

# Holographic representations of space-variant systems using phase-coded reference beams

Thomas F. Krile, Robert J. Marks II, John F. Walkup, and Marion O. Hagler

A new holographic implementation of a sampling technique permits, in principle, a straightforward representation of 2-D space-variant optical systems. The set of sample transfer functions required for the representation is recorded on a single holographic plate by utilizing phase coded reference beams. Because this approach does not depend on volume effects in the recording medium in an essential way, the holograms can be produced digitally, as well as optically. Basic concepts and preliminary experimental investigations related to this approach are presented and discussed.

## Introduction

A procedure has recently been described for using a sampling technique to represent space variant optical systems.<sup>1</sup> One proposed implementation of this technique relies on volume hologram effects to angle-multiplex various sample transfer functions of the system into a thick recording medium.<sup>2-4</sup> A problem arises with angle multiplexing, however, in that the extinction angle effect is, for practical purposes, 1-D in nature due to the resulting Bragg cones.<sup>3,5</sup> Thus, using volume holograms, only 1-D space-variant systems can be represented in a straightforward way, without undesirable cross talk upon playback. More straightforward techniques have been demonstrated elsewhere for 1-D space-variant system representation.<sup>6,7</sup>

This paper demonstrates a different holographic implementation technique for sampling theorem-based representations of 2-D space-variant systems. This implementation utilizes reference beam encoding of the various transfer functions via a random (or pseudo-random) phase diffuser. This approach eliminates, in principle, the need for a volume recording medium, so that thin (rather than thick) recording media may be used. It therefore opens the important possibility of simulating arbitrary 2-D space-variant operations using computer-generated holograms not possible with volume hologram representations.

Phase encoded reference beams have been used previously for color holography<sup>8,9</sup> and for the purpose of multiplexing a number of point source objects for

digital storage.<sup>10</sup> In the color holography application, simultaneous, but not coherent, readout of several different recordings (one for each primary color) is required. In the digital storage case, individual readout of each recorded object is desired with minimum background noise. The requirements in representing space-variant systems are more stringent in that simultaneous phase-coherent reconstruction of a number of recordings is desired with no significant objectionable cross talk between the various playback (reference) beams. As we will show, the random phase diffuser technique can satisfy these requirements.

## Recording and Playback Operations with Phase-Coded Reference Beams

The space-variant system recording procedure based on the sampling theorem, but without reference beam phase encoding, is shown in Fig. 1.<sup>2,3</sup> The system  $S$  for the moment is shown sampled by impulse functions (point sources)  $i_1$  to  $i_N$ , giving rise to the point spread functions  $h_1$  to  $h_N$  at the output plane of  $S$ . The lens  $L_1$  then produces fields proportional to the transfer functions  $H_1$  to  $H_N$  at the hologram plane, where  $H_j$  is the Fourier transform of  $h_j$ .<sup>11</sup> Now for each input point source  $i_j$  there is a corresponding reference point source  $r_j$  which is transformed by the lens  $L_2$  to form the reference plane wave  $R_j$  at the hologram plane.<sup>2,3</sup> The recording technique then consists of sequentially recording the interference patterns of the system function/reference plane wave pairs  $(H_j, R_j)$ . If we bias the medium so that the resulting hologram's amplitude transmittance function  $t$  is proportional to the exposing intensity pattern,

$$t = \sum_{j=1}^N |H_j + R_j|^2. \quad (1)$$

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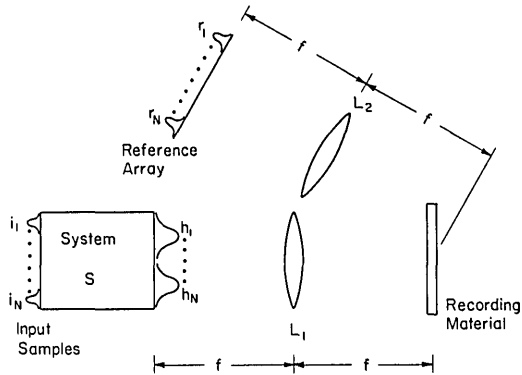


Fig. 1. Recording scheme for the holographic representation of the space-variant system  $S$ . The holograms for the  $(i_j, r_j)$  pairs are recorded sequentially. Transforming lenses  $L_1$  and  $L_2$  each have focal length  $f$ .

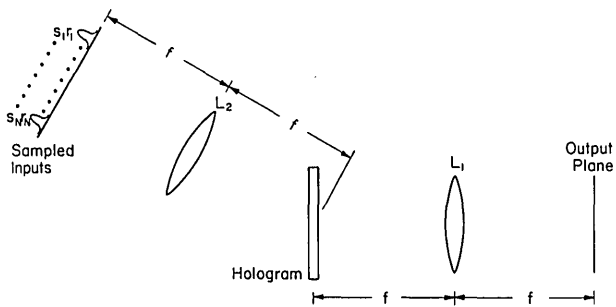


Fig. 2. Playback scheme for the holographic representation of the system  $S$ . The input is spatially sampled by a duplicate of the reference array.

The playback step is shown in Fig. 2. The input object is spatially sampled at the point source locations of the original reference array to produce sampled inputs  $s_1 r_1$  to  $s_N r_N$ , where  $s_j$  is the sampled value at the  $j$ th location. The desired output plane field is given by the sum

$$\begin{aligned} o &= \sum_{j=1}^N s_j \mathcal{F}^{-1}\{H_j\} \\ &= \sum_{j=1}^N s_j h_j, \end{aligned} \quad (2)$$

where we note that coherent addition of the simultaneous reconstructions is required. Also note that potential cross-talk terms have been neglected,<sup>12</sup> and that low pass filtering in the hologram plane will be required to obtain a continuous output.<sup>1</sup>

Since cross-talk terms have been neglected in writing Eq. (2), we have assumed that each input point sample sees only the desired transfer function upon playback. We will now analyze what happens in the special case of a simple two sample point system to show what the cross-talk problems are and to examine how utilization of the diffuser encoding technique in the reference beams can suppress cross talk.

After sequential recording of the two object-reference pairs  $(H_1, R_1)$  and  $(H_2, R_2)$ , the amplitude transmittance function stored in the hologram is given by

$$t = |H_1 + R_1|^2 + |H_2 + R_2|^2. \quad (3)$$

In the playback step, if the sampled input values are given by the constants  $s_1$  and  $s_2$ , the reconstructed wavefront just to the right of the hologram in Fig. 2 is

$$\begin{aligned} W &= (s_1 R_1 + s_2 R_2)t \\ &= s_1 R_1 R_1^* H_1 + s_2 R_2 R_2^* H_2 + s_1 H_2 R_1 R_2^* + s_2 R_1^* R_2 H_1 \\ &\quad + s_1 H_1 H_1^* R_1 + s_1 R_1 R_1^* R_1 + s_1 H_2 H_2^* R_1 + s_1 R_2 R_2^* R_1 \\ &\quad + s_2 H_1 H_1^* R_2 + s_2 R_1 R_1^* R_2 + s_2 R_2 R_2^* R_2 + s_2 H_2 H_2^* R_2 \\ &\quad + s_1 R_1 R_1 H_1^* + s_1 R_1 R_2 H_2^* + s_2 R_1 R_2 H_1^* + s_2 R_2 R_2 H_2^*. \end{aligned} \quad (4)$$

Here, the superscript  $*$  denotes a complex conjugate. Of the sixteen terms in Eq. (4), the first four have been diffracted by the hologram (see Fig. 2) and appear in the output plane. We will define these four terms as  $W'$ , so that

$$W' \triangleq s_1 R_1 R_1^* H_1 + s_2 R_2 R_2^* H_2 + s_1 R_1 R_2^* H_2 + s_2 R_1^* R_2 H_1. \quad (5)$$

Since  $s_1, s_2, R_1 R_1^*, R_2 R_2^*, H_1 H_1^*$ , and  $H_2 H_2^*$  are real numbers, we see that the second and third groups of four terms (each) in Eq. (4) are undiffracted terms which will not appear in the output plane of Fig. 2. Finally, the last four terms in Eq. (4) represent light which is diffracted to the opposite side of the playback system axis (analogous to a twin image effect) when compared with the first four terms which potentially appear in the output plane.<sup>11</sup> Thus we need only concern ourselves with the four terms listed in Eq. (5) when deciding which terms, after Fourier transformation by lens  $L_1$  of Fig. 2, will appear in the output plane.

Now suppose an ideal phase diffuser is placed in the reference beam side of Figs. 1 and 2, such that  $r_1$  and  $r_2$  are unit amplitude waves with complex phase fronts (i.e.,  $r_1 = \exp[j\phi_1(x, y)]$ ,  $r_2 = \exp[j\phi_2(x, y)]$ ). We may take the inverse Fourier transform<sup>12</sup> of Eq. (5) to show that the output (image) plane field is given by

$$\begin{aligned} o' &= \mathcal{F}^{-1}\{W'\} = s_1 h_1^*(r_1 \star r_1) + s_2 h_2^*(r_2 \star r_2) \\ &\quad + s_1 h_2^*(r_1 \star r_2) + s_2 h_1^*(r_2 \star r_1), \end{aligned} \quad (6)$$

where  $*$  represents convolution, and  $\star$  represents correlation. In obtaining Eq. (6) we have made use of the convolution and autocorrelation theorems of Fourier analysis.<sup>11</sup> Note that now, however,  $r_1$  and  $r_2$  represent the diffuser patterns seen at the two positions in the input sampling array. If the diffuser has the property that  $r_i \star r_i$  is effectively a Dirac delta function, while  $r_i \star r_j$  is a very broad, uniform spatial function, the output may effectively be expressed as

$$o' = s_1 h_1 + s_2 h_2 + \text{diffuse background noise}. \quad (7)$$

It should be noted that no extinction angle was assumed operative in obtaining the result of Eq. (7). In this approach, we are thus not restricted to working with thick recording media,<sup>2,3</sup> a potentially significant advantage. If, on the other hand, a volume hologram is used, the extinction angle effect can further reduce the average noise by restricting the cross talk to be only between input points lying along the loci defined by the

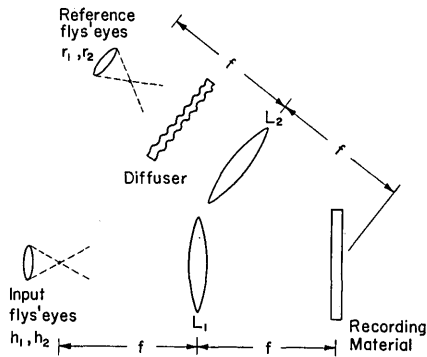


Fig. 3. Experimental setup for simulation of two-point magnifier. The elements of the fly's-eye arrays illuminated are vertical (in a line perpendicular to the plane of the drawing).

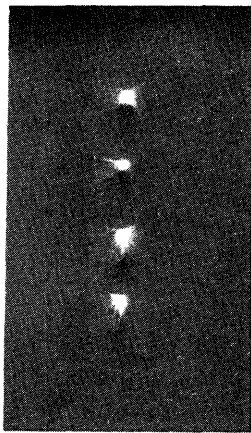


Fig. 4. Reconstructed output obtained with simulated two-point demagnifier and without diffuser-induced phase coding of reference beams. The two inner points are the desired output, while the two outer points represent cross talk.

relevant Bragg cones. Thus the existence of an extinction angle defined by the volume hologram, in addition to the cross talk-suppressing action associated with the diffuser, can effectively constrain cross-talk effects to  $N$  points at a time (assuming an  $N$  by  $N$  input sampling array) instead of the  $N^2$  interactions expected if one uses only a diffuser and a thin recording medium.

### Experimental Results

To test the theory presented above, an idealized, space-variant two-point magnifier was set up as shown in Fig. 3. Two fly's-eye lenses (3.8-mm diam each) having a vertical separation (in a line perpendicular to the plane of the figure) of 15.4 mm were used to form the reference point sources. The output of an idealized magnifier (with  $M = 1/2$ ) was simulated by using two fly's-eye lenses separated by 7.7 mm to form the object points. The transforming lenses  $L_1$  and  $L_2$  had 10-cm focal lengths. The results of four experiments which were performed are described below.

**Experiment 1:** In the first experiment no diffuser was placed in the reference beams. The  $(R_1, H_1)$  and  $(R_2, H_2)$  combinations were recorded sequentially on a Kodak 649F plate. Reconstruction was performed by moving  $L_1$  10 cm to the right of the hologram and observing the image plane, which is now the Fourier transform plane of  $L_1$  (see Fig. 2). The result is shown in Fig. 4 where the inner pair of points is the desired output (i.e., a pair of points separated by 7.7 mm), and the outer pair of points represents the cross-talk terms  $\mathcal{F}^{-1}(s_1 H_2 R_1 R_2^*)$  and  $\mathcal{F}^{-1}(s_2 H_1 R_1^* R_2)$  which we want to suppress.

It should be noted here that the geometry of this experiment was such that the extinction angle effect was not operative in the vertical direction, so that the points in the vertical exhibit maximum cross talk, as shown in Fig. 4.

**Experiment 2:** In this experiment, a shower glass diffuser was inserted in the reference beams, about 3 cm from the plane of the reference point sources so that each of the two beams would intercept a maximum diffuser area, but would not intersect the diffuser area intercepted by the other reference beam. The output plane result for this experiment is shown in Fig. 5. Here the two desired output points stand out clearly against a diffuse background of coherent noise. The design pattern induced by the particular shower glass diffuser used did not have a very sharp autocorrelation function, so that repositioning of the developed hologram was not too critical an operation (i.e., it took a movement of several millimeters to make the output points disappear). Note that the diffuser also had a spatially varying attenuation, so that the cross talk is not spread as uniformly as possible over the output field. Nevertheless the results appear to be encouraging.

**Experiment 3:** A third experiment was performed to demonstrate the 2-D capabilities of the diffuser encoding technique. A three-point demagnifier was simulated with the same basic configuration as in Fig. 3. However, three of the four lenses in a square subarray portion of the fly's-eye array were used. The result, when the diffuser was placed in the reference

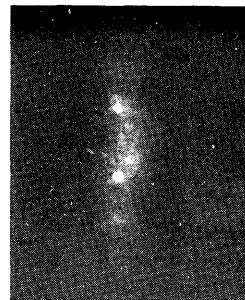


Fig. 5. Reconstructed output (Exp. 2) with phase coding of reference beams by shower glass diffuser. The cross-talk terms present in Fig. 4 have been spread out into diffuse background noise.

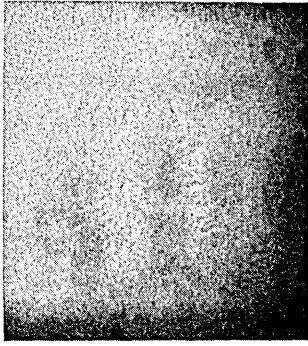


Fig. 6. Reconstructed output (Exp. 3) for the simulated three-point demagnifier, indicating diffuse background noise in both directions.

beams, is shown in Fig. 6. It can be seen that the multiple cross-talk points which would be expected without the diffuser encoding of the reference beams have been relegated to a diffuse background noise, while the three desired output points stand out clearly. Again it should be noted that since 649F has a rather large extinction angle relative to the angular separation of the reference beams used here, volume hologram effects were not responsible for eliminating either the horizontal or vertical cross talk expected without the diffuser.

Experiment 4: An experiment was performed to verify that even with the shower glass diffuser present in the path of the reference beams, coherent addition is obtained in the output plane as required based on Eq. (2). Basically the same setup as shown in Fig. 3 was used, with the exception that a single input point source was used in the object path (i.e., only  $h_1$  present), whereas both reference fly's eyes were used. Unfortunately, due to the complicated nature of the shower glass diffuser pattern, it was difficult to document that indeed coherent addition of the outputs corresponding to the two reference beam point source inputs was taking place in the playback step. Indications are, however, that coherent addition was taking place, and additional experiments are underway to provide documentation of this hypothesis.

### Conclusions

A technique has been demonstrated for holographically recording representations of 2-D space-variant systems using phase-encoded reference beams. The use of reference beam phase encoding for the purpose of multiplexing noninterfering holograms into a single thin recording medium, rather than attempting to use the extinction angle property of a thick recording medium to angle multiplex the multiple holograms, possesses a number of potential advantages. One would not be restricted to volume holograms for recording space-variant optical systems. Thus one could in principle construct computer-generated holograms for representing arbitrary 2-D space-variant processors.

It is clear that the correlation properties of the diffuser are important in determining the practical limitations of this space-variant recording process. Since a spatial sampling technique is used, one wants the diffuser autocorrelation function to be as nearly like a Dirac delta function as possible so that a high sampling rate can be achieved. This will in turn lead to precise positioning requirements for placement of the diffuser in the reconstruction step, suggesting *in situ* processing wherever possible. We also note that the cross correlation between any two different diffuser elements should be as broad as possible. This feature will optimally spread out the cross-talk noise spatially, so that one can multiplex as many samples as possible with (hopefully) a reasonable SNR on reconstruction. Additional investigations are underway in these and other areas relating to the technique presented in this paper.

As a final note, the scheme presented here is not limited to implementation of the sampling theorem. With minor changes in technique, other linear system representation models such as the piecewise isoplanatic approximation<sup>4</sup> and the orthonormal element response characterization method<sup>13</sup> can be similarly implemented.

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### Appendix: Representations Using Volume Holograms

If the reference point sources represented by  $r_1$  and  $r_2$  are sufficiently far apart so that the Bragg extinction angle property of a volume recording medium is operative, the third and fourth terms in Eq. (5) will not appear in the output plane, since those terms represent cross talk between the two reference waves.<sup>1,2</sup> In this case Eq. (5) effectively becomes

$$W' = s_1 R_1 R_1^* H_1 + s_2 R_2 R_2^* H_2.$$

Assuming, for illustration purposes, that  $R_1$  and  $R_2$  have unit amplitudes, the output plane field is then given by

$$\begin{aligned} o' &= \mathcal{F}^{-1}\{W'\} = s_1 \mathcal{F}^{-1}\{H_1\} + s_2 \mathcal{F}^{-1}\{H_2\} \\ &= s_1 h_1 + s_2 h_2, \end{aligned}$$

which, based on Eq. (2), is the desired result. If, on the other hand, the Bragg extinction angle affect is not operative (i.e., the two points  $r_1$  and  $r_2$  are too close together), the inverse transform of the cross-talk terms  $s_1 H_2 R_1 R_2^*$  and  $s_2 H_1 R_1 R_2^*$  present in Eq. (5) will appear in the output plane. As explained in the main text, however, the use of phase-encoded reference beams can greatly suppress cross-talk effects and of course has the advantage of permitting straightforward representation of 2-D inputs.

## References

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3. J. F. Walkup and M. O. Hagler, "Volume Hologram Representations of Space-Variant Optical Systems," in *Proceedings 1975 Electro-Optical Systems Design Conference* (Industrial and Scientific Conference Management, Inc., Chicago, 1975), pp. 31-37.
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8. Ref. 5, Chap. 17.
9. R. J. Collier and K. S. Pennington, *Appl. Opt.* **6**, 1091 (1967).
10. J. T. LaMacchia and D. L. White, *Appl. Opt.* **7**, 91 (1968).
11. J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, San Francisco, 1968).
12. In practice lens  $L_1$  in Fig. 2 takes a second direct Fourier transform, but aside from a coordinate reversal, we can represent this operation as an inverse transform as in Eq. (2) without loss of generality (see Ref. 11).
13. R. J. Marks II, J. F. Walkup, and M. O. Hagler, "Volume Hologram Representation of Space-Variant Systems," in *Applications of Holography and Optical Data Processing*, E. Marom *et al.* Eds. (Pergamon, Oxford, 1977).

## Meetings Calendar continued from page 3130

1978

- January**
- 16-18 **Integrated and Guided Wave Optics, IEEE/OSA mtg., Salt Lake City** J. W. Quinn, OSA 2000 L St. N.W., Washington, D.C. 20036
- 16-18 Optics in Missile Engineering, seminar, Los Angeles SPIE, P.O. Box 10, Bellingham, Wash. 98225
- 16-18 Utilization of Infrared Detectors, seminar, Los Angeles SPIE, P.O. Box 10, Bellingham, Wash. 98225
- 16-18 Practical Optics, seminar, Los Angeles SPIE, P.O. Box 10, Bellingham, Wash. 98225
- 20-26 American Physical Society, gen. mtg., San Francisco W. W. Havens, Jr., APS, 345 E. 45th St., New York, N.Y. 10017
- 24-26 Lasers and Applications, course, Arlington, Va. Continuing Education Courses, P.O. Box 245, McLean, Va. 22101
- 25-27 25th Annual Conference of the Western Spectroscopy Association, Asilomar A. Abu-Shumays, Varian Assoc., Palo Alto, Calif. 94303
- 26-28 3rd NASA Conference on Radiation Energy Conversion, Moffett Field K. W. Billman, NASA-Ames Res. Center, Mail Stop 230-3, Moffett Field, Calif. 94035
- February**
- 1 **Current Trends in the Contact Lens Field, sectional mtg.** E. M. Silverstein, Optical Society of Southern California, McDonnell Douglas, 5302 Bolsa Ave., Huntington Beach, Calif. 92647
- 1-3 San Diego Biomedical Symposium, San Diego P.O. Box 80543, San Diego, Calif. 92138
- 5-8 Conference on Objectives of Pictorial Color Reproduction, Williamsburg, Va. F. W. Billmeyer, Jr., Rensselaer Polytechnic Instit., Chem. Dept., MRC Room 217, Troy, N.Y. 12181
- 7-9 **IEEE/OSA Conference on Laser and Electro-Optical Systems, San Diego** J. W. Quinn, OSA, 2000 L St. N.W., Washington, D.C. 20036
- 7-9 **Inertial Confinement Fusion, topical mtg., San Diego** J. W. Quinn, OSA, 2000 L St. N.W., Washington, D.C. 20036
- 12-17 American Association for the Advancement of Science, natl. mtg., Washington AAAS Meetings Off., 1776 Massachusetts Ave. N.W., Washington, D.C. 20036
- 13-14 Laser Technology, seminar, Atlanta New York U., School of Continuing Education, 360 Lexington Ave., New York, N.Y. 10017
- 13-17 Infrared: Sources, Components, and Systems, course, Albuquerque Laser Instit. of America, P.O. Box 1744, Waco, Texas 76703
- 17-18 **Optical Fabrication and Testing Workshop, Orlando** N. Mohon, 3307 Balsam Drive, Winter Park, Fla. 32792
- 22-23 Light Scattering in Physics, Chemistry, and Biology, mtg., London A. D. Buckingham, University Chem. Lab., Lensfield Rd., Cambridge CB2 1EW, U.K.
- 22-23 20th Annual Conference on Aviation and Astronautics, Haifa Secretary of the Conference, Dept. of Aeronautical Engineering, Technion, Haifa, Israel
- 26-3 Mar. American Congress on Surveying and Mapping/American Society of Photogrammetry Spring Convention, Washington, D.C. W. J. Kosco, U.S. Geol. Survey Nat. Ctr., 510, Reston, Va. 22092
- 27-3 Mar. Pittsburgh Conference on Analytical Chemistry and Applied Spectroscopy, Cleveland F. A. Eggers, Jones and Laughlin Steel Corp., Graham Res. Labs., 900 Agnew Rd., Pittsburgh, Pa. 15230
- 27-3 Mar. Photographic Science Seminar, Rochester R. Francis, College of Graphic Arts & Technol., Rochester Instit. of Technol., 1 Lomb Memorial Dr., Rochester, N.Y. 14623
- March**
- 1 **Basic Optical Testing, sectional mtg.** E. M. Silverstein, Optical Society of Southern California, McDonnell Douglas, 5301 Bolsa Ave., Huntington Beach, Calif. 92647
- 1-2 Applications of Microprocessors in Measurement and Control, symposium, Utrecht, Netherlands R. G. Keiller, Sira Instit. Ltd., South Hill, Chislehurst, Kent BR7 5EH, U.K.
- 5-8 Waste Management '78, Tucson R. G. Post, U. of Ariz., College of Engineering, Tucson, Ariz. 85721

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