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Sampling Theorems for Linear Shift-Variant Systems

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***Abstract*—Sampling theorem concepts are applied to certain classes of linear shift-variant systems. Various sampling theorem characterizations arise from different bandlimiting assumptions on the system input and/or impulse response. In two of the three cases considered, the system output is also bandlimited and can be expressed in a sampling theorem expansion. The discrete characterizations arising from these two cases reduce to an identical computational form that can be evaluated with a generalized Z-transform treatment. The Fourier duals of the sampling theorems, wherein the system is characterized by its frequency rather than impulse response, are also presented.**

I. Introduction

PAST APPLICATIONS of sampling theorem concepts to linear shift-variant systems has been to evaluate the system impulse response rather than to characterize the input–output relationship [1], although adequate sampling rates have been discussed briefly [2]. In this paper, on the other hand, we present numerous conditions under which the superposition integral characterization of the input–output relation for linear shift-variant systems can be expressed in sampling theorem type expansions. Certain ramifications of these treatments, such as digital characterization of the system process without loss of information and generalized Z-transform treatment of discrete superposition relations, are also discussed. These linear system characterizations are the result of investigation of *space*-variant systems encountered in coherent optical processing. The sampling theorem for variation limited systems has previously been presented [3] and is included here for purposes of completeness and continuity.

Sampling-theorem expansions rigorously hold only for bandlimited signals. Unfortunately, a causal signal (zero for negative time) can never be rigorously bandlimited [4]. Nevertheless, there are many causal signals that can be considered to be "essentially bandlimited" so that familiar time–bandwidth product approximations can be applied if appropriate [5]. Since the causality considerations for the sampling theorems presented here are similar to those for the Whittaker–Shannon sampling theorem, for example, they are not discussed further in this paper. We note, however, that for spatial systems used in coherent optical processing, causality is not a constraint.

In the following section, certain preliminary notational and computational conventions are given which are necessary in the development of the sampling theorems. Sections III through V present three different sampling theorems corresponding to various bandlimiting assumptions

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on the system input and/or impulse response. We show Section VI that a direct consequence of these sampling theorems is infinite matrix characterizations of the system process which, except for sampling rates, are identical. The use of the Z-transform in treating these discrete characterizations is briefly discussed in Section VII. In Section VIII the Fourier duals of the sampling theorems are presented wherein sampling is largely performed in the frequency domain. Section IX contains some concluding remarks.

For clarity of presentation, attention will be restricted to one dimension. Generalization to multidimensional cases is straightforward.

II. Preliminaries

The response $y(t)$ of a linear system to an input $u(t)$ can be expressed by the superposition integral

$$y(t)=S[u(t)]=\int_{-\infty}^{\infty} h(t-\tau;\tau)u(\tau)\,d\tau \tag{1}$$

where $S(\cdot)$ is the system operator and the system impulse response is formally defined as

$$h(t-\tau;\tau)=S[\delta(t-\tau)] \tag{2}$$

where $\delta(t)$ denotes the Dirac delta. (For a causal system, $h(t-\tau;\ \tau)$ is zero for $t<\tau$.) This particular notation for the impulse response [6] apparently used first in the systems area by Kailath [1] and later in regard to optical systems by Lohmann and Paris [7], has the advantage of bringing us directly to the shift-invariant case when the system function does not depend on its second argument. Also, we can directly express the spectrum of the system output utilizing Fourier operators

$$\begin{aligned} Y(f)&=\mathfrak{F}_t[y(t)]\\ &=\int_{-\infty}^{\infty}\mathfrak{F}_t[h(t;\tau)]\exp(-j2\pi f\tau)u(\tau)\,d\tau\\ &=\mathfrak{F}_\tau\mathfrak{F}_t[h(t;\tau)u(\tau)]|_{\nu=f} \end{aligned} \tag{3}$$

where the Fourier operators are defined by

$$\mathfrak{F}_t[s(t;\tau)] \triangleq \int_{-\infty}^{\infty} s(t;\tau)\exp(-j2\pi ft)\,dt \tag{4}$$

and

$$\mathfrak{F}_\tau[s(t;\tau)] \triangleq \int_{-\infty}^{\infty} s(t;\tau)\exp(-j2\pi\nu\tau)\,d\tau. \tag{5}$$

In (3), the impulse response notation is consistent with that used in (2). An illustration of the transition from $h(t-\tau;\tau)$ to $h(t;\tau)$ is offered in Fig. 1. Note that for the shift-invariant case that $h(t;\tau)\rightarrow h(t)$. Equation (3) then takes on the familiar product form $Y(f)=\mathfrak{F}_t[h(t)]\mathfrak{F}_t[u(t)]$.

A transform of the impulse response which will be of interest is the *variation spectrum* defined as

$$H_\tau(t;\nu)=\mathfrak{F}_\tau[h(t;\tau)]. \tag{6}$$

The support of the variation spectrum is a measure of the manner in which the impulse response changes shape with

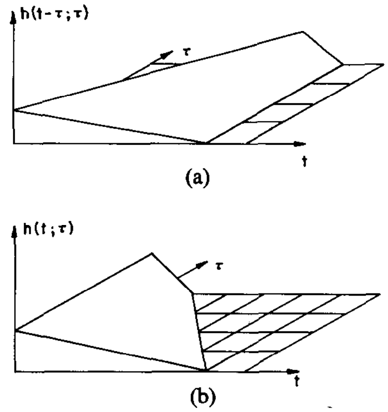

Fig. 1. An example of the transition between the system function forms (a) $h(t-\tau;\tau)$ and (b) $h(t;\tau)$ on the (t,τ) plane.

respect to τ. We consider here the low-pass case for which $H_\tau(t;\nu)$ is identically zero outside the interval $|\nu|\leq W_\nu$. Such systems will be referred to as *variation limited*. The quantity $2W_\nu$ is appropriately termed the *variation bandwidth* [1]. Note that a shift invariant system has a variation bandwidth of zero and is thus truly "invariant."

III. A Sampling Theorem for Variation Limited Systems

We now will develop a sampling theorem applicable to variation limited systems with bandlimited inputs. For the bandlimited input, we again consider the low-pass case where $u(t)$ has bandwidth $2W_u$. Consider, then, the term $h(t;\tau)u(\tau)$ which is the argument of the Fourier operator in (3). Multiplication in the τ domain corresponds to convolution in the ν domain. As such, if $u(\tau)$ has bandwidth $2W_u$ and $h(t;\tau)$ has a variation bandwidth of $2W_\nu$, then their product will have a bandwidth $2W_s$ equal to the sum of the component bandwidths:

$$2W_s=2W_\nu+2W_u. \tag{7}$$

The product $h(t;\tau)u(\tau)$ can thus be expanded in a uniformly converging [8] Whittaker–Shannon sampling theorem [9] in τ:

$$h(t;\tau)u(\tau)=\sum_n h(t;\tau_n)u(\tau_n)\cdot\operatorname{sinc}2W_s(\tau-\tau_n) \tag{8}$$

where $\tau_n=n/2W_s$ and $\operatorname{sinc}x \triangleq \sin\pi x/\pi x$. Substituting into (3) and simplifying gives

$$Y(f)=\frac{1}{2W_s}\sum_n H_t(f;\tau_n)u(\tau_n)\cdot\exp(-j2\pi f\tau_n)G\left(\frac{f}{2W_s}\right) \tag{9}$$

where our transfer function is defined by

$$H_t(f;\tau) \triangleq \mathfrak{F}_t[h(t;\tau)] \tag{10}$$

and $G(f)$ is the gate function:

$$G(f)=\begin{cases}1, & |f|\leq 1/2\\ 0, & |f|>1/2.\end{cases}$$

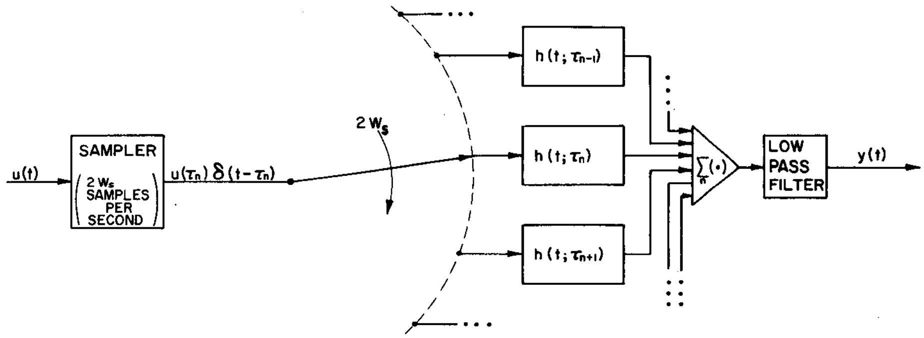

Fig. 2. An implementation of the sampling theorem presented in Section III. Sample values of the input are fed into a bank shift invariant filters each of which corresponds to a sample of the parent shift variant impulse response.

Inverse Fourier transforming (9) gives

$$y(t)=\sum_n h(t-\tau_n;\tau_n)u(\tau_n)*\text{sinc}\,2W_s t \qquad (11)$$

where "*" denotes the convolution operation.

We interpret (11) as follows: For bandlimited inputs, the output of a variation-limited system can be computed by 1) sampling the input, 2) multiplying each input sample by its corresponding sample impulse response, 3) summing the results, 4) passing the sum through a suitable low-pass filter. As is shown in Fig. 2, we can interpret this result as the representation of a variation-limited system by a bank of shift-invariant systems each of which corresponds to a sample impulse response. The switching mechanism required to feed each filter its corresponding sample value represents the shift variance of the overall system. Note that (11) is not optimal in the sense of utilizing minimum sampling rates. That is, $u(\tau)$ only requires a sampling rate of $2W_u$ and $h(t;\tau)$ a sampling rate of $2W_\nu$ in τ. Both are here being sampled at a rate of $2W_s$. As is shown in [3], however, the expression resulting from *individual sampling* of the input and impulse response at their corresponding minimum rates yields a computationally less attractive expression.

IV. An Alternate Sampling Theorem

In the previous section, $h(t;\tau)$ was assumed to be bandlimited in τ. Note that this restriction does not necessarily assure that $h(t-\tau;\tau)$ is also bandlimited in τ. As such, we can derive an alternate sampling theorem for the case where $\mathcal{F}_\tau[h(t-\tau;\tau)]$ is zero outside of the interval $|\nu|\leqslant W_h$. If our input has bandwidth $2W_u$, then the product $h(t-\tau;\tau)u(\tau)$ has bandwidth $2W_d=2W_u+2W_h$ in τ and can be expressed in the sampling theorem expansion:

$$h(t-\tau;\tau)u(\tau)=\sum_n h(t-\tau_n;\tau_n)\cdot u(\tau_n)\,\text{sinc}\,2W_d(\tau-\tau_n) \qquad (12)$$

where, here, $\tau_n=n/2W_d$. Substituting into the superposition integral (1) gives

$$y(t)=\sum_n h(t-\tau_n;\tau_n)u(\tau_n)\int_{-\infty}^{\infty}\text{sinc}\,2W_d(\tau-\tau_n)\,d\tau = \frac{1}{2W_d}\sum_n h(t-\tau_n;\tau_n)u(\tau_n). \qquad (13)$$

Our expansion here is similar to that in (11) except for the sampling rate and the fact that no low-pass filtering is required. We thus note that the system output in this case need not necessarily be bandlimited.

V. A Third Sampling Theorem

The sampling theorems thus far discussed require sampling in the τ or input domain. A third sampling theorem that utilizes output sampling occurs when $h(t;\tau)$ is bandlimited in t with (low pass) bandwidth of, say, $2W_t$. (Note that this condition is equivalent to $h(t-\tau;\tau)$ being bandlimited in t.) Such a condition holds when the system response to an impulse input is bandlimited irrespective of the location of the input delta function. Under this bandlimited assumption, we can immediately express the impulse response in the sampling theorem expansion:

$$h(t;\tau)=\sum_n h(t_n;\tau)\,\text{sinc}\,2W_t(t-t_n) \qquad (14)$$

where $t_n=n/2W_t$. Substituting into (3), followed by simplification, leaves

$$Y(f)=\frac{1}{2W_t}\sum_n \mathcal{F}_\tau[h(t_n;\tau)u(\tau)]\Big|_{\nu=f}\cdot\exp(-j2\pi f t_n)G\left(\frac{f}{2W_t}\right). \qquad (15)$$

Inverse transforming yields

$$y(t)=\sum_n [h(t_n;t-t_n)u(t-t_n)]*\text{sinc}\,2W_t t. \qquad (16)$$

As before we have reduced the system characterization to a summation of convolutions. In this case, however, we do not have to place any bandlimiting constraints on our input. Note that due to the convolving sinc, our output is bandlimited.

We can interpret (16) as shown in Fig. 3. Our input is fed into a tapped delay line that serves as the shift variance of the overall system. The outputs at various points along the delay line are then multiplied by the appropriate sample responses. All these products are then summed and passed through an appropriate low pass filter to give the corresponding system output. Kailath [1] has also utilized tapped delay lines in shift-variant system synthesis.

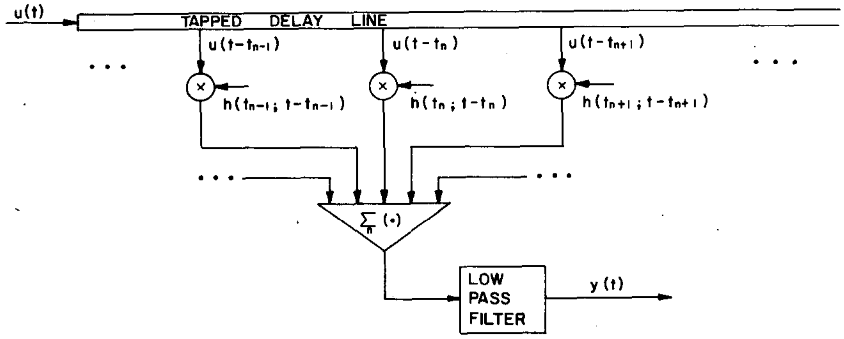

Fig. 3. An implementation of the sampling theorem presented in section V. Delayed versions of the input are multiplied by corresponding sample impulse responses, summed, and passed through a low pass filter to give the output of the parent shift variant system.

VI. Discrete Characterization

Two of the three sampling theorems thus far presented (11), (16) yield bandlimited outputs which also can be expressed in a sampling theorem expansion. In these cases, we can thus completely characterize the system output by its sample values which, in turn, can be directly evaluated by the sample input and impulse response values.

1) Consider first the variation limited system with bandlimited input. From (11), we define the low-passed system impulse response as

$$\hat{h}(t-\tau_n;\tau_n) \triangleq 2W_s h(t-\tau_n;\tau_n) * \text{sinc}\, 2W_s t. \tag{17}$$

Equation (11) can now be written as

$$y(t) = \frac{1}{2W_s}\sum_n \hat{h}(t-\tau_n;\tau_n)u(\tau_n). \tag{18}$$

It follows immediately that

$$y(t_m) = \frac{1}{2W_s}\sum_n \hat{h}(t_m-\tau_n;\tau_n)u(\tau_n). \tag{19}$$

From (9), $y(t)$ has a bandwidth of $2W_s$. Thus we require that $t_m = m/2W_s$. Note that (19) can be straightforwardly expressed in an infinite matrix form.

Suppose we now make the additional constraint that $h(t;\tau)$ is bandlimited in t with bandwidth $2W_t$. If $W_s > W_t$, then the low-passed impulse response in (17) is the same as our actual impulse response:

$$\hat{h}(t-\tau_n;\tau_n) = h(t-\tau_n;\tau_n); \qquad W_s > W_t. \tag{20}$$

Then, (19) becomes

$$y(t_m) = \frac{1}{2W_s}\sum_n h(t_m-\tau_n;\tau_n)u(\tau_n), \qquad W_s > W_t \tag{21}$$

where $t_n = \tau_n = n/2W_s$.

2) Secondly, consider the sampling theorem expansion in (16) where the output has bandwidth $2W_t$. The corresponding mth-output sample here is given by

$$y(t_m) = \sum_n \left[h(t_n;t)u(t) * \text{sinc}\, 2W_t t\right]\big|_{t=t_m-t_n} \tag{22}$$

where, now, $t_m = m/2W_t$.

A more computationally attractive form of (22) occurs when, in addition to being bandlimited in t, the system is variation limited and the input is bandlimited such that $2W_s < 2W_t$. In this case, (15) becomes

$$\begin{aligned} Y(f) &= \frac{1}{2W_t}\sum_n \mathcal{F}_t[h(t_n;t)u(t)] \\ &\quad \cdot \exp(-j2\pi f t_n) G\left(\frac{f}{2W_t}\right) G\left(\frac{f}{2W_s}\right) \\ &= \frac{1}{2W_t}\sum_n \mathcal{F}_t[h(t_n;t)u(t)] \\ &\quad \cdot \exp(-j2\pi f t_n), \qquad W_s < W_t. \end{aligned} \tag{23}$$

Inverse transforming and evaluation at $t = t_m$ gives

$$y(t_m) = \frac{1}{2W_t}\sum_n h(t_m-\tau_n;\tau_n)u(\tau_n), \qquad W_s < W_t \tag{24}$$

where $t_n = \tau_n = n/2W_t$.

Inspection of the results of the two discrete characterizations above (21), (24) reveals computationally identical forms. Our assumptions in both cases are 1) $u(\tau)$ is bandlimited and 2) $h(t,\tau)$ is bandlimited in both variables. We can combine these discrete characterizations into a single matrix-type expression:

$$y(t_m) = \frac{1}{2W}\sum_n h(t_m-\tau_n;\tau_n)u(\tau_n) \tag{25}$$

where

$$W \triangleq \max(W_t, W_s) \tag{26}$$

and $t_n = \tau_n = n/2W$. We again stress that sampling of both the input and impulse response is here performed at a rate above the required minimum allowable sampling rate.

VII. Z-Transform Treatment

For shift-invariant systems, (25) takes on the form of a discrete convolution which is frequently treated with the Z transform [9]. We will now show that, due to our choice of impulse response notation, such treatment can be generalized to the shift variant case.

We define two Z transforms of a two variable discrete sequence $s(m,n)$ by

$$Z_n[s(m,n)] = \sum_n z^{-n}s(m,n) \tag{27}$$

and

$$Z_m[s(m,n)] = \sum_m z^{-m}s(m,n). \tag{28}$$

Note the similarity of the spirit of these definitions to the Fourier transform operations in (4) and (5). Denote the Z transform of $y(m/2W)$ by $\hat{Y}(z)$. From (25), it follows that

$$\begin{aligned}\hat{Y}(z)&=Z_m[y(t_m)]\\&=\frac{1}{2W}\sum_n\Big[\sum_m z^{-m}h(t_m-\tau_n;\tau_n)\Big]u(\tau_n)\\&=\frac{1}{2W}\sum_n Z_m[h(t_m;\tau_n)]z^{-n}u(\tau_n)\\&=\frac{1}{2W}Z_nZ_m[h(t_m;\tau_n)u(\tau_n)].\end{aligned}\tag{29}$$

This is the Z-transform treatment of a discrete shift-variant process. We present it here primarily to point out a generalization. Applicability of (29) to discrete analysis has yet to be established. Note, however, as was in the case of (3), the result reduces to the more familiar product form for the shift-invariant case.

VIII. Fourier Dual Sampling Theorems

The sampling theorems thus far presented can also be applied in a Fourier dual sense to the frequency domain. The corresponding constraints here take on a physically different meaning and thus widen the class of systems which can be characterized in sampling theorem type expansions.

To change the computational form of the superposition integral, we apply Parseval's theorem to (1):

$$y(t)=\int_{-\infty}^{\infty}k(t-c\nu;\nu)U(\nu)\,d\nu \tag{30}$$

where $U(\nu)=\mathfrak{F}_\tau[u(\tau)]$ and

$$k(t-c\nu;\nu)=\int_{-\infty}^{\infty}h(t-\tau;\tau)\exp(j2\pi\nu\tau)\,d\tau. \tag{31}$$

The kernel $k(\cdot,\cdot)$ is recognized as the system frequency response:

$$k(t-c\nu;\nu)=S[\exp(j2\pi\nu t)]. \tag{32}$$

The constant c is included simply to maintain dimensional consistency between the time variable t and frequency variable ν.

1) Consider first the Fourier dual of the sampling theorem for variation limited systems presented in Section III. Here, we require that

$$\mathfrak{F}_\tau^{-1}[k(t;\nu)]=\int_{-\infty}^{\infty}k(t;\nu)\exp(j2\pi\nu\tau)\,d\nu \tag{33}$$

be identically zero outside the interval $|\tau|\leqslant T_\nu$. Also $U(\nu)$ must be "bandlimited." That is, our input $u(\tau)$ must be nonzero only over the interval $|\tau|\leqslant T_u$. The resulting sampling theorem, then, is simply the Fourier dual of (11):

$$y(t)=\frac{1}{c}\sum_n k(t-c\nu_n;\nu_n)U(\nu_n)*\mathrm{sinc}\,(2T_st/c) \tag{34}$$

where $2T_s=2T_u+2T_\nu$ and $\nu_n=n/2T_s$.

2) Consider next the Fourier dual of the sampling theorem in Section IV. Here, we require $\mathfrak{F}_\tau^{-1}[k(t-c\nu;\nu)]$ is zero for $|\tau|>T_h$ and, again, that $u(\tau)$ is zero for $|\tau|>T_u$. The Fourier dual of the sampling theorem in (13) follows immediately as

$$y(t)=\frac{1}{2T_d}\sum_n k(t-c\nu_n;\nu_n)U(\nu_n) \tag{35}$$

where $2T_d=2T_h+2T_u$ and $\nu_n=n/2T_d$.

3) Lastly, we inspect the Fourier dual of the sampling theorem presented in Section V. Our constraint in this case is that $k(t,\nu)$ is bandlimited in t with bandwidth $2W_t$. Note that this constraint is the same as requiring $h(t;\tau)$ to be bandlimited in t. The resulting sampling theorem expansion corresponding to (16) is

$$y(t)=\frac{1}{c}\sum_n k\left[t_n;\frac{t-t_n}{c}\right]\cdot U\left(\frac{t-t_n}{c}\right)*\mathrm{sinc}\,2W_tt \tag{36}$$

where $t_n=n/2W_t$.

The three sampling theorems presented here can obviously be placed in discrete form as was done in Section VI. For brevity, these discrete cases will not be presented but can be straightforwardly derived utilizing previous notions.

IX. Conclusions

We have presented several sampling theorems applicable to various classes of shift-variant systems involving certain bandlimiting constraints on the system impulse response and/or input. The system output, in certain instances, is bandlimited and the computational form required to evaluate the values for its sampling theorem expansion was shown to result in an infinite matrix relation. The computational forms in the two cases considered are identical differing only in sampling rate. The matrix-type relationship was shown to be able to be evaluated in a generalized Z-transform treatment. Fourier duals of the sampling theorem, where sampling is largely performed in the frequency domain, were also presented. Possible areas of application of the sampling theorems include signal and image processing as well as shift-variant system synthesis with a number of shift-invariant systems and/or tapped delay lines. Investigation into implementation of the sampling theorems with coherent optical processors is also presently under way [10].

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