# Proceedings of the 1978 International Optical Computing Conference

September 5-7, 1978 London, England

# DIGEST OF PAPERS



Sponsored by the Computer Society of the Institute of Electrical and Electronic Engineers in cooperation with the Office of Naval Research and the Society of Photo-Optical Instrumentation Engineers

78CH 1305-2C

M. O. Hagler, E. L. Kral and J. F. Walkup Department of Electrical Engineering Texas Tech University Lubbock, Texas 79409

and

R. J. Marks II Department of Electrical Engineering University of Washington Seattle, Washington 98195

In certain linear coherent processing techniques, a temporal signal is spatially encoded as an amplitude transmittance which serves as the processor input. In this paper, a technique is presented whereby the temporal signal is alternately used to amplitude and/or phase modulate a raster scan of the processor's input plane. Using the temporal integrating and summing properties of a hologram placed in the processor's output plane, one can then regain the identical processor output which would have arisen from the spatial encoding technique. Preliminary experimental results are presented along with the theory of the input scanning technique.

#### Introduction

In certain coherent processing schemes, a processor input is received as a temporal electronic signal. Conventionally, this signal is spatially encoded as a two-dimensional amplitude transmittance which then serves as the processor input. It is, however, usually the corresponding processor output which is of interest.

In this paper, we present a scheme whereby one can achieve an identical linear processor output by utilizing the temporal signal to amplitude and/ or phase modulate the field amplitude of an input raster scan. The time-varying field amplitude at the system's output is then temporally integrated and summed using holographic techniques. Upon playback, the hologram is shown to produce a diffracted term which is identical to that which would be obtained by placing a corresponding input field amplitude transmittance mask at the processor's input. This scheme, then, eliminates the necessity of spatially encoding the input. Use of erasable photographic media suggests possible implementations near real-time.

Input scanning has been used extensively in incoherent processing to add the temporal degree of freedom to the already available spatial variables. Various incoherent processors and corresponding references are given in the excellent re-

view paper by Monahan, Bromley and Bocker<sup>1</sup>. Scanning techniques have also been extensively applied in holography  $2^{-7}$  as have the effects of time-varying field amplitudes.

### Theory

We limit our scanning technique to those systems which are linear. Such systems can formally be expressed via the superposition integral:

$$g(\mathbf{x},\mathbf{y}) = S[u(\mathbf{x},\mathbf{y})] = \iint_{-\infty}^{\infty} u(\xi,\eta)h(\mathbf{x}-\xi,\mathbf{y}-\eta;\xi,\eta) d\xi d\eta$$
(1)

where g is the system output corresponding to an input u into a system  $S[\cdot]$ . The point spread function is defined as

$$h(x-\xi,y-\eta;\xi,\eta) = S[\delta(x-\xi,y-\eta)]$$
 (2)

where  $\delta(\cdot, \cdot)$  denotes the Dirac delta. We are here using the Lohmann-Paris point spread function (impulse response) notation.  $^{11,12}$ 

Consider, then, the scanning geometry shown in Fig. 1. For the fixed value of  $\eta$  =  $\eta_m,$  we scan the

input plane over  $\xi$  at speed v. Modeling the scanning point as an amplitude and/or phase modulated delta function, the field amplitude to the right of the input plane at time t is

$$u(vt,\eta_{0})\delta(\xi-vt,\eta-\eta_{0})$$
 (3)

From Eq. (2), the corresponding complex field amplitude incident on the output plane is

$$u(vt,\eta_)h(x-vt,y-\eta_;vt,\eta_)$$
(4)

Placed in the output plane is a photosensitive medium on which is also incident a planar reference

beam  $exp(jk\alpha y)$ , where  $\alpha$  is a direction cosine.<sup>9</sup> The corresponding intensity at time t is thus given by

$$I_{m}(x,y;t) = |u(vt,\eta_{m})h(x-vt,y-\eta_{m};vt,\eta_{m}) + \exp(jkay)|^{2}$$
(5)

Assuming the resulting hologram's amplitude transmittance is proportional to the exposing intensity function, we have, for one scan, an amplitude transmittance of  $^{8-10}$ 

$$\hat{t}_{m}(\mathbf{x},\mathbf{y}) = \int_{t=0}^{T} I_{m}(\mathbf{x},\mathbf{y};t) dt$$
(6)

where T is the exposure time for a single scan. For M scans corresponding to various values of  $\eta_m$ ,

the hologram's amplitude transmittance is

$$\hat{t}(x,y) = \sum_{m=1}^{M} \hat{t}_{m}(x,y) = t_{1} + t_{2} + t_{3}$$
 (7)

where

$$\begin{aligned} t_{1} &= \sum_{m} \left[ \int_{0}^{1} u(vt, \eta_{m}) h(x-vt, y-\eta_{m}, vt, \eta_{m}) dt \right] \exp\left(-jkay\right) \\ t_{2} &= t_{1} \star \\ t_{3} &= TM + \sum_{m} \int_{0}^{T} \left[ u(vt, \eta_{m}) h(x-vt, y-\eta_{m}, vt, \eta_{m}) \right]^{2} dt \end{aligned}$$

Here, "\*" denotes complex conjugate. It is the t<sub>1</sub> term in which we are interested. Making the variable substitution  $\xi = vt$  and assuming each scan covers the entire input pupil at  $\eta = \eta_m$  gives

$$t_{1} = \frac{1}{v} \sum_{m} \left[ \int_{-\infty}^{u} (\xi, \Pi_{m}) h(x-\xi, y-\Pi_{m}; \xi, \Pi_{m}) d\xi \right] \exp(-jk\alpha y)$$
(9)

Playback is performed as shown in Fig. 2. The playback beam gives rise to three diffracted terms. The term  $t_3 e^{jk\alpha y}$  is the zero order through beam and  $t_2 e^{jk\alpha y}$  is the twin image conjugate component. The term of primary interest is

$$t_{1} \exp(jk\alpha y) = \frac{1}{v} \sum_{m}^{\infty} \int_{-\infty}^{\infty} u(\xi, \eta_{m}) h(x-\xi, y-\eta_{m}; \xi, \eta_{m}) d\xi$$
(10)

This expression is recognized as a semi-discrete version of the superposition integral in Eq. (1). That is, the integral over  $\eta$  is approximated by a summation. In some instances, Eq. (10) will be an adequate approximation for the true system output.

Under certain bandlimited assumptions on the input and point spread function, we can obtain a better approximation by performing a low pass filtering operation in the y direction. This stems from space-variant system sampling theory.<sup>13,14</sup> If

$$\int_{-\infty}^{\infty} u(\xi,\eta) \exp(-j2\pi\eta \nu) d_{\Pi} \approx 0 \text{ for } |\nu| > w_{U}$$
(11)

and

$$\int_{-\infty}^{\infty} h(\mathbf{x}, \mathbf{y}; \boldsymbol{\xi}, \boldsymbol{\eta}) \exp(-j2\pi\eta \boldsymbol{\nu}) d_{\boldsymbol{\eta}} \simeq 0 \quad \text{for } |\boldsymbol{\nu}| > w_{\boldsymbol{\nu}} \quad (12)$$

then the desired low pass filter is unity over the frequency band

$$-(w_{v} + w_{u}) \leq v \leq (w_{v} + w_{u})$$
(13)

As shown in Fig. 2, this filtering can be performed by conventional spatial filtering techniques.<sup>9</sup>

# Experiment

To illustrate the temporal integration capa-

bilities of the hologram, we consider the system in Fig. 3. A point source makes a single scan agross the one-dimensional double-slit input aperture a(x). The linear processor in this example is the familiar Fourier transformer which consists of the single lens  $L_1$ . The scan is performed along the line  $\eta = 0$ .

Following the previous model development, the field incident on the photosensitive medium is

$$a(vt) exp(-jkvtx/f) + exp(jkax)$$
 (14)

where f is the focal length of lens  $L_1$ . Under the previously stated recording assumptions, the resulting holographic field amplitude is

$$t(x,y) = t_1 + t_2 + t_3$$
 (15)

where

$$t_{1} = \int_{-T/2}^{T/2} a(vt) \exp(-jkvtx/f) dt \exp(-jk\alpha x)$$
  

$$t_{2} = t_{1} \star$$
(16)  

$$t_{3} = T + \int_{-T/2}^{T/2} |a(vt)|^{2} dt$$

When the hologram is played back, the diffracted term immediately to the right of the hologram corresponding to  $t_1$  is

$$\frac{1}{v} \int_{-\infty}^{\infty} a(\xi) \exp(-jk\xi x/f) d\xi$$
(17)

where we have made the variable substitution  $\xi = vt$  and have assumed the scan completely covered both input pulses. Equation (17) is recognized as the one-dimensional Fourier transform of a(x). Thus we should be able to regain a(x) by an additional Fourier transform. This is accomplished by a single cylindrical lens. The result of playback is shown in Fig. 4 and, as can be seen, compares quite favorably with the theory. Similar results for a single pulse (slit) input are given in Fig. 5.

# Conclusions

We have demonstrated a technique whereby temporal signals can be linearly processed without first being spatially encoded as an amplitude transmittance. The scheme makes use of the temporal integration and summation properties of the hologram.

This technique is potentially applicable to all linear coherent processors. By using a scanning modulated line source, it is also directly applicable to the recently presented class of linear one-dimensional coherent processors.

## Acknowledgements

The authors want to acknowledge the assistance with the experiments provided by Mr. Mike I. Jones, and the typing of the manuscript by Ms. Judy Clare. This research was supported by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Grant AFOSR-75-2855.

# References

- M. A. Monahan, K. Bromley and R. P. Bocker, "Incoherent Optical Correlators," <u>Proceedings</u> of the IEEE, <u>65</u>, p. 121, 1977.
- J. C. Palais, "Scanned Beam Holography," <u>Applied Optics</u>, <u>9</u>, p. 709, 1970.
- J. C. Palais and I. C. Vella, "Some Aspects of Scanned Reference Beam Holography," <u>Applied</u> <u>Optics</u>, <u>11</u>, p. 481, 1972.
- D. Kermisch, "Partially Coherent Image Processing by Laser Scanning," <u>Journal of the</u> Optical Society of America, 65, p. 887, 1975.
- D. Kermisch, "Principle of Equivalence Between Scanning and Conventional Optical Imaging Systems," Journal of the Optical Society of America, 67, p. 1357, 1977.
- B. K. Yap, "Effects of Object and Detector Sizes on the Spatial Frequencies of a One-Dimensionally-Scanned Optical System," <u>Applied</u> Optics, 14, p. 567, 1975.
- F. O. Huck and S. K. Park, "Optical-Mechanical Line-Scan Imaging Process: Its Information Capacity and Efficiency," <u>Applied Optics</u>, <u>14</u>, p. 2508, 1975.
- J. W. Goodman, "Temporal Filtering Properties of Holograms," <u>Applied Optics</u>, <u>6</u>, p. 857, 1967.

0.000

- J. W. Goodman. Introduction to Fourier Optics. New York: McGraw-Hill, 1968, p. 247, Eq. 8-88.
- R. J. Collier, C. B. Burckhardt and L. H. Lin. <u>Optical Holography</u>. New York: Academic Press, 1971, p. 440, Eqs. 15.12 through 15.14.
- 11. A. W. Lohmann and D. P. Paris, "Space-Variant Image Formation," Journal of the Optical Society of America, 55, p. 1007, 1965.
- R. J. Marks II, J. F. Walkup and M. O. Hagler, "Line Spread Function Notation," <u>Applied Optics</u>, <u>15</u>, p. 2289, 1976.
- R. J. Marks II, J. F. Walkup and M. O. Hagler, "A Sampling Theorem for Space-Variant Systems," Journal of the Optical Society of America, 66, p. 918, 1976.
- R. J. Marks II, J. F. Walkup and M. O. Hagler, "Sampling Theorems for Linear Shift-Variant Systems," <u>IEEE Transactions on Circuits and</u> Systems, CAS-25, p. 228, 1978.
- J. W. Goodman, P. Kellman and E. W. Hansen, "Linear Space-Variant Processing of 1-D Signals," <u>Applied Optics</u>, <u>16</u>, p. 733, 1977.
- R. J. Marks II, J. F. Walkup, M. O. Hagler and T. F. Krile, "Space-Variant Processing of One-Dimensional Signals," <u>Applied Optics</u>, <u>16</u>, p. 739, 1977.



Figure 1. General input scanning geometry.



Figure 2. Playback using a low pass filter.



Figure 3. Experimental scanning configuration.



Figure 4. Experimental output for a double slit input.

8



Figure 5. Experimental output for a single slit input.