. You encountered the first case in step 3 and the second case in step 4. Note that it is possible to determine which condition you have by noting whether the galvanometer deflection increases or decreases as the slider is moved from the zero end (point A) to the 1000 div end (point B). There is a special case of the second condition which is worth noting, namely, the situation in which there is no current in the uniform wire, a condition easily obtained by failing to plug in the power supply. In this case, all emfs will appear to be too high even if they are correctly connected.

A black box is provided which has an emf somewhere between 0 and 2 volts. The positive terminal is not indicated. See if you can with your potentiometer determine which is the positive terminal. After you have done this, determine the unknown emf.

6. The potentiometer was calibrated in step 1 so that is essentially direct reading. Once you obtain a balance for an unknown emf, you need only to multiply the dial reading by 2 to obtain the value of the unknown emf (of course you have to decide where the decimal point goes). This manner of calibrating the potentiometer is convenient, but not necessary. All that is required is that the current in the uniform wire be constant and of such a magnitude that one can obtain a balance both with the standard cell and with the unknown emf. To illustrate this case, turn the knob which varies R a couple of turns in either direction. There is now some unknown current flowing in the wire. Connect the dry cell that you used in step 2 to the X terminals of the potentiometer, and move the sliding tap until the galvanometer reads zero when switch 0 is held down momentarily. Record the reading of the dial and let us refer to it as D_x. Now rotate switch SW_3 to the SC position and adjust the sliding tap until the galvanometer doesn't deflect when 0 is held down momentarily. Record this dial reading and let us refer to it as D_{sc}. Now calculate the emf of the dry cell from

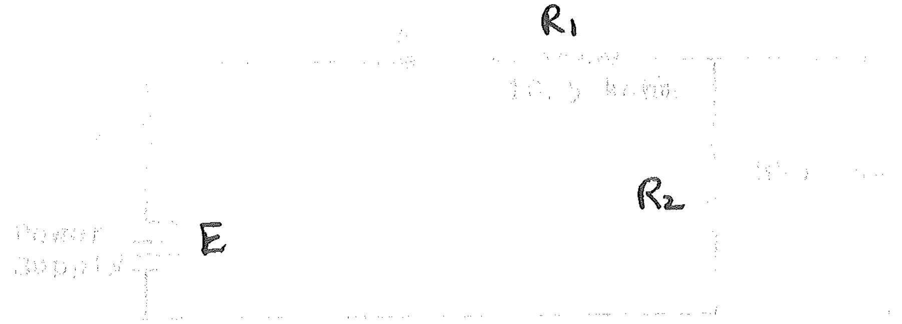
$$\frac{\text{emf of dry cell}}{D_x} = \frac{\text{emf of standard cell}}{D_{sc}}$$

This computed value should compare favorably with the value for the emf of the dry cell that you calculated in step 2.

7. Using the technique suggested in step 6, make three more measurements of the emf of the dry cell, each using a different unknown current. Calculate the average of all five values and the standard deviation of the mean.

[illegible] ... the scale ... [illegible]

... as shown below and measure the ... [illegible] ... ammeter and again using the ... [illegible]



Assuming the internal resistance of the power [illegible] to be negligible compared to the other resistances [illegible] the circuit, what should voltage V_{BC} be according to theory (roughly - the source voltage is only approximately 6, 12 or 36 volts depending on the supply given [illegible] the instructor). How do the voltages measured [illegible] voltmeter and potentiometer compare with this [illegible] explain discrepancies and show that the internal [illegible] of the voltmeter can be obtained from the relation

$$R_V = \frac{R_1 R_2}{R_2 (E/V) \quad (R_1 \quad R_2)}$$

where V is the voltage measured with the voltmeter [illegible] R_1 and R_2 are as shown in the diagram and E [illegible] of the power supply (d-c). Assume that the meter [illegible] calibrated correctly. From your data find the [illegible]

E

8\. Connect the unknown source ("black box" which [illegible] presumably) to the decade resistance box and [illegible] shown in the diagram below. Leave the switch [illegible]

[illegible]

Measure the terminal voltage V_T of the black box with a [illegible] while drawing current from it (set the decade resistance box to 300 ohms). Repeat with R = 200, 100, 70, 50, 40, 30, 20 and 10 ohms checking the calibration (standard cell) occasionally (and be very careful to leave the switch open while turning dials on the resistance box—an accidental resistance of less than 5 ohms across the terminals will draw enough current to burn out the box)

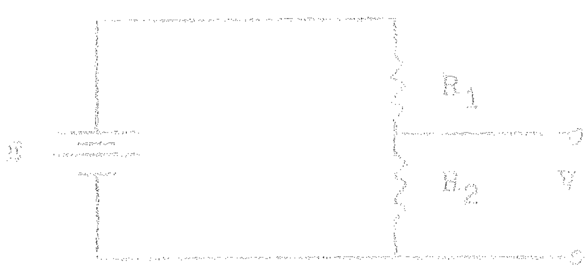
5. Plot a graph of the terminal voltage V_T of the "black box" versus the current I it supplies to the series loop shown above. Since

$$V = E - Ir = IR$$

this graph should be a straight line with slope (-r) and intercept E (if it is not, perhaps E is changing as current is drawn from the box). Determine the value of the internal resistance of the unknown from the slope of the line or, if it is curved, from the slope of the tangent at I = 0. Record the values of E and r for the unknown.

EXERCISES:

Given a circuit such as that shown below.



1. Show that the potential difference $V = \left(\frac{R_2}{R_1 + R_2}\right)E$

If V is to be equal to (1/10)E, what fraction must R_2 be of R_1? This circuit arrangement is called a voltage divider.

2. If R_1 = 1800 ohms, R_2 = 100 ohms and E = 100 volts, what is the smallest resistance that an accurately-calibrated voltmeter can have if it is to measure the voltage V with an error of less than 5%?

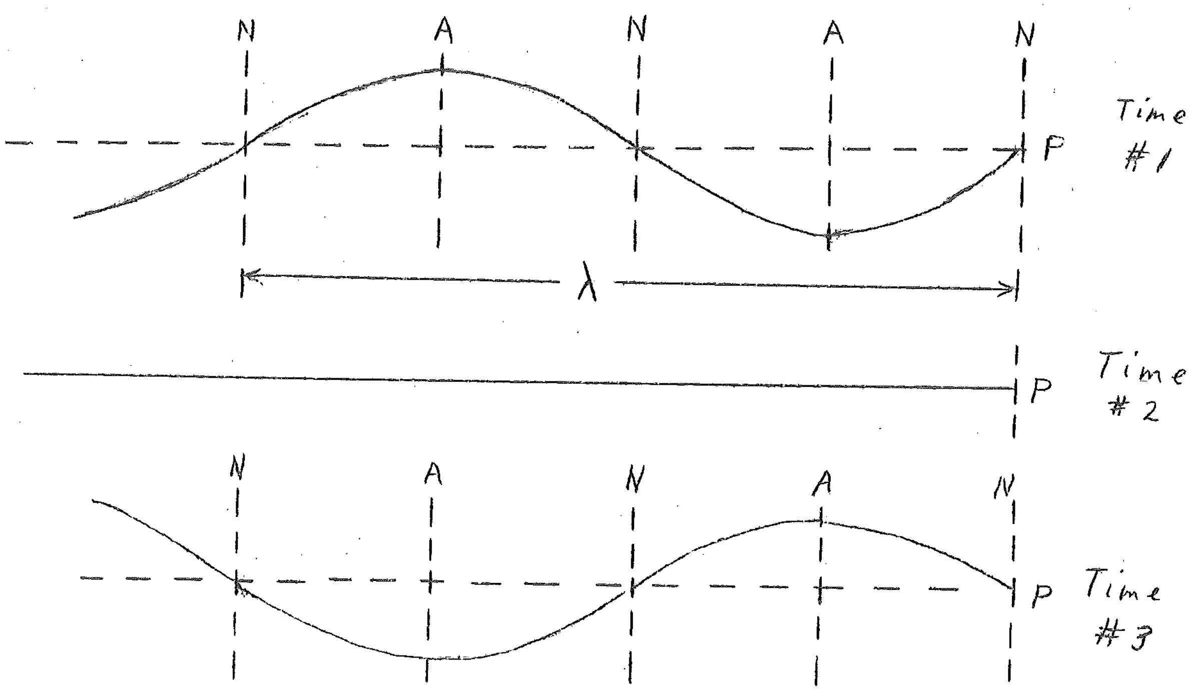
WAVES IN STRINGS

OBJECT: To study the conditions necessary for the production of standing waves in a string.

THEORY: When a string of mass per unit length μ and under tension T is disturbed in a regular way, waves travel in both directions at a speed

$$c = \sqrt{T/\mu} \qquad (1)$$

If these waves come to a point in the string where it is not free to move they are reflected and travel back in the opposite direction, combining with any "incoming" waves to produce the resultant displacement of the string at each point. Under certain conditions the waves may be produced in such a way that the incoming and reflected waves are sine curves of wavelength λ . In this case the resultant shape of the string will also be a sine curve of wavelength λ . However, this resultant sine wave remains fixed in position longitudinally while the amplitude fluctuates, rather than moving to the right or left as its component traveling waves do. The wave is therefore called a "standing" or "stationary" wave. Successive "snapshots" of the string near a fixed point P might look as shown below.



Certain points N along the string are stationary at all times and are called nodes. Halfway between the nodes are the antinodes A where the vibrations have the largest amplitude. <u>The distance between two successive nodes (or two antinodes) is equal to half a wavelength</u>. In the preceding figure the point P must of course be a node since it is not free to move.

Suppose that a string of length L, fixed at both ends and under tension T, is disturbed in such a way that sine waves travel along it in both directions and are reflected at each end. Standing waves such as those described above can result only if the wavelength λ is of such a magnitude as to make the two fixed ends of the string nodes. This means that standing waves can occur only if they (and their component traveling waves) have a wavelength λ such that L equals an integral number of half-wavelengths.

$$L = n \lambda/2 \qquad n = 1, 2, 3, ...$$

or

$$\lambda = 2L/n$$

Since the speed of the traveling wave components c is related to their frequency and wavelength by the expression $c = f\lambda$, one can say that standing waves may be set up in the string by vibrations of a number of different frequencies

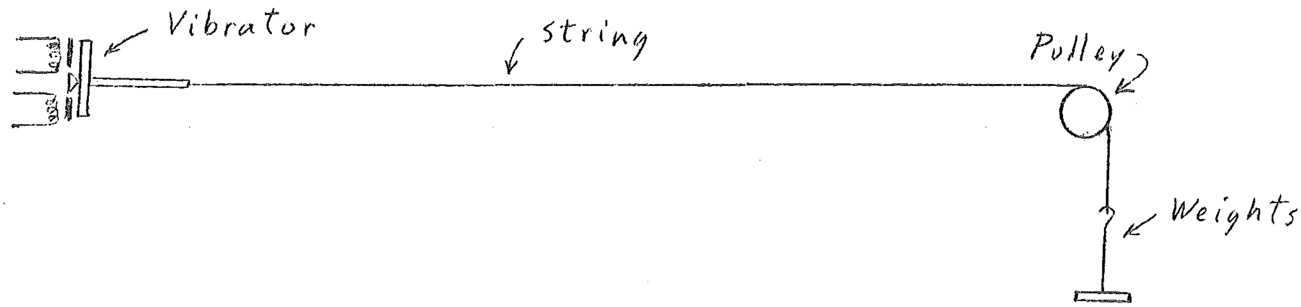
$$f_n = c/\lambda = nc/2L \qquad (2)$$

Making use of equation (1), the frequencies f_n which give standing waves, are related to the tension T in the string by the equation

$$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad (3)$$

For a given tension T the lowest allowed frequency (n=1) is called the fundamental or first harmonic, the next highest frequency (n=2) the second harmonic, the next (n=3) the third harmonic and so forth. If the string is disturbed in a regular but arbitrary fashion the resultant wave is composed of a combination of sinusoidal waves each having one of the allowed frequencies, rather than a standing wave of a single frequency.

In this experiment one end of the string is attached to a rod which vibrates sinusoidally at a single fixed frequency (the amplitude is small enough that the end of the string may be considered fixed). In general, waves are set up with no well defined nodes and small amplitudes but when the tension T is adjusted so that equation (3) is satisfied for an integral value of n, standing waves of large amplitude and definite, although not perfect, nodes result. The tension in the string is varied to obtain standing waves for several different values of n.



INSTRUCTIONS:

(1) Arrange the apparatus as shown in the figure.

(2) Vary the tension by adding or subtracting weights at the pulley end. Find and record those values of T (in newtons) for which well defined standing waves of maximum amplitude are set up. For each standing wave determine the distance between a node near the pulley and a node near the vibrating rod and divide this distance by the number of "loops" between these points to obtain $\lambda/2$. Make a table showing the values of T and λ for the standing wave modes. (Note: to obtain some of the smaller tensions that will be required, it will be necessary to use the 5 gram weight hanger provided. Use this smaller hanger only for weights of less than 75 grams.)

(3) With the string under one of the tensions that produced standing waves in (2), cut out a section 1 meter long and determine its mass in kilograms. Record this mass per unit length μ.

(4) From equation (1) and the fact that the speed of a traveling wave is equal to the product of its frequency and wavelength, show that for a fixed frequency the quantity λ^2 is directly proportional to the tension T in the string. Plot a graph of T versus λ^2 using the corresponding values of T and λ from part (2). Determine the frequency f from the slope of the curve.

EXERCISE:

Make estimates of the precision of T, μ, and λ and calculate the corresponding precision in the value of f obtained.

[illegible] frequency [illegible] only [illegible]
[illegible] given by the eq. [illegible]

$\nu = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$ Prove This! [illegible]

[illegible] constant until T is varied until [illegible]
[illegible] produced. These values of f correspond to [illegible]

[illegible] in this experiment, f remains constant [illegible]
[illegible] vibration at only one frequency)

[illegible] the moment, consider the equation $f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$
[illegible] f_1, f_2, f_3 etc. (f remains constant), the [illegible]
[illegible] to excite the fundamental frequency and its [illegible]
[illegible], third, etc. harmonics) is to vary the tension
[illegible] string until one obtains a large amplitude standing [illegible]

$f_1 = \frac{1}{2L}\sqrt{\frac{T_1}{\mu}}$, $f_2 = \frac{2}{2L}\sqrt{\frac{T_2}{\mu}}$, $f_3 = \frac{3}{2L}\sqrt{\frac{T_3}{\mu}}$, etc.

Note: $\frac{1}{2L} < \frac{2}{2L} < \frac{3}{2L} < \frac{4}{2L} <$ etc.

[illegible] for the frequency to remain constant [illegible]

[illegible] $T_1 > T_2 > T_3 > T_4 >$ etc.

[illegible] $f_1 = f_2 = f_3 = f_4 = f_5 = f_6 =$ etc.

[illegible] $\sqrt{T_1} = 2\sqrt{T_2} = 3\sqrt{T_3} = 4\sqrt{T_4} = 5\sqrt{T_5} = 6\sqrt{T_6} =$ [illegible]

[illegible] $4T_2 = 9T_3 = 16T_4 = 25T_5 = 36T_6$ [illegible]

[illegible], the tension that corresponds to n [illegible]
[illegible] as the tension that corresponds [illegible]
[illegible] upon the [illegible]

[illegible] $\frac{4}{9}T_2 = T_3$, $T_4 = \frac{9}{16}T_3$ [illegible] $\frac{1}{16}$ etc.

How does your Data compare with that predicted by theory?

(Example: (Courtesy of Holcomb & Wright).

Tension which produce max. amp. st. wave (nt.)		
.069	T_5	$n=5$
.176	T_4	$n=4$
.284	T_3	$n=3$
.490	T_2	$n=2$
1.127	T_1	$n=1$

Let's assume that for $n=1$, $T_1 = 1.127$ nt. (ie. you assume that a tension of 1.127 nt. will produce a fundamental frequency). Then $\frac{T_1}{4}$ will give the tension required to produce the second harmonic ($n=2$).

Calculate this tension and see how it compares with the exp.

	theory	~~actual~~ (exp.)
$T_2 = \frac{1.127}{4}$	.282	.490
$T_3 = \frac{1.127}{9}$	.125	.284

So you can see, our initial assumption is wrong.

Next, assume that a tension of 1.127 nt. corresponds to $n=2$, the second harmonic.

.069	T_6	$n=6$
.176	T_5	$n=5$
.284	T_4	$n=4$
.490	T_3	$n=3$
1.127	T_2	$n=2$
?	T_1	$n=1$

	theory	exp.
$T_3 = \frac{4}{9} T_2$	.500	.490
$T_4 = \frac{9}{16} T_3$	.282	.284
$T_5 = \frac{16}{25} T_4$	.180	.176

This time, our assumption is correct. What tension will produce the fundamental mode or frequency?

By comparing the values obtained in the laboratory with those obtained from theory, perhaps a better understanding of the experiment will result.

The Hall Effect

Introduction

Fig. 1 shows 2 equal point charges, both positive. Points M and N are positioned as indicated. Which point, M or N is at the higher potential? Now refer to Fig. 2. Which point, M or N is at the higher potential?

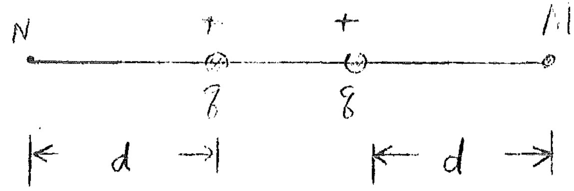


Fig 1

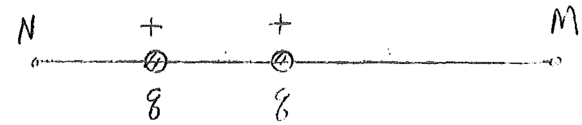


Fig 2

Suppose current is flowing through a rectangularly shaped conductor as indicated in Fig. 3. As indicated in the diagram (conventional) current is

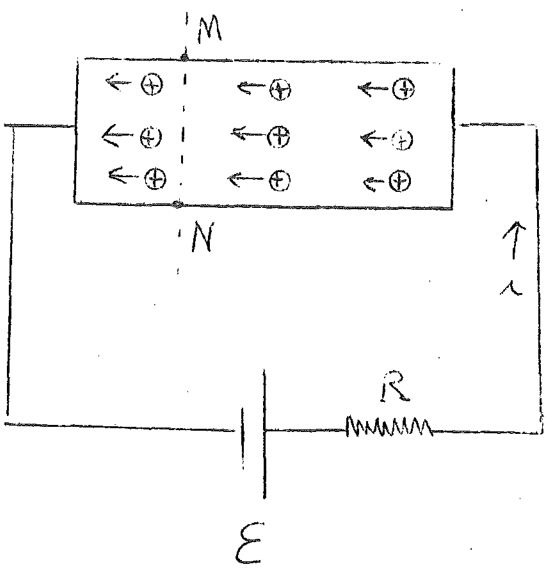


FIG 3

NO MAGNETIC FIELD

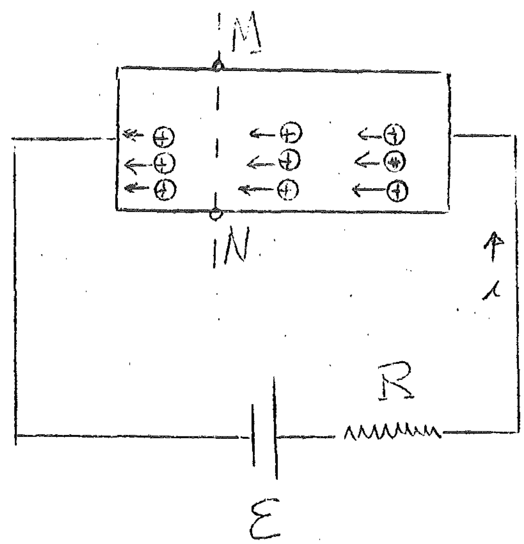


FIG 4

MAGNETIC FIELD $\vec{B}$ DIRECTED INTO PLANE OF FIGURE flowing from right to left in the sample. Let us assume that in the sample the current is due to positive charges flowing from right to left as indicated. Since like charges repel each other, the positive charges will be arranged approximately uniformly across a given cross-section of the conductor, as indicated. Because of this uniform arrangement, one would expect that two points such as M and N would be at the same potential. Suppose that we now establish a uniform magnetic field of magnitude B which is directed into the plane of the figure. Since the positive charges are moving with a velocity $\vec{v}$ to the left, there will be an additional force on them given by

$$\vec{F} = q \vec{v} \times \vec{B}$$

Since q is +, the right hand rule indicates that this force is directed downward. As a result of this magnetic force, the moving charges are no longer distributed uniformly over a given cross-section but will be crowded into the lower portion of the sample as suggested in Fig. 4. As a consequence we would now expect N to be at a higher potential than M. Experimentally, one finds that for some samples this is indeed the case; however, for other samples just the reverse is true, i.e. one finds M to be at the higher potential, even though the direction of the conventional current in the sample and the direction of $\vec{B}$ are the same. One concludes that for those samples for which N is higher in potential than M, that conduction in the sample is actually due to positive carriers moving in the direction of the conventional current. For those samples in which M is higher in potential than N, one concludes that conduction in the sample is due to negative charges moving opposite in direction to the conventional current. (You should be able to convince yourself that negative charges moving from left to right would also be forced downward when a magnetic field is applied which is directed into the plane of the paper.)

The difference in potential, $V_N - V_M$, that is produced between two points such as M and N when a sample carrying current is placed in a magnetic field directed at right angles to the current, is called the Hall voltage V_H. On the basis of the explanation of the Hall effect given above, would you expect the Hall voltage measured for a given sample to increase, decrease, or remain the same if one increased the magnitude of the magnetic field, keeping everything else constant?

Experimentally, it is found that for a given sample

$$V_H = R_H \frac{B\,i}{t} \tag{1}$$

where B is the magnitude of the magnetic field, i is the current in the sample, t is the thickness of the sample (see Fig. 5), and R_H is a constant, characteristic of the sample. R_H is called the Hall coefficient of the sample and is considered to be positive if the charge carriers are positive and negative if the charge carriers are negative. As is evident from equation (1), R_H can be determined if one measures V_H, i, B and t. In this experiment we are going to measure V_H, i and B. The thickness t has already been measured carefully and is marked on the sample holder. Measurements of B and i are reasonably straightforward. The measurement of V_H is somewhat complicated due to the fact that experimentally points M and N do not always lie exactly on the same cross-section. Consider for example Fig. 6 which shows points M and N misaligned. With the (conventional) current flowing as indicated by the arrow, point N will be higher in potential than M even without a magnetic field. It is higher because conventional current always flows in a conductor from a higher to a lower potential. If the carriers are positive and flowing as indicated, then the application of a magnetic field directed into the paper would force the positive carriers downward, and make N even higher in potential than before. It is this increase in the potential difference which is the true Hall voltage. The true Hall voltage V_H is most easily determined by making two measurements of the difference in potential between M and N, one with the field directed into the paper and one with the field directed out of the paper. The true Hall voltage then is

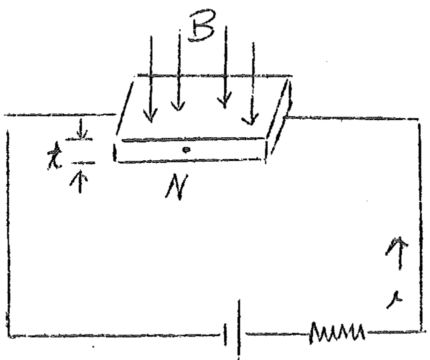
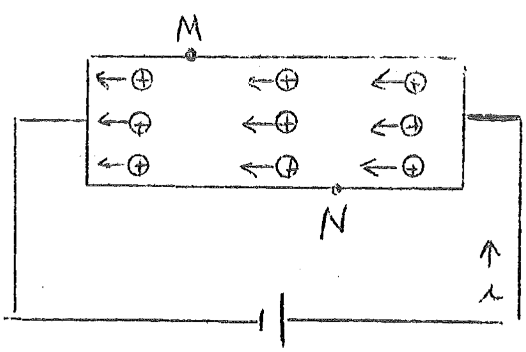


Fig 5



$$V_H = [(V_N - V_M) - (V_N - V_M)^1] / 2 \qquad (2)$$

where $V_N - V_M$ is the first of these two measurements, and $(V_N - V_M)^1$ is the second of these two measurements. Note that these two quantities may have different signs, depending on the extent of the misalignment of M and N.

Procedure

1. Since you will use a potentiometer to measure $V_N - V_M$ as well as to determine the current flowing in the circuit, it will be necessary for you to refresh your memory as to its operation and method of calibration. (Reread, if necessary, the experimental write-up dealing with the use of a potentiometer.) Since the voltages you will measure in this experiment will not exceed 2 volts, it will be helpful to calibrate your potentiometer so that a dial reading of 1000 division corresponds to a potential of 2.000 volts. Make this calibration before proceeding.

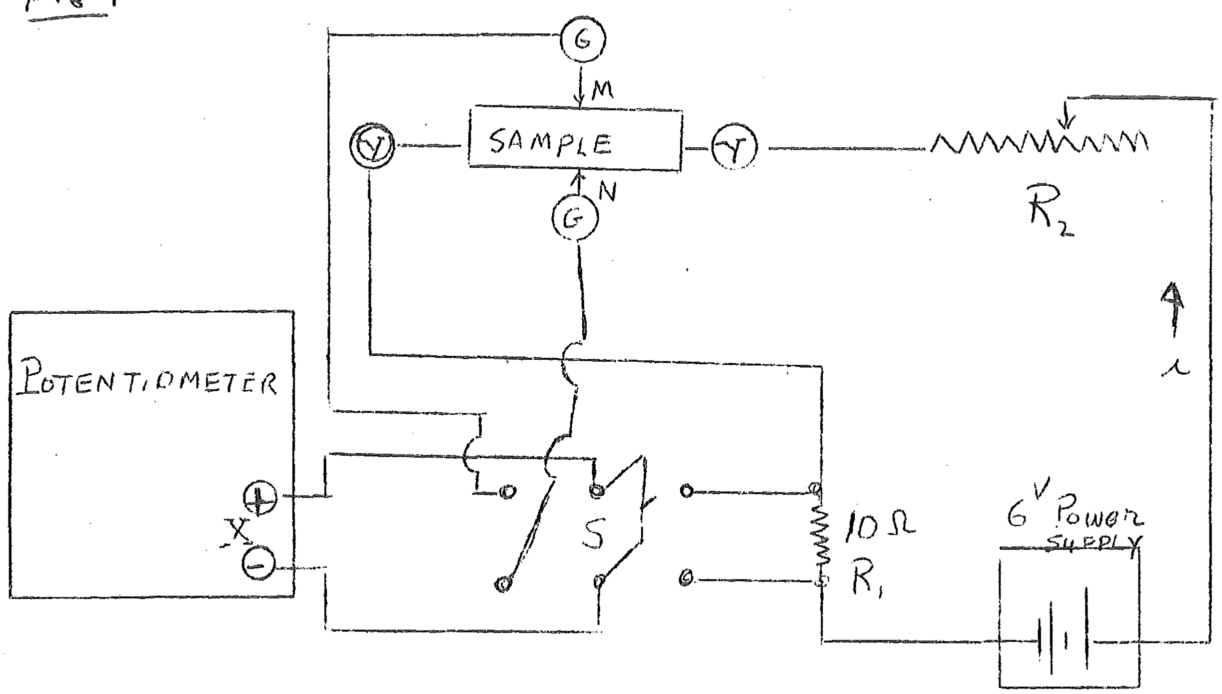
2. Your sample is encased in a piece of plastic to protect the leads. Note that points M and N are brought out to green terminals and that the yellow terminals are connected to the <u>ends</u> of the sample.

3. Wire up the circuit as indicated in Fig. 7.

R_1 ≡ PRECISION RESISTANCE BOX

R_2: RHEOSTAT (MAX RES. > 200 Ω)

FIG 7



<u>Do not plug in your 6 volt power supply until the instructor OK's your circuit.</u> Set the sliding contact on the rheostat about midway between the two ends. Note that with the switch S thrown to the right the potentiometer is connected across the two ends of the 10 ohm. precision resistor, while, when it is thrown to the left, it is connected to points M and N.

4. After the instructor OK's your circuit and you have calibrated your potentiometer, plug in the power supply and throw S to the left. Orient the board holding the sample and permanent magnet

so that the north pole (marked N) of the magnet is nearest you. The magnetic field B between the poles will now be directed away from you as indicated in Fig. 8. Orient the sample so that it can be

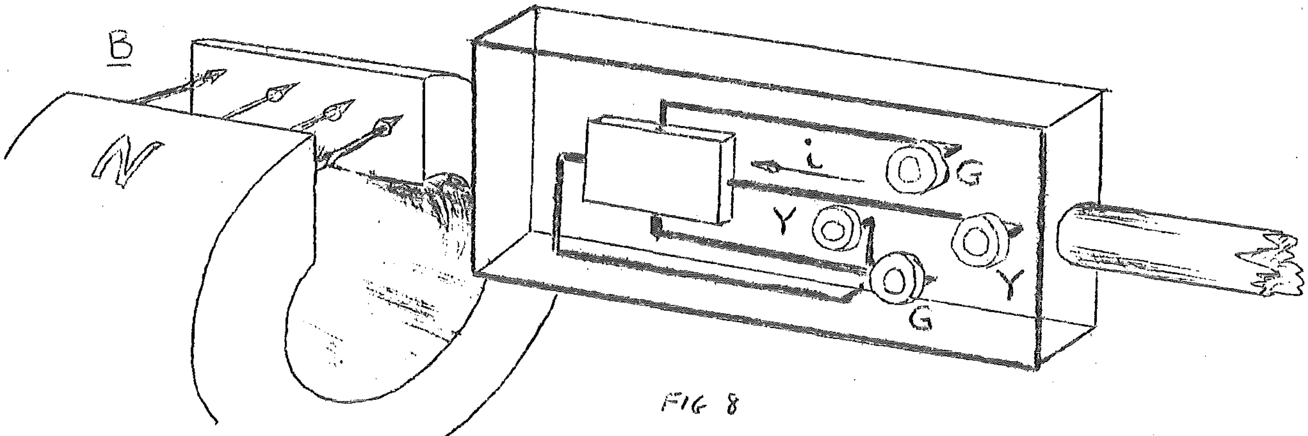


FIG 8

slipped into the field. With the sample in this position but still out of the field, carefully trace the circuit starting from the positive (red) terminal of the 6 volt power supply to determine if the conventional current is flowing from right to left through the sample as you view it. If necessary, reverse the connections to the two yellow terminals so that the current does flow from right to left. Carefully trace the lead from the top end of the sample (call this point M) to see if it goes to the potentiometer terminal marked + or that marked -. Tie a string or rubber band around this lead to remind you it is the lead that goes to M. With the sample still out of the field see if you can detect with your potentiometer any difference of potential between points M and N. Since you do not know in advance whether M or N is at the higher potential, and since it is impossible to obtain a balance on the potentiometer unless the point of higher potential is connected to the + terminal of the potentiometer, it may be necessary to reverse the leads going to the potentiometer, in order to get a reading. If you are able to detect a difference of potential, record this and note whether it is M or N which is at the higher potential. If you are unable to detect any difference of potential between M and N, you can conclude that the misalignment is very small, and the information you obtain in step 5 will suffice to determine the sign of the charge carriers.

5. Insert the sample into the field. You should now be able to measure a difference of potential between M and N although again it may be necessary to reverse the leads at the potentiometer to obtain a reading. Once you are able to get a balance, record the reading and again note whether M or N is at the higher potential.

6. From the information you obtained in steps 3 and 4 and the explanation given in the introduction you should be able to decide whether the charge carriers are positive or negative.

7. Throw switch S to the right and determine with the potentiometer the voltage across the 10 ohm resistance (again, it may be necessary to reverse the leads going to the potentiometer to get a balance.) If this voltage is less than 0.5 volts move the sliding contact on the rheostat a small amount in the direction which decreases the resistance, and recheck the voltage across the 10 ohms. Continue this process until the voltage across the 10 ohm resistor is somewhere between 0.5 and 1.0 volt.* Measure and record this voltage. Throw the swtich S to the left, insert the sample in the field and carefully measure and record $V_N - V_M$. Now remove the sample from the field, turn it over and reinsert it. Carefully measure and record $(V_N - V_M)^1$.

8. Move the slider on the rheostat about 1 to 1½" in the direction which increases the resistance in the circuit. Leaving it in this position, measure the new voltage across the 10 ohm resistor, and the new values of $(V_N - V_M)$ and $(V_N - V_M)^1$ as indicated in step 6.

9. Repeat step 7 three more times. It will be helpful perhaps to make a table as follows:

Trial	Voltage across 10 ohm resistor volts	current $i = \frac{V}{10}$ (amp)	V_N-V_M volts	$(V_N-V_M)^1$ volts	V_H volts	R_H $\frac{m^3}{coul}$
I						
II						
III						
IV						
V						

10. Measure the magnetic field B between the poles of the magnet, using the Gauss meter. The instructor will show you how to use this instrument.

11. For each of the trials calculate V_H from equation (2) and R_H from equation (1). Calculate the average* and the average deviation. (If you prefer, calculate the standard deviation.) The average or standard deviation can be used as a measure of the precision of your measurement of R_H.

* Or until slider is at the end of its range.

12. In this experiment, B and t were held constant. Equation (1) predicts that V_H should be proportional to i. You might like to check this by plotting V_H against i.

13. Can you explain why equation (2) gives the true Hall voltage?

---

*A "better" average could be obtained by "weighting" each R_H by an amount proportional to the current in the sample. Can you see why this is so?

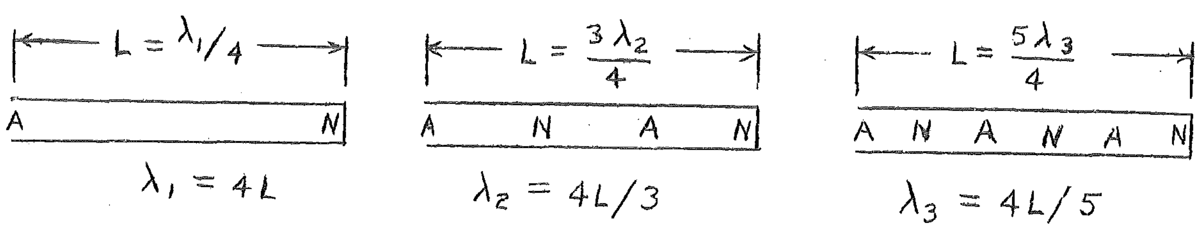
RESONANCE IN AIR COLUMNS

OBJECT:

To study the conditions which give rise to longitudinal standing waves in an air column.

THEORY:

Longitudinal waves traveling along a tube are reflected at the ends in the same way as are transverse waves in a string. In this case also, the waves traveling to the right and those traveling to the left may combine to form standing waves having a large amplitude at certain natural (resonant) frequencies. As in the case of the vibrating string, a node should exist at a closed end since the molecules aren't free to vibrate longitudinally. If the tube is narrow compared with the wavelength an antinode will occur at an open end. Thus by drawing the possible standing wave patterns, it is seen that the wavelengths of the first, second, and third harmonics are as shown below for a tube closed at one end and open at the other.



The corresponding resonant frequencies ($f = c/\lambda$) are therefore

$$f_n = \frac{(2n-1)c}{4L} \qquad n = 1, 2, 3, \ldots.. \tag{1}$$

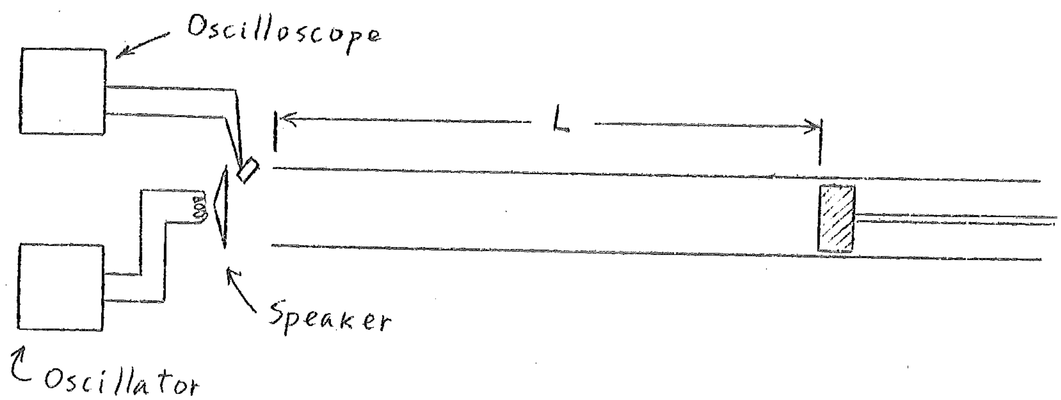
For a tube open at both ends resonant frequencies can be predicted by assuming antinodes at both ends.

Suppose we have a closed tube which may be varied in length by positioning a plug in the tube. If we hold the frequency f fixed then according to equation (1) we should have resonance whenever

$$L = \frac{(2n-1)c}{4f} \tag{2}$$

is satisfied for any integral value of n. The smallest value of L is c/4f, the next larger value is 3c/4f, the next 5c/4f, etc. Thus if the L values which produce resonance are plotted against the numbers 1, 3, 4, 7, etc., the resulting points should lie on a straight line of slope c/4f.

APPARATUS:



INSTRUCTIONS:

(1) Record an identifying number for the oscillator used. Arrange the apparatus as shown in the preceding figure. Set the oscillator dial at 1000 cycle/sec. Starting with the plug near the end of the tube farthest from the speaker, move the plug slowly toward the speaker. Note those positions where the sound intensity increases to a maximum. (Note: Just beyond the resonant points the intensity drops rapidly. These sudden changes may be more readily detected than the actual maximums.) Measure carefully the distance these maximum positions are from the speaker end of the tube. Since the location of resonant positions is somewhat a matter of judgment, locate them independently a number of times.

(2) Repeat step (1) with the oscillator set at 1500, 2000, and 2500 cps. For one of the frequencies determine if the lengths L measured depend upon the position of the speaker relative to the tube.

(3) Determine the exact frequency of the oscillator at each of the above settings by hooking it up to the E-put meter via the auxiliary terminals on the supply panel. The E-put meter is being used by several other groups, so it will be necessary to coordinate your activities with theirs.

(4) Using the data obtained in the previous steps plot, for each of the four frequencies, a graph of the length of the tube to the resonant positions against the numbers 1, 3, 5, 7, 9, 11. Plot the distance to the resonant position nearest the speaker above the 1 on the horizontal scale, the distance to the next resonant point above the 3, etc. According to equation (2) the points plotted in this manner should lie in a straight line. Draw the most representative lines through your plotted points and determine the slopes and intercepts for each frequency.

(5) From the slopes determined in the previous step and the frequencies found in step (3), determine the velocity of sound in the tube (average the four values).

(6) According to equation (2) the graphs plotted in part (4) should go through the origin. Does your experimental data fit equation (2) and, if not, how would you modify equation (2) to make it fit your data? Can you think of any physical reason why equation (2) might need to be modified?

PRISM SPECTROMETER - DISPERSION CURVE FOR GLASS

OBJECT: To measure as a function of wavelength the index of refraction of a glass sample.
Ref - H & R Chap 41 (see example 3)(see prob. 7)

THEORY: The glass sample, in the form of a prism, is illuminated by a beam of light from a source S as shown in Fig. 1. Light is admitted to the collimating tube by a slit and is converted into a parallel beam by a lens at the other end of the tube. This beam then strikes the prism, is refracted at the two surfaces, and enters a telescope which is used to observe the light after refraction.

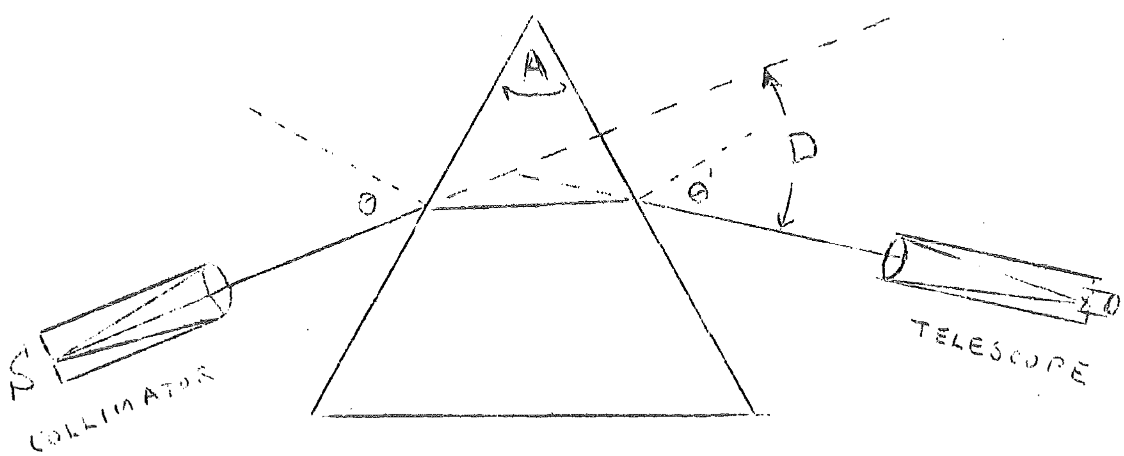


Fig. 1

The light beam strikes the first surface of the prism at angle of incidence θ and is refracted twice, leaving the second surface at angle θ' with the normal. The angle of deviation D is the angle which the rays emerging from the prism make with the rays which are incident on the first surface. If the prism is rotated so as to vary the angle of incidence θ, the deviation angle D will change and one must move the telescope in order to view the beam. There will be one particular angle of incidence for which the deviation D has a minimum value D_m. Using the laws of refraction and geometry, one can show that the index of refraction of the glass is given by

$$n = \frac{\sin \frac{A + D_m}{2}}{\sin \frac{A}{2}} \qquad (1)\ \text{(Text)}$$

The above discussion assumes that the source S emits light waves of only one frequency (i.e. a monochromatic source), the index of refraction calculated from equation (1) being the index of the glass for this frequency. If the source is not monochromatic but contains a number of discrete frequencies, each frequency will be deviated through a different angle D and several beams will emerge from the prism - one for each frequency of light emitted by the source. This occurs because the refraction of light at the two surfaces depends on the index of refraction n of the material (Snell's law) and the index is different for light of different frequencies. One can find the value of n for any of the frequencies emitted by the source, simply by determining the minimum angle of deviation D_m for that particular frequency and using equation (1). A plot of the index of refraction of a substance as a function of light frequency f or wavelength (in air) λ is called a dispersion curve.

APPARATUS: The apparatus used for this experiment is called a spectrometer. It consists essentially of a collimator which is simply a tube with a slit at one end and a lens at the other, a telescope and a prism table. The telescope and prism table are arranged so they may be rotated independently about a vertical axis. A circular scale permits one to measure the angle through which the telescope is rotated.

To make accurate measurements with a spectometer the following preliminary adjustments must be made:

1. The collimator and telescope must be adjusted so that the beam emerging from the collimator is parallel and so that the objective lens of the telescope brings this light to a focus exactly in the plane of the cross hairs.

2. The axes of the collimator and telescope must be adjusted so they are perpendicular to the axis of rotation.

3. The refracting surfaces of the prism must be made parallel to the axis of rotation.

These adjustments have already been made.

DO NOT TOUCH THE PRISM. DO NOT ADJUST THE POSITION OF THE COLLIMATOR LENS, NOR THE OBJECTIVE LENS OF THE TELESCOPE. DO NOT DISTURB THE LEVELING SCREWS OF THE PRISM TABLE, TELESCOPE OR COLLIMATOR.

Before using the instrument, please read the following description. The first applies to the spectrometers made by Ealing Co., the second to those made by Gaertner.

Ealing Spectrometer:

A large thumb screw directly beneath the collimator and about one inch from the base of the instrument clamps the prism table and scale. A similar thumb screw directly beneath the telescope clamps the telescope. When this is tightened, the telescope can still be rotated a small amount by the vernier screw located just above and to the right of the telescope clamping screw. A small thumb screw at the slit end of the collimator adjusts the width of the slit. The four thumb screws mentioned in this description are the only ones that should be manipulated by the student.

Gaertner Spectrometer

A large thumb screw near the axis of the instrument and about one inch above the divided scale clamps the telescope. A second large thumb screw at the same height but at the base of the telescope support arm permits a fine adjustment of the telescope position. The prism table assembly can be clamped by means of the small thumb screw located near the axis of the instrument and about 1-1/2 inches below the top of the prism table. Note that the prism table assembly rides on a collar which is clamped in position by means of a second small thumb screw. When it is necessary to raise or lower the prism table assembly, both small thumb screws should be loosened and both the collar and prism table assembly moved at the same time. At the slit end of the collimator is a small thumb screw which adjusts the width of the slit. This thumb screw along with the other ones that are mentioned in this description are the only ones that should be manipulated by the student.

INSTRUCTIONS:

1. Look into the telescope and move the small tube containing the eye lens in or out until the cross hairs may be seen distinctly.

2. Loosen the screw which clamps the prism table and rotate the prism table until the apex of the prism points toward the collimator. Re-tighten the screw to keep the prism in this position. Illuminate the slit using the small 110 volt lamp provided. Rotate the telescope into such a position that the image of the slit formed by light reflected from one face of the prism can be seen in the telescope (position B, Fig. 2). If you can't find the image, ask the instructor for assistance---do not move the prism around or adjust the leveling screws. Clamp the telescope in this position and use the fine adjustment screw so as to set the cross hairs exactly on the image of the slit. Read and record this position of the telescope on the circular scale. Loosen the telescope clamping screw and rotate the telescope into B'. Reclamp and use the fine adjustment screw to set the cross hairs on this image. Read and record this position of the telescope on the circular scale.

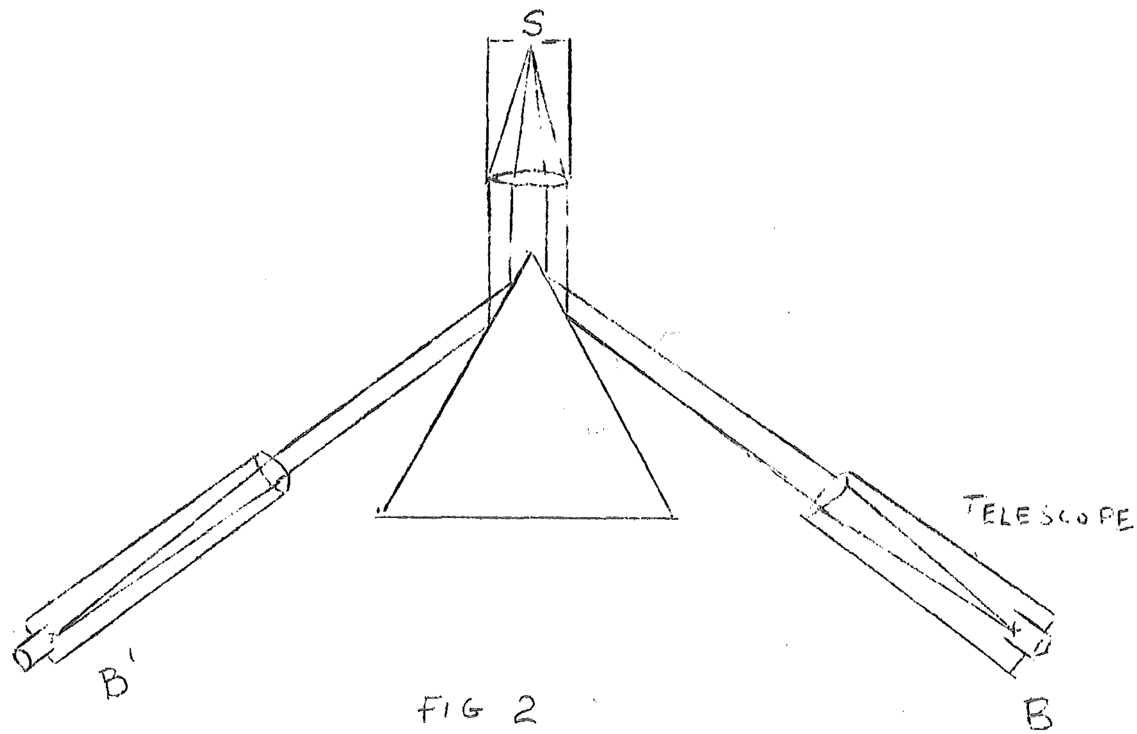


FIG 2

The angle through which the telescope has been rotated from position B to position B' is twice the refracting angle A of the prism. Prove this! Since the decision as to when the cross hairs coincide exactly with image is a matter of judgment, it would be wise to determine the readings corresponding to positions B and B' several times independently. The average deviation of these individual settings will give you some idea of how precisely you can measure angle A.

3. Loosen the screw which clamps the prism table and rotate the table until the prism is approximately in the position shown in Fig. 1. Mount the Hg vapor lamp so that it will illuminate the slit of the collimator. With the telescope approximately in the position shown in Fig. 1, look through the telescope and adjust the position of the telescope and/or prism table until you see in the telescope a number of colored images of the slit. These will be spaced approximately as shown in Fig. 3 (all of the lines may not be visible for any one position of the telescope, and it may not be possible for some students to see all of the violet lines).

4. Focus your attention on the 1st Yellow line of the spectrum. Keeping this line in view in the telescope, slowly rotate the prism table in the direction which decreases the angle of deviation for this line. Note that there is some position of the prism table for which the angle of deviation is minimum and if the prism table is turned in either direction from this position, the deviation will increase. (With the prism table set in the position for minimum deviation for the line in question, clamp the prism table and carefully adjust the position of the telescope until the cross hairs are centered on the line.) Read the position of the telescope on the circular scale. From here on the procedure will vary depending on which instrument you are using.

FOR THOSE USING EALING INSTRUMENT

5. Leaving the prism table clamped, rotate the telescope until it is directly opposite the collimator. The prism can be moved aside without rotating the table so that you should be able to see a direct image of the slit. Set the telescope carefully so that the cross hairs fall on the image and record the position. The difference between this reading and the first one is the minimum angle of deviation for the line in question.

6. Unclamp the prism table and repeat steps 4, 5 and 6 for each of the other lines in the spectrum of the source. Finally, set up the small white light at the slit and note the spectrum due to this source.

FOR THOSE USING THE GAERTNER INSTRUMENT

5. Unclamp the prism table and rotate the table so that it is in the position of minimum deviation for the second line of the spectrum. Clamp the table and carefully adjust the position of the telescope so that the second line falls on the cross hairs. Record this position of the telescope. Repeat for all the lines in the spectrum. Set up the small white light source at the slit. Note the spectrum produced by this type of source.

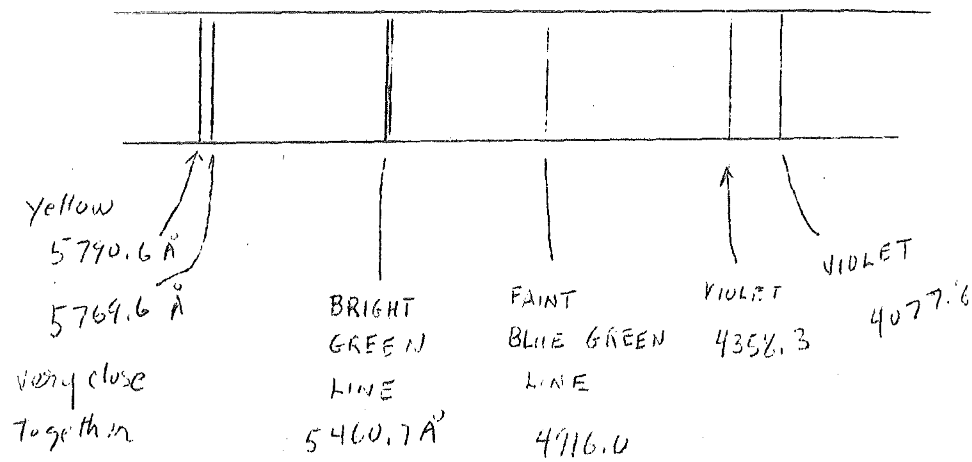
6. Lower the prism table assembly after first loosening the two small set screws. Swing the telescope to a position directly opposite the collimator. It should be possible to see a direct image of the slit. Set the telescope so that this image is centered on the cross hairs and record the reading on the circular scale. Raise the prism table assembly to its original height. You can check this by holding the small white light at the eye piece end of the telescope and examining the beam emerging from the front end to determine if it is completely intercepted by the prism face.

ANALYSIS AND RESULTS:

1. Calculate for each wavelength the index of refraction using equation (1).

2. Draw a dispersion curve showing the index of refraction n in this type of glass for various light wavelengths (in air) λ.

3. For a wavelength of 6000 A, the indices of refraction of dense flint, light flint and crown glass are respectively 1.65, 1.58 and 1.52. Based on your data, of which type of glass is the prism made?



$$Z_{12} = \frac{Z_{12} Z_L}{Z_{22} + Z_L}$$

The Prism Spectrometer

If a transparent prism is illuminated by a beam of light from a source S as shown in the figure below, the beam strikes the first surface of the prism at angle of incidence θ and is refracted twice, leaving the second surface at angle ϕ with the normal.

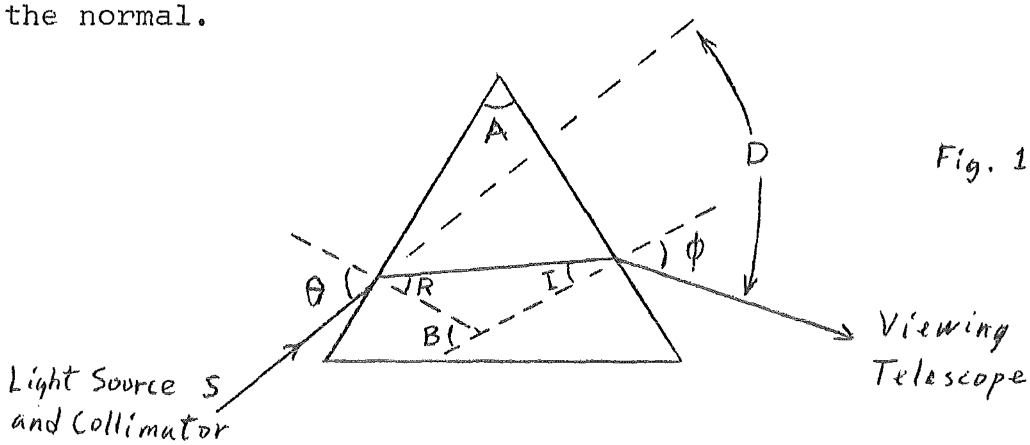


Fig. 1

The angle of deviation D of the light depends on the angle of incidence θ, and on the frequency of the radiation since the index of refraction is different for different frequencies in "dispersive" media such as that from which the prism is made. It can be shown geometrically (do this) that:

$$D = \theta + \phi - A \quad \text{or} \quad \phi = D + A - \theta \qquad (1)$$

$$A = B = R + I \qquad (2)$$

Applying Snell's law at each refracting surface:

$$R = \arcsin\left[(\sin\theta)/n\right]$$

$$I = \arcsin\left[(\sin\phi)/n\right]$$

Substituting into (2) we get

$$A = \arcsin\left[(\sin\theta)/n\right] + \arcsin\left[(\sin\phi)/n\right]$$

or, bringing all three terms to one side of the equals sign and calling the sum of these three terms F(n):

$$F(n) = \arcsin\left[(\sin\theta)/n\right] + \text{arsin}\left[(\sin\phi)/n\right] - A = 0$$

where equation (1) allows the replacement of angle ϕ by $D + A - \theta$. Thus if angles θ and A are held constant, equation (3) expresses implicity how the deviation angle D depends on the index of refraction n and vice versa. The prism will have different indices of refraction n for the different frequencies, which will then be deviated by different amounts D for light incident at a fixed angle θ on a prism of a certain apex angle A.

An equation like (3) which is not solved explicitly for one variable in terms of another (e.g. $D = f(n)$) but has the variables "mixed up", is called a "transcendental" equation. Solutions to this type of equation may be obtained by a method of successive approximations called the Newton-

Raphson method (see McCracken, "Fortran with Engineering Applications", Chapter 10). Suppose that you wanted to solve an equation F (x) = 0 such as that shown in the figure below for the values of x which make F (x) = 0 (the roots of the equation).

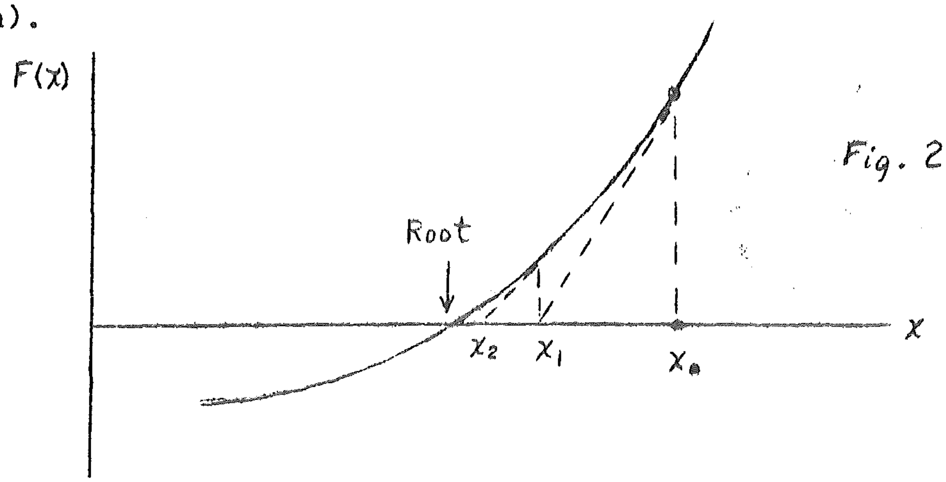


Fig. 2

To start the computation a guess x_o is made as a rough approximation to the value of the root. The value of F (x) and the slope of the F (x) curve at x_o is obtained and a better approximation to the root is calculated from the equation for the slope

$$F'(x_o) = F(x_o) / (x_o - x_1)$$

or

$$x_1 = x_o - F(x_o) / F'(x_o)$$

where x_1 is the second approximation to the root. Then, in the same way $F(x_1)$ and $F'(x_1)$ are calculated and a better approximation

$$x_2 = x_1 - F(x_1)/F'(x_1)$$

and so forth until there is a negligible increase in precision for any further iterations. In order to use this procedure on equation (3) it is necessary to have an expression for the slope of the F (n) curve, obtained by taking the derivative of equation (3) with respect to n while holding other variables constant.

$$F'(n) = \frac{-(\sin\theta)/n^2}{[1-(\sin^2\theta/n^2]^{1/2}} + \frac{-(\sin\phi)/n^2}{[1-(\sin^2\phi)/n^2]^{1/2}} \quad (4)$$

Experiment:

I. Using the clamping screw which locks the prism table, fix the prism in a position such that its apex points toward the collimator and white light source as shown. (It isn't possible or necessary to align the base of the prism exactly perdendicular to the beam).

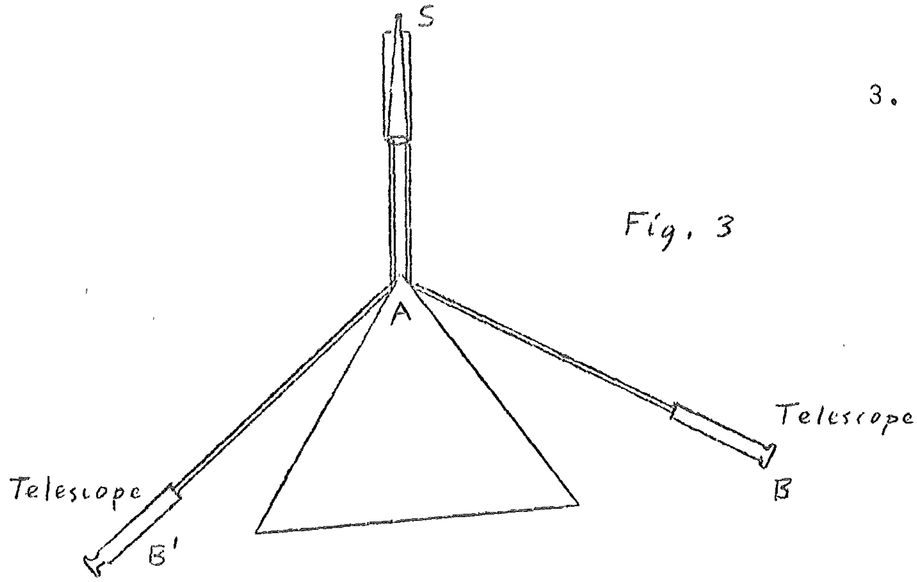


Fig. 3

Use the fine adjustment screw to set the telescope cross hairs exactly on the reflected image of the slit at B and B', recording both positions on the telescope circular scale. Repeat this measurement several times and determine the average values and standard deviations. Prove that the angle through which the telescope is rotated from B to B' is twice the prism apex angle A and calculate A and its standard deviation.

II. Using the mercury vapor lamp source rotate the prism table until it is in the position shown in Fig. 1. Adjust the beam slit opening and the position of the telescope and prism table until you see a number of sharp narrow vertical lines of different colors through the telescope. These are images of the slit opening formed by light of the wavelengths given below.

Yellow	5791 Å and 5770 Å
Green	5461 Å
Bluegreen	4916 Å
Blue	4358 Å
Violet	4078 Å and 4047 Å

By trial and error position the prism so that the green line has the smallest deviation angle D (see Fig. 1) that it can have for any prism setting. This "minimum deviation" angle for the green light is the one for which θ and ϕ are equal for light of this wavelength, and these angles can thus be easily determined from the position of the telescope when lined up on the refracted beam, using equation (1). Leave the prism table clamped in the same position (θ will be the same for all lines in the refracted spectrum) and record the position of each of the visible lines on the telescope scale. Each reading should be repeated independently at least three times. In order to get a line in the red part of the spectrum change your source to a gas discharge hydrogen or helium tube and determine the position of one of the following lines without changing the prism orientation. (Note: BEWARE OF THE HIGH VOLTAGE - SEVERAL THOUSAND VOLTS - USED ON THE GAS DISCHARGE TUBES - THINK!!)

Helium Red line 6678 Å

Again keeping the prism in the same orientation as before determine the positions of some lines of unknown wavelength (for example other lines from hydrogen, helium, cadmium, oxygen, nitrogen or neon). Finally, rotate the telescope until it is directly opposite the collimator and move aside the prism without unclamping the table (Ealing instrument) or lower the prism table assembly after first loosening the two small set screws (Goertner instrument). Record the position of the unrefracted light beam on the telescope scale. This reading will be necessary in order to get the deviation D associated with each of the previous readings. Calculate angle θ and angle D for each of the lines on which you made measurements.

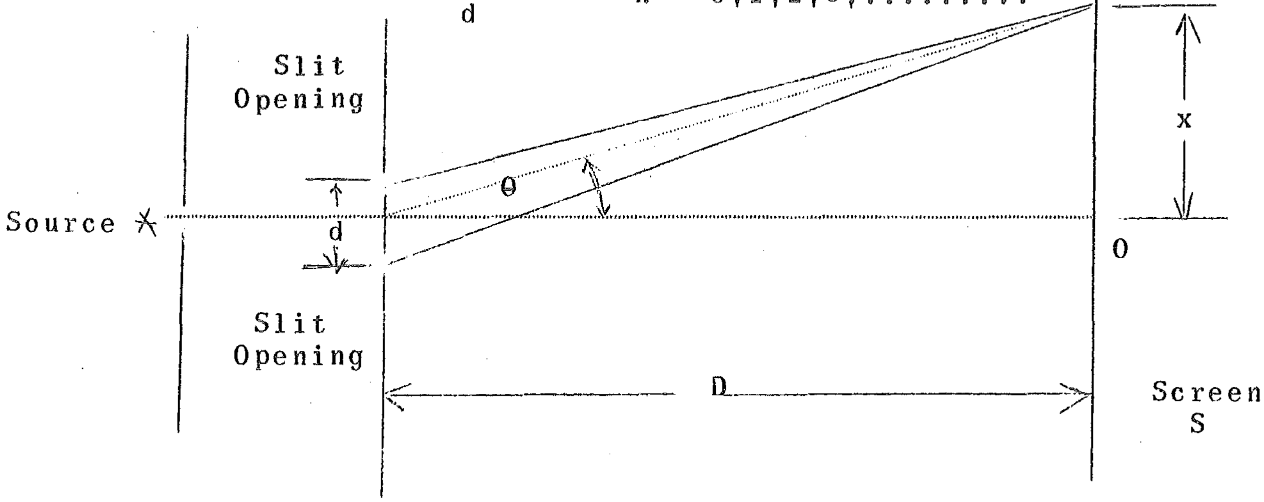
III. There will be a computer program available in the laboratory for solving by the Newton-Raphson method for the index of refraction that the prism has for light of each of the wavelengths present in the spectral lines. The program repeats the successive approximations to the value of each n until the difference between a computed value and the preceding value is less than .00001.

IV. Plot a graph showing the prism index of refraction as a function of the wavelength of the refracted light. Use the graph to determine the wavelengths of the lines for which λ is unknown. A more precise way of determining these wavelengths would be to determine the coefficients in a least-squares fit of a polynomial $y = A + Bx + Cx^2 + Dx^3 + \ldots\ldots\ldots$ to the graph of index n versus wavelength λ or vice versa. Any wavelength could then be determined by putting the corresponding value for index of refraction n into the equation. The computing center has subroutines for doing such a least-squares fit (see "Scientific Subroutines" section of their manual for subroutines "GDATA, ORDER, MINV, and MULTR")

INTERFERENCE AND DIFFRACTION

According to Huygens' principle, each point along a wavefront may be regarded as a new source of waves. Whenever something obstructs part of the wavefronts, interference between "wavelets" emanating from different parts of the unobstructed wavefronts produce a diffraction pattern which is characteristic of the geometry of the obstruction (or opening in object which blocks the light) and of the wavelength of the light. It is shown in nearly all introductory physics textbooks, for example (see Resnick and Halliday, section 43-1), that when light waves pass through a double slit arrangement like that shown below they interfere constructively and destructively at different positions to form fringes on the screen S such that intensity maximum appear at positions

$$x = n\left(\frac{D\lambda}{d}\right) \qquad n = 0,1,2,3,\ldots\ldots\ldots$$



In somewhat the same way wavelets passing through different parts of a single slit interfere to produce a single slit diffraction pattern with destructive interference causing diffraction minima at angles θ such that (see Resnick and Halliday, section 44-2)

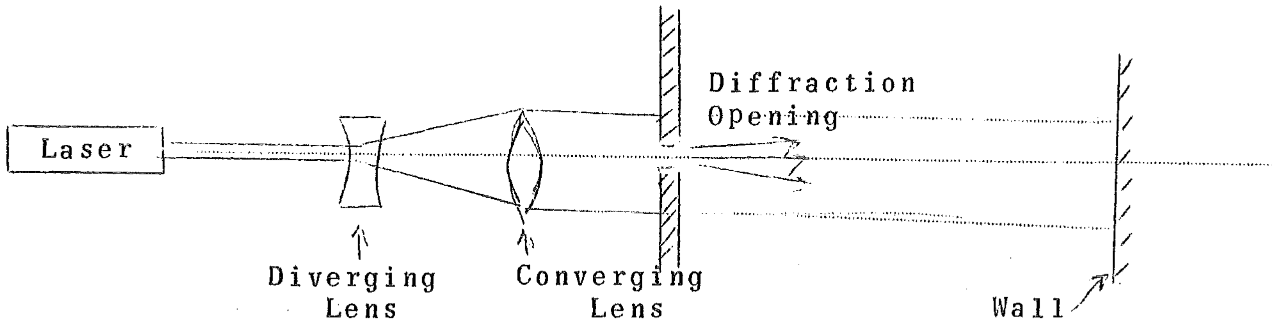
$$a \sin\theta = m\lambda \qquad m = 1,2,3,\ldots\ldots\ldots$$

with maxima approximately half way between (the exact intensity expressions are given in section 44-3), where a is the slit width. A circular aperture of diameter d results in fringes having circular symmetry with the first minimum appearing at a distance from the center such that (see Resnick and Halliday, section 44-5)

$$\sin\theta = 1.22\ \lambda/d$$

Experiment: (Due to a limitation on equipment, half the groups will have to start on part (2) below and do (1) at the end of the experiment.)

The lasers used for a light source are helium-neon gas discharge lasers and put out a beam of wavelength $\lambda = 6328$ Å. The apparatus is set up as shown below.



The converging lens is moved close enough to diverging lens to keep the beam diameter constant from the converging lens to the wall (beam must be wide enough to completely fill the diffraction opening.

(1) You will be given an IBM card. Use the comparator to determine the dimensions of a punch hole and the spacing (center to center) of adjustment holes. Measure the distance from your diffraction openings to the wall.

(2) Put a single punch opening in the beam as the diffraction opening and measure the distances to the various diffraction peaks in the two directions. Repeat with the triple punched opening.

(3) Repeat part (2) with a single narrow slit, a circular opening, and a screen.

(4) Compare theory and experiment using your measurements of parts (1) and (2) which involve the single and triple punch holes in the IBM cards. What is the percentage difference between the length and width of a single rectangle calculated from measurements on its diffraction pattern and the length and width measured directly?

(5) Discuss qualitatively the appearance of the diffraction patterns of the circular opening and the screen of part (3) and determine the slit width, circular hole diameter, and screen mesh spacing from the diffraction pattern measurements. The bright disc at the center of the diffraction pattern for the circular hole is called the airy disk. You may want to compare this disk diameter (really the diameter of the dark ring surrounding the bright center) to the theoretical value given in the text (section 44-5 in Halliday & Resnick).

The Diffraction Grating

Ref. Halliday, Resnick, Sections 45-1,2,3.

In the prism spectrometer experiment, a spectrum of mercury was obtained by allowing light from a mercury lamp to be refracted by a prism. In this experiment a spectrum of mercury will be produced by diffraction; a grating replaces the prism.

The arrangement is sketched in Figure 1. If your grating is labelled B you may be able to observe only the zeroth and first order spectrum. If it is marked A, several orders may be visible. The angle θ at which a given wavelength λ is observed is related to the wavelength as

$$\sin\theta = m\lambda / d$$

where d = grating spacing (distance between lines ruled in grating from which replica has been made)

m = an integer, the "order" of the observed diffraction maximum.

Procedure

Use first order spectrum for B grating, second order if you have A grating. Carefully position the telescope so that the intense violet Hg line is centered on the cross-hair intersection and record the position of the telescope. Repeat for each of the other lines, keeping a tabular record. Then take readings for the same lines, same order, on the other side of the zero-order slit image. Suggested format:

Color	Wavelength	R_{left}	R_{right}	$\theta = \frac{R_L - R_R}{2}$	$\sin\theta$
MERCURY					
violet	4047 A				
faint violet-blue	4358				
blue-green	4916				
faint green	5461				
$yellow_1$	5770				
$yellow_2$	5791				
NEON					
yellow					

Upon finishing the Hg observations, place the neon source (orange) in front of the slit and take readings on the brightest yellow line in its spectrum.

Ask someone with a grating label (A,B) different from yours for a look at his Hg spectrum. Does A or B have a smaller grating spacing? (If curious, analyse a few higher order lines.)

Analysis

Using a full sheet of graph paper and scales which make full use of the precision of your measurements, plot sin θ vs. wavelength. Since the grating equation above predicts a linear relationship between sin θ and λ, use a ruler to draw in the line which best fits your data. Extrapolate on your plot to find the wavelength of the neon yellow line. From the graph, find d. Include quantitative estimates of the experimental uncertainties in your reported results.

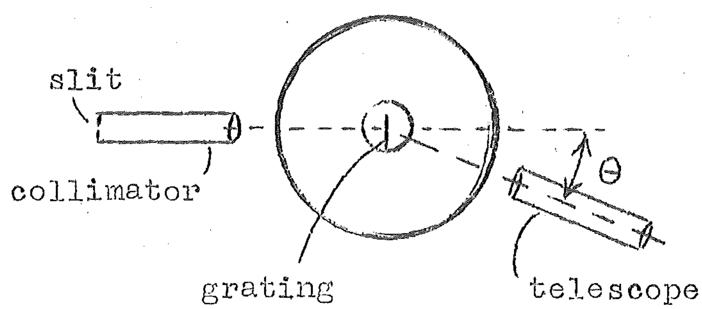
Optional questions

1. Can a "line which best fits" be found by more sophisticated techniques? Are these warranted here?

2. Two yellow lines in the Hg spectrum, separated by about 20 Angstrom units, are well resolved. In first order ($m = 1$), how close can two red lines be in order that they be resolved by your grating? (Base your estimate on resolving power R (text) considering the Rayleigh criterion, i.e., diffraction effects at the grating. This ignores other effects, such as diffraction at the entrance slit and lens effects.

3. Look up your prism report and comment on the relative resolving power of the two "dispersing" devices, prism and grating.

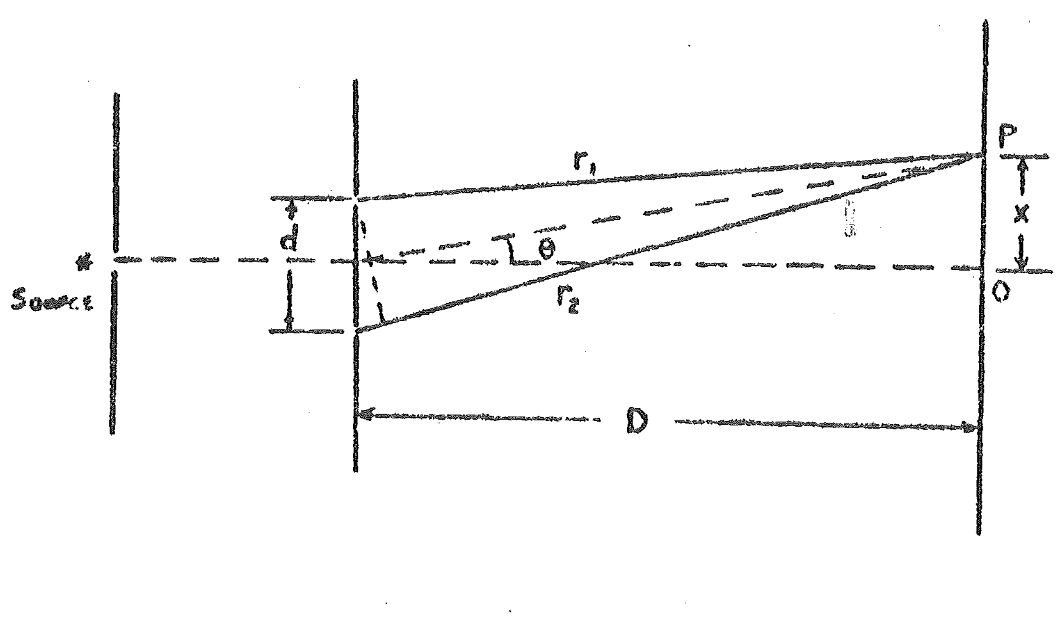
Fig. 1



INTERFERENCE & DIFFRACTION

OBJECT: To observe the effect of interference of light waves and to measure the wavelength of the light emitted by a monochromatic source using Young's experiment.

THEORY: In Young's "double slit" experiment an approximately monochromatic light source illuminates a slit, and light from this slit is allowed to fall on two slits separated by a spacing d as shown in the figure below. If these two slits are sufficiently narrow, each will act as a point source sending out new wavelets (Huygens' principle) toward the screen at the right. The illumination intensity at any point P can be found by superimposing the waves arriving at the point from the two slits; however this "adding" of the waves yields different results at different points P. For example, consider light waves which reach the point P shown in the figure from the two slits. The two waves start in phase at the slits but since one has to travel a greater distance $r_2 > r_1$ it lags behind the other at P. If the lag amounts to one, two, three etc. wavelengths at point P, adding the two waves gives rise to a wave having an amplitude which is the sum of the amplitudes of the individual waves. This condition is referred to as constructive interference and results in a high intensity at P. If point P is a point where the path difference $r_2 - r_1$ is such as to make one wave arriving at the point lag the other by an odd number of half-wavelengths, the wave formed by superposition has almost zero amplitude. This condition is referred to as destructive interference and results in very low intensity at P.



It is shown in a number of texts (e.g. Sears and Zemansky) that for points P for which the angle θ is small, the path difference $(r_2 - r_1)$ is given the equation

$$r_2 - r_1 = \frac{d}{D} x$$

and hence constructive interference occurs at points P for which

$$\frac{d}{D} x = n\lambda \quad \text{where } n=0,1,2,3,\ldots\ldots$$

It follows that there will be bright interference fringes at the following values of x:

$$x_0 = 0 \qquad x_2 = \frac{2D\lambda}{d}$$

$$x_1 = \frac{D\lambda}{d} \qquad x_3 = \frac{3D\lambda}{d} \qquad \text{etc.}$$

Whenever a part of the wavefronts coming from a point source are cut off by some obstacle, fringes occur as the result of interference between Huygens' wavelets emanating from various parts of the unobstructed wavefronts. For example a small circular object when placed in the light from a point source produces circular fringes, alternately bright and dark. The process involved is present in every wave but this effect is observable only if some part if the wave is cut off and is called diffraction.

APPARATUS: A long box with a slit at one end and the film holder of a Polaroid Land camera at the other, a monochromatic (approximately) light source, double slits, a fine wire, and a razor blade.

Instructions:

(Important Note: Please keep the cameras wrapped up and inside the boxes when not in use. Do not pull open metal shutter until you are ready to expose the film. In each of the following parts remove the camera and observe the interference pattern through the magnifier lens before photographing the pattern. Rotate the primary slit (directly in front of source) to that position where the fringes are most distinct (parallel to the double slits or the diffracting obstacle). Then replace the camera and make an exposure. An exposure is started by pulling the thin sheet metal diaphragm up to the line inscribed on it..... not all the way out of its slot. To stop the exposure push this diaphragm back down. Note that there are three notches cut in the flat plate to which the film holder is fastened. The bottom of the plate and these three notches determine four fixed vertical positions of the camera. Four exposures are to be taken on a single film. Each group is expected to take only one picture but will be allowed one more if a mistake is made on the first.)

(1) Each group will be given two double slit systems with different spacing d between the slits. Place the double slit with the smaller spacing in the support near the center of the box. After adjusting for the most distinct fringes as described above, make a three minute and an eight minute exposure on the same picture (different vertical position of camera).

(2) Using the same film repeat part (1) with the other double slit. Ask the instructor about the procedure for developing and removing the film. After the film has been removed from the camera, coat it with the applicator provided.

(3) Mount the film on a comparator (if they are both in use go on to steps 4 and 5). Observe the pattern by means of the microscope provided and determine the distance between fringes for the two different double slits. (On the larger comparator one full turn of the main dial produces a carriage displacement of 1 mm.) Since the fringes are presumable equally spaced, one can obtain precision in determining the distance between two successive fringes by measuring the distance between two widely separated fringes and dividing by the appropriate number. Make some kind of estimate of the precision of this determination of the distance between successive fringes.

(4) Measure the distance from the slits to the metal diaphragm on the front of the film holder. The distance from this diaphragm to the film position is 2.70 cm. Use the comparator to measure the spacing between the slits.

(5) Replace the double slits with a fine wire. Observe the diffraction pattern with a magnifier. Make this as sharp as possible by rotating the primary slit a few degrees to the right or left. Sketch the pattern in your notebook. Repeat for the straight edge (razor blade).

ANALYSIS & QUESTIONS:

1. Calculate the wavelength of the light used from your measurments of D, the distance between successive fringes, and the values of slit spacing d.

2. What accuracy would you claim for the wave length? Justify your answer.

Ultrasonic Double Source Interference

The arrangement of equipment is illustrated in Fig. 1.

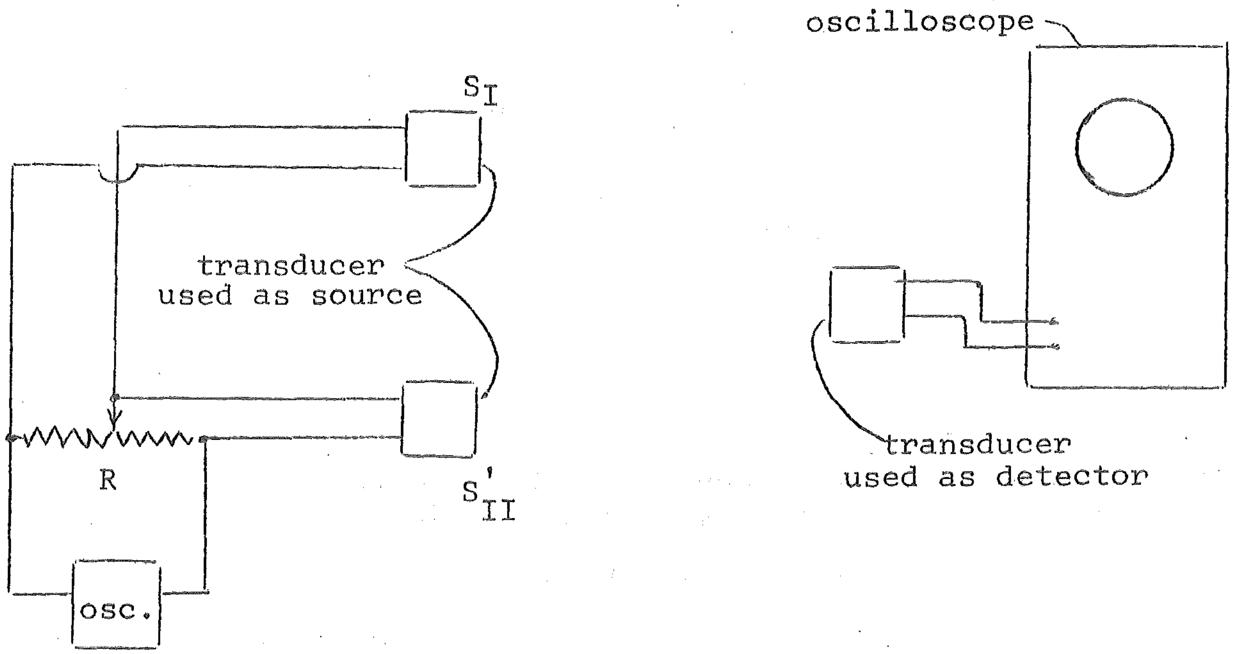


Fig. 1

S_I and S_{II} are surfaces which are forced to move in simple harmonic motion at a frequency f determined by the oscillator setting.

With the circuit shown, the motions of S_I and S_{II} are exactly 180^o out of phase, so they may be represented mathematically by

$$Y_{SI} = A \sin \omega t$$

$$\text{where } \omega = 2\pi f$$

$$Y_{S\,II} = -B \sin \omega t$$

Both surfaces, S_I and S_{II} radiate sound waves of frequency f. Referring to Fig. 2, the waves sent out in a given direction, say x, by source S_I may be represented by the equation

$$Y_I = A \sin \omega(t - \frac{x}{c}) = A \sin (\omega t - kx)$$

and those sent out by S_{II} in a direction specified by X as

$$Y_{II} = -B \sin \omega(t - \frac{X}{c}) = -B \sin (\omega t - kX)$$

Here $k = \frac{\omega}{c}$ and c is the velocity of sound in the medium (air).

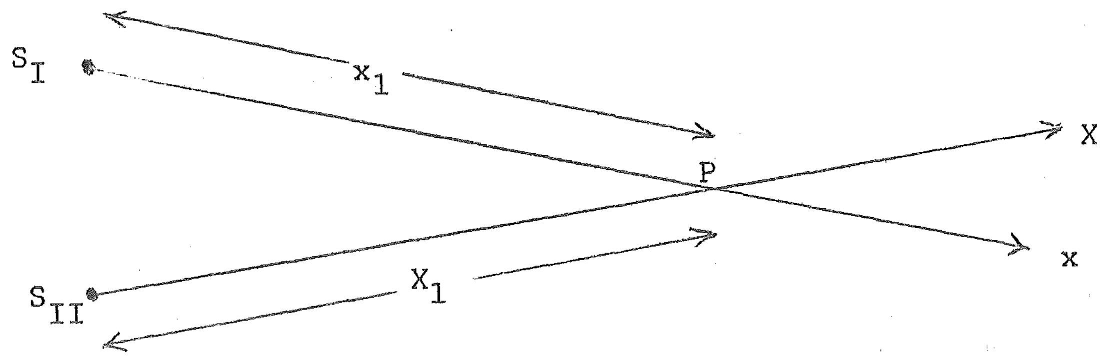


Fig. 2

The disturbance (motion) produced by the wave from source I at any point such as P, a distance x_1 from S_I is simply

$$y_{I_P} = A \sin (\omega t - kx_1)$$

Similarly, the disturbance (motion) at P due to the source S_{II} is given by

$$y_{II_P} = -B \sin (\omega t - kX_1)$$

It is generally assumed that at any point such as P where the two wave trains cross, that the disturbance is simply the sum of the motions that each wave would produce separately, i.e.,

$$y_P = A \sin (\omega t - kx_1) - B \sin (\omega t - kX_1)$$

(A superposition principle applies here.) We consider two special cases. Suppose P is situated so that the difference between X_1 and x_1 is some whole number of wave lengths of the sound wave, i.e.,

$$X_1 - x_1 = n\lambda = n \frac{c}{f} = n \frac{2\pi c}{\omega} = n \frac{2\pi}{k}$$

where n = 0, 1, 2, 3, ... If this holds then

$$X_1 = x_1 + n\lambda \qquad (1)$$

and for this special case

$$y_P = A \sin (\omega t - kx_1) - B \sin \left[\omega t - k \left(x_1 + n \frac{2\pi}{k}\right)\right]$$

$$= A \sin (\omega t - kx_1) - B \sin \left(\omega t - kx_1 - n (2\pi)\right)$$

$$= A \sin (\omega t - kx_1) - B \sin (\omega t - kx_1)$$

$$= (A - B) \sin (\omega t - kx_1)$$

This last equation represents a simple harmonic motion of amplitude (A - B). If one considers the special case where P is so situated that

$$X_1 - x = (2n - 1)\frac{\lambda}{2} = (2n - 1)\frac{\pi}{k} \qquad (2)$$

$$n = 1, 2, 3$$

then it is easily shown that the motion at P will be given by

$$y_P = (A + B)\sin(\omega t - kx_1)$$

This equation represents a simple harmonic motion of amplitude (A + B).

Suppose that the two sources are arranged as in Fig. 3, and a detector (which responds to the resultant (sum) of the disturbances produced by the two waves at any point) is moved along 0' 00".

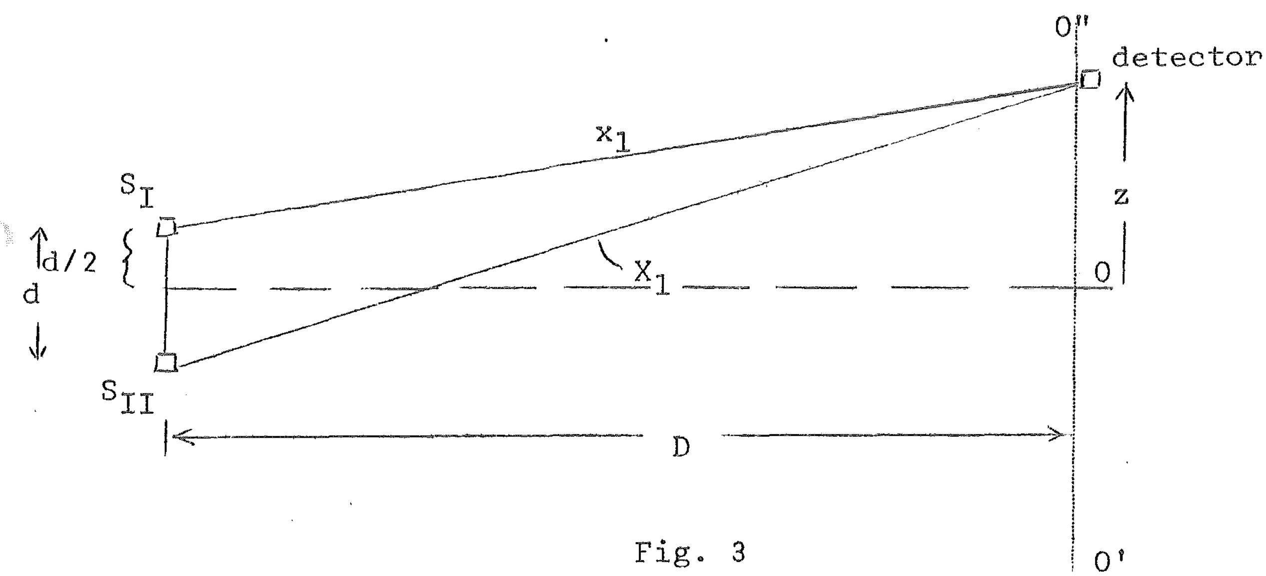


Fig. 3

There should be some points along this path where condition (1) is satisfied and some points for which condition (2) is satisfied. It should be apparent that the point 0 is one of these points for which condition (1) is satisfied. If the first point between 0 and 0" for which condition (1) is again satisfied has a coordinate z it should be easy to show that

$$\lambda = \sqrt{D^2 + (z + \frac{d}{2})^2} - \sqrt{D^2 + (z - \frac{d}{2})^2}$$

Hence, by measuring z, d, and D, one could determine λ, and if one can measure the frequency f one can determine c, the velocity of sound in air.

Hints:

(i) The transducers are resonant systems and work best at their resonant frequency which is around 40,000 hertz.

(ii) A and B in the above equations can be made equal by adjusting the slider or resistance R. This makes it possible to get very nearly zero amplitude at points (nodes) for which (1) is satisfied.

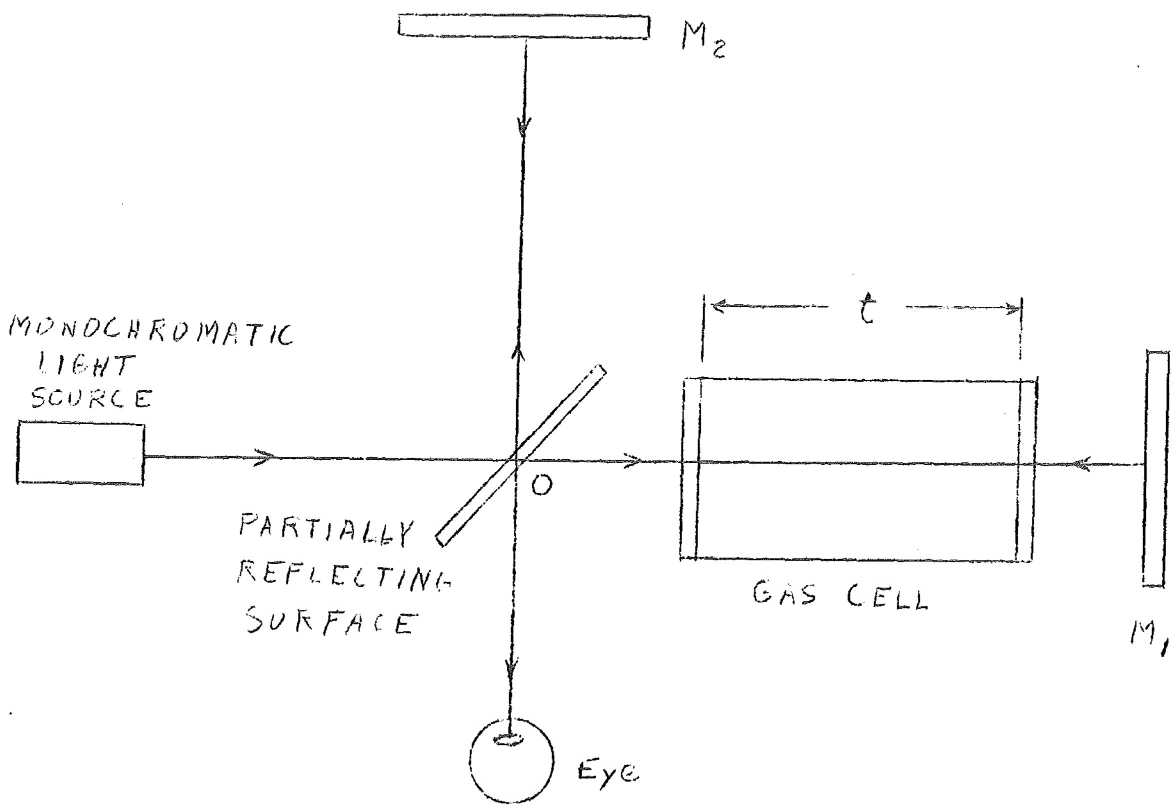
(iii) The distance d in Fig. 3 can be changed. You may want to see what effect this has on z.

(iv) The transducers radiate most strongly in the forward direction.

Index of Refraction of a Gas

Object: To measure the index of refraction of air using a modified Michelson Interferometer.

Apparatus: The construction of the interferometer is shown in the figure. A single wave from the light source is partially transmitted and partially reflected at O,



giving rise to two waves, one of which travels to mirror M_1 and back and the other to M_2 and back. Part of each of these waves reaches the eye and they are brought to a focus at the same spot on the retina. If the wave traveling path OM_1O arrives at the retina in phase with the wave traveling path OM_2O, that spot will be bright. Another wave originating at a different place on the monochromatic source will travel different paths to the eye and will be focused at a different spot on the retina. The waves from this spot on the source may arrive at the retina in phase or out of phase depending on their paths (i.e. starting point). The result is that one sees a series of alternately bright and dark circular fringes, the bright fringes caused by waves which arrive in phase and interfere constructively and the dark fringes by waves which are out of phase and interfere destructively.

Theory: The condition that waves (from a single point on the source) traveling paths OM_1O and OM_2O arrive at the retina in phase is that their paths differ by an integral number of wavelengths λ . However the wavelength of an electromagnetic wave of a given frequency f depends on the medium through which it is traveling and can be expressed

$$\lambda = \lambda_o/n$$

where λ_o is its wavelength in vacuum and n is the index of refraction of the medium for a wave of this frequency. Thus if one wanted to express the number of wavelengths in distance OM_1 he would write

$$N_1 = \frac{(D_1 - t - 2t_g)}{\lambda_o / n_a} + \frac{t}{\lambda_o/n_1} + \frac{2t_g}{\lambda_o/n_g}$$

where D_1 is the actual distance from O to M_1, n_1 is the index of refraction fo the gas in the cell, t_g is the thickness of each glass window, n_g is the index of reffraction of the glass windows, and n_a is the index of refraction of air. Similarly the number of wavelengths in distance OM_2 is

$$N_2 = \frac{D_2}{\lambda_o/n_a}$$

The difference in the number of wavelengths along paths OM_1O and OM_2O is therefore

$$2N_2-2N_1 = \frac{2D_2\ n_a}{\lambda_o} - 2\left[\frac{n_a(D_1-t-2t_g)}{\lambda_o} + \frac{n_1 t}{\lambda_o} + \frac{2n_g t_g}{\lambda_o}\right]$$

$$= \frac{1}{\lambda_o}\left[2n_a\ (D_2 - D_1 + t + 2t_g) - 2n_1\ t - 4n_g t_g\right]$$

Letting $K = 2n_a\ (D_2 - D_1 + t + t_g) - 4n_g t_g$

one obtains for the difference in the number of wavelengths along the two paths,

$$2N_2 - 2N_1 = \frac{1}{\lambda_o}\ (K - 2n_1 t)$$

It is this difference which must be an integral number m = o, 1, 2, 3, ... if the two waves are to arrive in phase at the retina. Thus the condition for constructive interference of the two waves is

$$K - 2n_1 t = m\,\lambda_o$$

The index of refraction n_1 of any gas is a function of its density and hence varies with the pressure of the gas. Suppose that for a given pressure the above condition is satisfied with $m = m_1$ (some integral number) for a particular spot on the retina so that

$$K - 2n_1 t = m_1 \lambda_o \qquad (1)$$

If n_1 is allowed to increase slightly by adding a small amount of gas, the two waves will no longer arrive exactly in phase and the spot will be reduced in intensity. When n_1 reaches a value n_1' such that

$$K - 2n_1' t = (m_1 - 1/2) \lambda_o$$

the spot will be dark, since this is the condition that the two waves will be exactly out of phase. When n_1 reaches a value n_1'' such that

$$K - 2n_1'' t = (m_1 - 1) \lambda_o \qquad (2)$$

the spot will again be bright since $(m_1 - 1)$ is also an integral value of m. Subtracting (2) from (1) we obtain

$$n_1'' - n_1 = \frac{\lambda_o}{2t}$$

It follows then that if we start with condition (1) and allow gas to leak in slowly, counting the number of times N the spot changes from bright to dark and back again, we can write for the change in the index of refraction of the gas during this process

$$\Delta n = N \frac{\lambda_o}{2t}$$

Therefore if we start with the cell evacuated so that $n_1 = 1$, we can determine the index of refraction of any gas at any pressure just by counting the number of times the spot changes while the gas is allowed to leak into the cell until the proper pressure is obtained. The wavelength λ_o of the light in vacuum and the length t of the gas cell must be known.

$t = (8.000 \pm 0.002)$ cm

Instructions

1. Using a mercury lamp with a filter for the 5461 A green line adjust the mirrors so that bright circular fringes can be seen clearly.

2. All groups should connect their cells and the manometer to the vacuum pump (open stopcocks) and evacuate them at the same time. Note the pressure in the cells as indicated by the difference in level of the two mercury columns in the manometer. Turn off the valve to the manometer.

3. Close off your cell from the pump and open the stop cock to the controlled leak, allowing air to leak slowly into the cell as you count the new fringes appearing as bright spots in the center of the pattern. Continue to count until the cell again contains air at atmospheric pressure. Try to estimate the total number of fringes to a fourth of one fringe. Repeat this procedure.

4. Open one cell only to the pump and evacuate this cell, again reading the manometer. Close off the pump, but not the manometer, from the cell, and again allow air to leak slowly into the cell. When 20 new fringes have appeared in the center of the pattern shut off the leak and read the manometer.

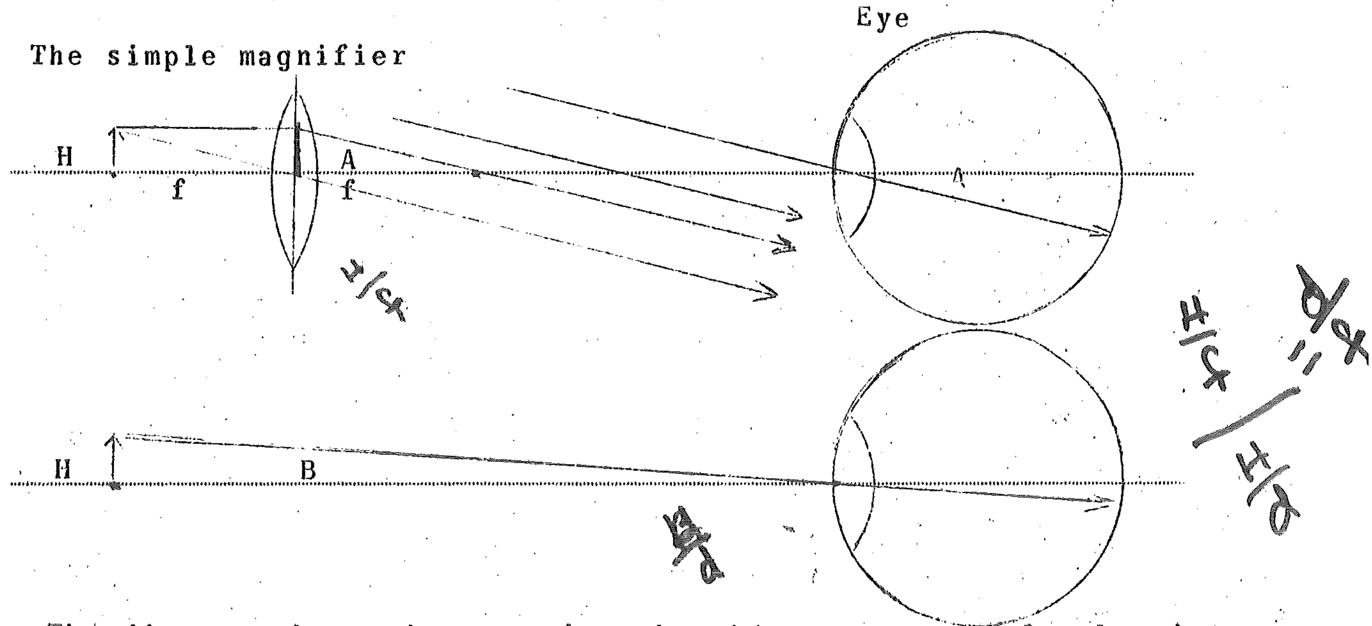
Analysis

1. Calculate the index of refraction of air at atmospheric pressure and at the pressure obtained in part 4.

2. Estimate the precision of this method.

LENSES II

The simple magnifier



The diagram shows that putting the object near the focal point of a converging lens causes rays to come out of the lens nearly parallel to each other at angle A to the axis. One of these goes through the center of the eye lens undeflected and forms an image on the retina. If the magnifier were not present, the object rays would go through the eye lens undeflected at an angle B. SHOW that the angular magnification for this case is the distance of the object from the eye divided by the focal length of the lens (this is only true for distances in the range of a foot or so from the eye, where some rays from the lens at angle A would be able to reach the pupil of the eye). Angular magnification is the ratio A/B, but for small angles we can take this ratio to be (tan A/tan B), and this is what one should use in the proof above.

EXPERIMENT

For each of the two shortest focal length lenses, put a 3x5 card at approximately the focal length of the lens, and look through the lens at the card. Adjust the lens-to-card distance until you see a sharp image of the lines on the card (you may want to draw one or two additional lines on the card). Measure the magnification by looking with both eyes at both the card and the lens and determine how much larger the line spacing appears as viewed in the lens. Compare this value to that given by the theory for each lens used.

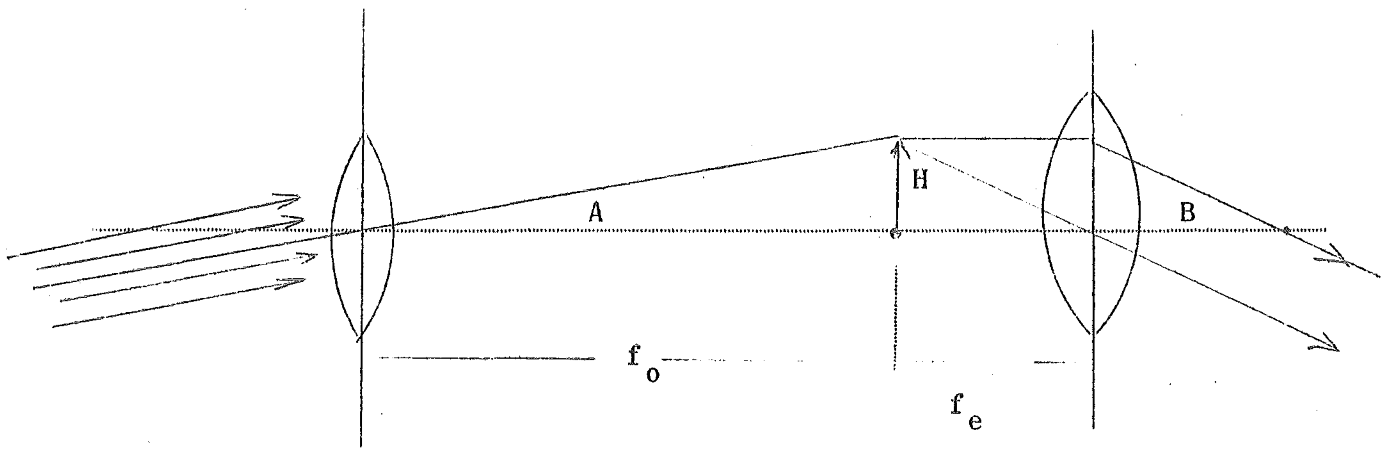
THE TELESCOPE

In this part of the experiment, you will study the simplest types of telescopes, made out of two lenses. At the end of the telescope nearest the object to be viewed is placed a lens (called the objective lens) with a long focal length. The other lens is close to the eye of the observer and is called the eyepiece. The distance between the lenses is the sum of their focal lengths. If the eyepiece is a converging lens, we have an 'astronomical telescope', but if the eyepiece is a diverging lens we have a 'Galilean telescope' or 'opera glass'. Your task in this part of the experiment is to construct one astronomical telescope and one Galilean telescope. For each telescope, you should measure the magnification. To do this, you will have to figure a way to measure the apparent enlargement of the object. You may want to do this by taking the cover of a book and viewing it in the telescope until a particular line of print becomes just legible. The distance of the object from your eye can then be compared to the distance from your naked eye for the same line of print to be barely readable. Or you may want to draw equally spaced lines on the blackboard and view the lines simultaneously, one eye looking directly at the blackboard, and the other looking through the telescope. From the relative sizes, one can judge magnification. Be sure to measure the distance between lens and compare this to the algebraic sum of their focal lengths (the diverging lens focal length is negative).

The theory of the telescope is that rays come in from a distant object and are focused at the focal point of the objective lens. This point is also the focal point of the eyepiece, so the rays come out of the eyepiece parallel to themselves. The eye focuses these parallel rays on the retina. The diagram shows the geometry for an astronomical telescope. The ratio of the outgoing angle B to the incoming angle A is called the angular magnification and for small angles this turns out to be equal to the ratio of the focal lengths:

$$\text{magnification} = (f_{\text{objective}})/(f_{\text{eyepiece}})$$

Compare the results of your magnification measurements to values given by this formula for both the Galilean and astronomical telescopes.



$$\tan A = \frac{H}{f_o}$$

$$\tan B = \frac{H}{f_e}$$

$$\frac{\tan B}{\tan A} \cong \text{angular magnification} = \frac{f_o}{f_e}$$

[illegible]

[illegible] to your spectrometer and prism.

Spectrometer A B C D E

Prism A B C D E

[illegible] deviation for the green line [illegible] doublet of the sodium source. [illegible] ± 3' of our carefully measured [illegible]

$\theta^{green}_{min.dev.}$ = ____________

[illegible] angle of your prism is 60.0 ± .01°. [illegible] of the prism for this green [illegible]

n = ____________

[illegible] Name: ______________________

A. Focal length measurement.

You are to find the focal length of this lens by using the formula $\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$. Form a focused image of the small hole in your light source on the card, and show your measurements of object and image distances, and calculations on this page.

You MAY USE THE WANG CALCULATOR if it is available.

____
focal length =
____

B. Magnification ($m = - i/o$) measurement.

TURN OFF YOUR LIGHT SOURCE when part A is completed. Now set up your lens and card to produce a magnified image of an object which will be placed 10 cm from whichever end of your meter stick you choose (the instructor will bring over the object and place it at this position when you tell him you are ready.) The lens and card should be spaced so that when the instructor places the light source in position and turns it on the image on the card will be focused and [ ] times larger than the object.

This part for the instructor.

Focused? [ ]

Experimental magnification [ ]

Name ____________________

DIFFRACTION GRATING

Circle the letter corresponding to your spectrometer and grating.

Spectrometer A B C D E

Grating A B C D E

Using the hydrogen source and the strongest red line (λ = 6563 Å) do an approximate measurement from which you can calculate the grating spacing d.

d = __________ Å

Using this value of d and the same experimental technique find the wavelength of the blue line in this hydrogen spectrum.

λ_{blue} = __________ Å

Name: ______________________

DIFFRACTION GRATING

Circle the letter corresponding to your spectrometer and grating

Spectrometer	A	B	C	D	E
Grating	A	B	C	D	E

Using the sodium source and assuming the yellow doublet has a mean value of λ = 5893 Å do an approximate measurement using this yellow line from which you can then calculate the grating spacing d. (Your result should be within 200 Å of our carefully measured value.)

d = ______________ Å

Using value of d and the same experimental technique find the wa ngth of the red line in this sodium spectrum.

red = ______________ Å

NAME ______________________________

[illegible]

1 [illegible]
1 He-Ne laser, λ = 6328 Å
2 [illegible]
1 Numbered Al plate containing a small circular hole
1 [illegible] screen.

[illegible] serial number of the Al plate.
[illegible] the laser beam.
[illegible] diffraction pattern produced on the screen by the hole in the Al plate.
By [illegible] this pattern determine as accurately as you can the diameter of the hole.

For circular apertures the angular position, θ, of the first diffraction minimum is given by

$$\sin\theta = 1.22\,\frac{\lambda}{d},$$

where d is the aperture diameter.

The distance from the front edge of the table to the distant screen is [illegible] ± 0.04 m

[illegible]

d = __________ ± __________

[illegible] Name: ______________________

A. Focal length measurement.

You are to find the focal length of this lens by using the formula $\frac{1}{i} + \frac{1}{o} = \frac{1}{f}$. Form a focused image of the small hole in your light source on the card, and show your measurements of object and image distances, and calculations on this page. You MAY USE THE WANG CALCULATOR if it is available.

focal length = ____________

B. Magnification ($m = -i/o$) measurement.

TURN OFF YOUR LIGHT SOURCE when part A is completed. Now set up your lens and card to produce a magnified image of an object which will be placed 10 cm from whichever end of your meter stick you choose (the instructor will bring over the object and place it at this position when you tell him you are ready.) The lens and card should be spaced so that when the instructor places the light source in position and turns it on the image on the card will be focused and [ ] times larger than the object.

This part for the instructor.

Focused? [ ]

Experimental magnification [ ]