Maximizing Lifetime in an Energy Constrained Wireless Sensor Array Using Team Optimization of Cooperating Systems

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Abstract - For omnidirectional wireless broadcast, if a node has sufficient power to broadcast to another node, it also has the ability to broadcast to all closer nodes. This is the wireless advantage. For the broadcast problem, one node (the source) is required to communicate to all other nodes, by a single transmission to the farthest node or using intermediate hop nodes. For a given node constellation, there exist many wireless connection trees to do this. For a known node constellation, the maximum lifetime of a single tree is equal to the minimum battery life of all the nodes. The battery life is determined by the power, if any, expended by each node. Reaching nodes far removed takes more power. The broadcast lifetime can be significantly increased by using a plurality of trees switching from tree to tree in accordance to a prescribed duty cycle. Using *the viability lemma*, we propose an evolutionary *team optimization of cooperating systems* to determine the best team of broadcast trees - along with specified duty cycles - to maximize the lifetime of a given node constellation with specified battery reserves.

I. Introduction

From a specified source node, broadcast to all other nodes in an array using omnidirectional transmission, either directly or by multi-hop, is the wireless broadcast problem. A broadcast tree wherein the source node communicates to all other nodes is said to be *viable*. Many such trees exist for a given geometry. To specify one, choice of nodes as being transmitting or leaves in the connection tree must be established. If a node transmits, either as the source or a hop node, the node transmission power must be identified.

The status of one tree being better than another is determined by the optimization metric placed on the tree. Using a tool dubbed the *the viability lemma* [1], such optimization can be crafted into an fitness function to be maximized using evolutionary search. Assumptions in formulating solutions commonly include:

1. the power required for one node to reach another is proportional to the separating distance raised to a power α,
2. broadcasting is omnidirectional, and
3. while power is required to transmit, no power is used to receive.

Evolutionary optimization has been presented for the problem of finding the tree which minimizes the tree power [1].

The problems addressed in this paper are more difficult. Each node is assumed to have a non-renewable energy source, for example, a battery. The first problem is to find the single viable connection tree which maximizes the *tree lifetime*. The tree lifetime is defined as the time from initialization to when the first node battery in the tree is totally discharged. Since there is assumed a single tree for the entire life of the broadcast, this will be called the *static* maximal lifetime wireless optimization problem. The *dynamic* maximal lifetime optimization problem allows the use of more than one connection tree. Using multiple connection trees with correspondingly assigned duty cycles will generally increase the lifetime of the broadcast before the first battery in the system fails. We propose evolutionary *team optimization of cooperating systems* (TOCS) [2] for solving the dynamic maximal lifetime problem. In certain cases, the optimal solution requires a plurality of batteries to simultaneously go dead at the end of the composite team tree broadcast.

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II. The Viability Lemma

Recently, a tool useful for wireless multihop broadcasting optimization, dubbed *the viability lemma* has been proposed [1]. A total of N nodes are dispersed and characterized by an $N \times N$ distance matrix, $\mathbf{D}$. The element $(\mathbf{D})_{24} = (\mathbf{D})_{42}$, for example, is the distance between the second and fourth node. One node in the constellation is designated the source node. Using omnidirectional broadcast for all nodes and the wireless advantage, we desire to communicate the message emanating from the source node, either directly or by hop, to all remaining nodes. This is the wireless broadcast problem. The power required to communicate between nodes is proportional to the distance between them raised to the power of α. The *power matrix*, $\mathbf{P}$, is obtained by raising each of the elements in the distance matrix, $\mathbf{D}$, to the power α. The element $(\mathbf{P})_{24} = (\mathbf{P})_{42}$ is proportional to the power required for node 2 to communicate with node 4. Three viable trees for wireless broadcast in a 7-node constellation with C being the source node are shown in Figure 1.

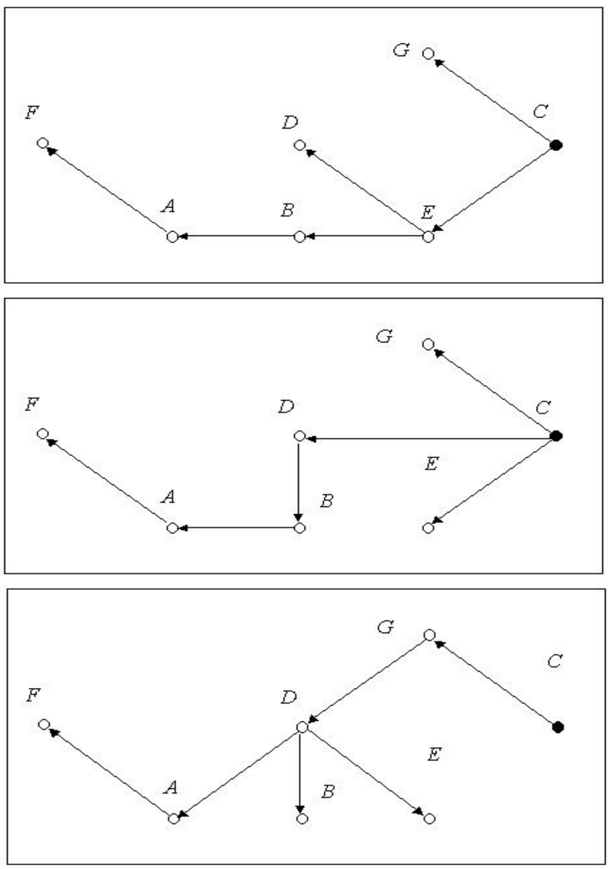

Fig. 1. An example of three different methods of covering the nodes when C is the source node. Each individual tree is a static solution for wireless broadcast. They may also act in concert for a dynamic solution wherein broadcast is switched from tree to tree in accordance to a prescribed duty cycle.

Assuming $\alpha = 2$, the power matrix for the constellation in Figure 1 is:

$$\mathbf{P} = \left[\begin{array}{ccccccc} 0 & 1 & 10 & 2 & 4 & 2 & 8 \\ 1 & 0 & 5 & 1 & 1 & 5 & 5 \\ 10 & 5 & 0 & 4 & 2 & 16 & 2 \\ 2 & 1 & 4 & 0 & 2 & 4 & 2 \\ 4 & 1 & 2 & 2 & 0 & 10 & 4 \\ 2 & 5 & 16 & 4 & 10 & 0 & 10 \\ 8 & 5 & 2 & 2 & 4 & 10 & 0 \end{array}\right] \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \quad (1)$$

The *rank matrix*, $\mathbf{R}$, is obtained by ranking each row of the power matrix, $\mathbf{P}$, from smallest to largest. For the power matrix in (1), the rank matrix is:

$$\mathbf{R} = \left[\begin{array}{ccccccc} 0 & 1 & 2 & 2 & 4 & 8 & 10 \\ 0 & 1 & 1 & 1 & 5 & 5 & 5 \\ 0 & 2 & 2 & 4 & 5 & 10 & 16 \\ 0 & 1 & 2 & 2 & 2 & 4 & 4 \\ 0 & 1 & 2 & 2 & 4 & 4 & 10 \\ 0 & 2 & 4 & 5 & 10 & 10 & 16 \\ 0 & 2 & 2 & 4 & 5 & 8 & 10 \end{array}\right] \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \quad (2)$$

A *cut vector*, $\vec{\chi_R}$, referenced to the rank matrix, is an N-element vector (where N is the number of nodes in the network) containing integers between 1 and N. Each element of a cut vector specifies a corresponding element of the rank matrix. For example, if the second element (corresponding to node B) of a cut vector is '3', the third element of the second row of the rank matrix is chosen. This value is the power level of node B for that cut vector and is used to threshold all elements in the second row of the power matrix. Those elements below the threshold are set to zero and those equal or above to one. The resulting matrix of ones and zeros is the *transfer matrix*, $\mathbf{H}$. For the cut vector

$$\vec{\chi_R} = \left[\begin{array}{c} 3 \\ 3 \\ 3 \\ 1 \\ 4 \\ 1 \\ 1 \end{array}\right] \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \quad (3)$$

the transfer matrix is:

$$\mathbf{H}[\vec{\chi_R}] = \left[\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \quad (4)$$

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To illustrate interpretation of (4), the third row, corresponding to node C, dictates that node C has sufficient power to reach nodes C, E and G.

The viability lemma is a computationally efficient tool which indicates whether the power settings at different nodes are sufficient for the source node to communicate, either directly or by hopping, to all other nodes. When this is true, there exists an associated viable broadcast tree. Each cut vector also has a corresponding cost. For the minimum power broadcast tree problem, the cost is equal to the total tree power corresponding to the optimal cut vector. The cut vector, over which search can be performed, provides a monotonic search structure. If a cut is viable, increasing one or more elements in the cut vector also corresponds to a viable cut. Conversely, if a cut is not viable, decreasing one or more elements in the cut vector results in a non-viable cut. A viable cut is on the *viability boundary* when reduction of any element in the cut by one renders the cut unviable. For many optimization criteria, including minimum power and maximum lifetime, the optimal solution lies on the viability boundary.

Let the source node be node n. One manifestation of the viability lemma states that the cut is viable if the nth row of $\mathbf{H}$ raised to the power of $N-1$ in a Boolean fashion[1] contain all ones. Since this is true for the transfer matrix in (4), the cut in (3) is viable. The upper most tree in Figure 1 is a possible manifestation of this cut. More in-depth explanation and alternate more computationally efficient versions of the viability lemma exist [1].

III. Static Energy Constrained Wireless Broadcasting

Maximizing the lifetime of a system for a single viable tree is the *static energy constrained wireless broadcasting* problem. The system assumptions are the same as minimal power broadcasting with the addition that each node has a battery with known energy. The tree is chosen to maximize the time over which the system can operate before a node in the tree goes dead. The optimization problem is similar to that of minimal power broadcasting, except a different cost function is used.

A. Node Energy

The initial battery energy of each node is known and can be expressed as an N dimensional vector, $\vec{E}(0)$ with units of joules. For the $N=7$ node example in Figure 1 a possible energy vector is:

$$\vec{E}(0) = \begin{bmatrix} 15 \\ 12 \\ 16 \\ 20 \\ 44 \\ 11 \\ 13 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} \qquad (5)$$

B. Lifetime Matrix

For a node transmitting at a constant power level P, a linear model of the residual energy in its battery at time t is:

$$E(t) = (E(0) - Pt)\,\mu\,(T-t)$$

where $\mu(\cdot)$ denotes a unit step. The battery lifetime [2] is:

$$T = \frac{E(0)}{P}.$$

Using the power matrix $\mathbf{P}$, a battery lifetime matrix, $\mathbf{T}$, can be computed as follows:

$$\mathbf{T}_{mn} = E_m / \mathbf{P}_{mn};\ 1 \leq m, n \leq N$$

where E_m is the mth element of the vector $\vec{E}(0)$ in (5). For the power matrix in (1) and the energy vector in (5), the battery lifetime matrix is:

$$\mathbf{T} = \begin{bmatrix} \infty & 15 & \frac{3}{2} & \frac{15}{2} & \frac{15}{4} & \frac{15}{2} & \frac{15}{8} \\ 12 & \infty & \frac{12}{5} & 12 & 12 & \frac{12}{5} & \frac{12}{5} \\ \frac{8}{5} & \frac{16}{5} & \infty & 4 & 8 & 1 & 8 \\ 10 & 20 & 5 & \infty & 10 & 5 & 10 \\ 11 & 44 & 22 & 22 & \infty & \frac{22}{5} & 11 \\ \frac{11}{2} & \frac{11}{5} & \frac{11}{16} & \frac{11}{4} & \frac{11}{10} & \infty & \frac{11}{10} \\ \frac{13}{8} & \frac{13}{5} & \frac{13}{2} & \frac{13}{2} & \frac{13}{4} & \frac{13}{10} & \infty \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} \qquad (6)$$

C. System Lifetime

How long will a viable cut last until the first battery totally discharges? This can be answered straightforwardly by point-by-point consideration of the battery lifetime matrix with respect to the transfer matrix in (4). The

[1] Multiplies are replaced by logical AND's and additions by logical OR's.

[2] Assuming a constant P profile and a continuous packet transmission process.

values in $\mathbf{T}$ corresponding to a '1' in the transfer matrix are:

$$\mathbf{T}[\vec{\chi_R}] = \begin{bmatrix} \infty & 15 & - & \frac{15}{2} & - & \frac{15}{2} & - \\ 12 & \infty & - & 12 & 12 & - & - \\ - & - & \infty & - & 8 & - & 8 \\ - & - & - & \infty & - & - & - \\ - & 44 & 22 & 22 & \infty & - & - \\ - & - & - & - & - & \infty & - \\ - & - & - & - & - & - & \infty \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} \quad (7)$$

Node E can communicate with node C for 22 minutes or node B for 44 minutes. The cut under inspection is required to reach node C, however, and the lifetime of the battery for node E is 22 minutes. The lifetime for each node, captured in the battery lifetime vector, $\vec{\tau}$, is revealed as the minimum of each row of $\mathbf{T}[\vec{\chi_R}]$.

$$\vec{\tau} = \min_{\text{row}} \mathbf{T}[\vec{\chi_R}]. \quad (8)$$

For the running example in (7),

$$\vec{\tau} = \begin{bmatrix} 7.5 \\ 12 \\ 8 \\ \infty \\ 22 \\ \infty \\ \infty \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} \quad (9)$$

The values for nodes D, F and G are ∞ because, as leaves, they expend no energy.

The lifetime of the system for this cut, $\tau(\vec{\chi_R})$, is the minimum entry in $\vec{\tau}$.

$$\tau(\vec{\chi_R}) = \min \vec{\tau} = \min_{all} \mathbf{T}[\vec{\chi_R}]$$

A useful computational equivalent is:

$$\tau(\vec{\chi_R}) = \min_{all \neq 0} [\mathbf{T} \odot \mathbf{H}]$$

where $\odot$ denotes element-by-element multiplication.

The system lifetime for our example is $\tau(\vec{\chi_R}) = 7.5$.

D. Optimization

Optimization for static energy constrained wireless broadcasting is similar to that for minimum power broadcasting [1]. The problem is to find the viable cut which maximizes the system lifetime, $\tau(\vec{\chi_R})$.

The search through different cuts obey the following properties[3]

- **Lifetime Monotonicity**: For a given source, if $\vec{\chi_{R2}} \geq \vec{\chi_{R1}}$, then $\tau(\vec{\chi_{R2}}) \leq \tau(\vec{\chi_{R1}})$.
- **Cut Viability Monotonicity**: For a given source, if $\vec{\chi_{R3}}$ is viable, and $\vec{\chi_{R4}} \geq \vec{\chi_{R3}}$, then $\vec{\chi_{R4}}$ is viable.
- **Cut Non-Viability Monotonicity**: For a given source, if $\vec{\chi_{R5}}$ is not viable, and $\vec{\chi_{R6}} \leq \vec{\chi_{R5}}$, then $\vec{\chi_{R6}}$ is not viable.
- **The Optimal Lifetime Cut**: The optimal cut that maximizes the system lifetime will be on the viability boundary.

IV. Dynamic Energy Constrained Wireless Broadcasting

The static energy constrained wireless broadcasting problem in Section III assumes that a single connection tree is used throughout the broadcast operation. Once the first non-leaf node loses its battery power, that connection tree can no longer be used. There are two ways by which the system life can be extended.

1. Reconfiguring by optimizing around the remaining nodes.
2. Using multiple connection trees during the broadcast operation to prolong overall system life.

We dub the latter approach *dynamic energy constrained wireless broadcasting*. Note that, under the set of assumptions, a node with a dead battery can still act as a leaf in a completely connected tree. At the current stage of research, it is unclear whether reconfiguration or dynamic energy constrained wireless broadcasting will, in general, result in the longest composite system life.

A. Analysis and Optimization

Let there be Θ distinct viable cuts with assigned connection trees. The cuts are numbered $\{\theta | 1 \leq \theta \leq \Theta\}$. During broadcast, cut θ will be used $100 \times \pi_\theta$ per cent of the time. Clearly

$$\sum_{\theta=1}^{\Theta} \pi_\theta = 1 \quad (10)$$

The quantity π_θ is referred to as the *duty cycle* of cut θ.

For a given set of duty cycles, pre-specified time intervals can be subdivided into intervals proportional to π_θ. During each interval, the corresponding tree is used. For equally long packets, route variation dictated by the duty cycles can be accomplished by tagging each packet with

[3]By $\vec{\chi_{R2}} \geq \vec{\chi_{R1}}$ we mean each element in $\vec{\chi_{R2}}$ is greater than or equal to each element of $\vec{\chi_{R1}}$.

θ in order to specify, in route, which connection tree to follow. Packets are labeled θ for $100 \times \pi_\theta$ per cent of the time, or can be labeled stochastically with probability π_θ.

The *expended energy* of the nth node in the dynamic case is

$$X_n(t) = \sum_{\theta=1}^{\Theta} \pi_\theta P_n^\theta t \mu(\tau_n - t)$$

where P_n^θ is the power expended by node n in cut θ and τ_n is the lifetime of the nth node in the dynamic system. Clearly, at time $t = \tau_n$, all of the battery energy is expended. Thus

$$X_n(\tau_n) = E_n(0) = \sum_{\theta=1}^{\Theta} \pi_\theta P_n^\theta \tau_n.$$

Solving for the node lifetime gives:

$$\begin{aligned} \tau_n &= \frac{E_n(0)}{\sum_{\theta=1}^{\Theta} \pi_\theta P_n^\theta} \\ &= \frac{1}{\sum_{\theta=1}^{\Theta} \pi_\theta / \tau_n^\theta} \end{aligned}$$

where

$$\tau_n^\theta = \frac{E_n(0)}{P_n^\theta}$$

is the lifetime of node n in static case θ. The dynamic system lifetime is:

$$\tau = \min_n \tau_n.$$

Our goal, then, is to find a set of duty cycles, π_θ which will maximize

$$\tau = \min_n \left(\frac{1}{\sum_{\theta=1}^{\Theta} \pi_\theta / \tau_n^\theta} \right) \qquad (11)$$

Equation (11) can be alternately expressed as a minimax optimization problem as follows.

$$\begin{aligned} &\max \left[\min_n \left(\frac{1}{\sum_{\theta=1}^{\Theta} \pi_\theta / \tau_n^\theta} \right) \right] \\ &= \min \left[\frac{1}{\min_n \left(\frac{1}{\sum_{\theta=1}^{\Theta} \pi_\theta / \tau_n^\theta} \right)} \right] \\ &= \min \left[\max_n \left(\sum_{\theta=1}^{\Theta} \pi_\theta / \tau_n^\theta \right) \right] \qquad (12) \end{aligned}$$

Standard minimax optimization routines (*e.g.*, MATLAB's *fminimax* routine) can be used to solve for the optimal duty cycles in (12).

Once the optimal lifetime τ is found, the residual battery life at the end of the dynamic system lifetime can be calculated as:

$$E_n(\tau) = E_n(0) - \sum_{\theta=1}^{\Theta} \pi_\theta P_n^\theta \tau. \qquad (13)$$

The system can be reconfigured from the remaining battery power if desired.

B. The Teamwork Principle

For dynamic energy constrained wireless broadcasting the Θ best cuts need not be the same as the best Θ cuts [3], *i.e.* the best team does not necessarily consist of the best individual players. Thus, in choosing Θ cuts to constitute the dynamic system, one is not assured of optimality by choosing the best Θ static cuts.

B.1 TOCS

As generically illustrated in Figure 2, a *team optimization of cooperating systems* (TOCS) approach [2] is appropriate for solution of the dynamic energy constrained wireless broadcasting. Values of cuts are aggregated to the fitness, τ, from which substitutions for the next iteration of cuts and duty cycles emerges.

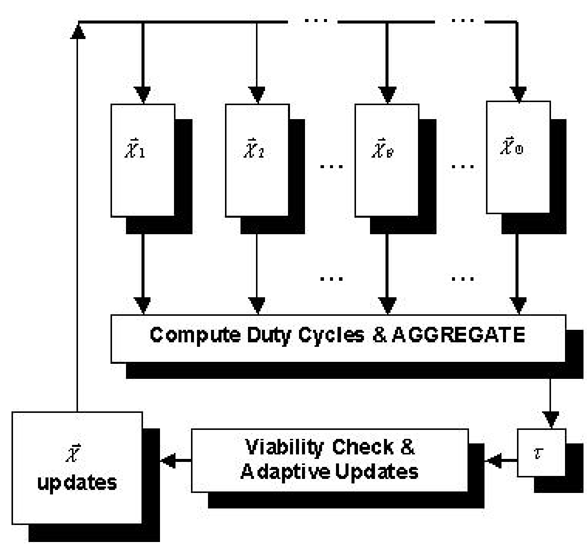

Fig. 2. Proposed procedure for team optimization of cooperating systems (TOCS) solution of the dynamic energy constrained wireless broadcasting problem.

TOCS in Figure 2 begins with Θ viable cuts. The choice of Θ is fixed. As a pedagogical example, suppose there are $N = 7$ nodes and $\Theta = 3$ trees. The optimal

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solution may reveal that tree number 3 does not add to the system lifetime. In such cases, the duty cycle of the third tree will be zero thereby erasing it seamlessly from the solution.

The Θ viable cuts in Figure 2 are used to compute corresponding duty cycles using minimax search. The result is the team fitness, τ. Based on the fitness, the cuts are adaptively updated into a new set of Θ viable cuts. The process is iteratively repeated until an imposed stop criterion is satisfied. For this procedure, evolutionary search is effective [2].

B.2 Examples of Dynamic Energy Constrained Wireless Broadcasting Optimization

To illustrate evaluation of the TOCS fitness values, consider the $\Theta = 3$ different trees in Figure 1.

1. If the initial battery energy in (5) is assumed, we find that the static lifetimes of the three trees, from top to bottom, are $\vec{\tau}_{static} = [7.5\ 7.5\ 6.5]$.
   Application of TOCS to this problem (12) reveals that making the system dynamic does not increase the lifetime. The reason is that, in the top and middle trees in Figure 1, node A has the first dead battery while, in the bottom cut, the battery in node G first goes dead.
2. Consider the same system with the connection trees in Figure 1 except that the initial battery energy, instead of (5), is

$$\vec{E}(0) = [20\ 10\ 25\ 15\ 5\ 0\ 10] \qquad (14)$$

   The static lifetimes of the three trees, from top to bottom, are $\vec{\tau}_{static} = [2.5\ 7.5\ 5]$.
   A dynamic system, in this case, gives a lifetime better than any individual static case. Using the duty cycle vector $\vec{\pi} = [0.25\ 0.5\ 0.25]$, the dynamic system lifetime is obtained to be $\tau = 10$. The energy left in the batteries at the end of the dynamic system lifetime, using (13), is

$$\vec{E}(\tau) = \begin{bmatrix} 0 \\ 7.5 \\ 5.0 \\ 0 \\ 0 \\ 0 \\ 5.0 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \end{matrix} .$$

   Thus, the batteries at nodes A, D and E go dead at the same time - all at the end of the system lifetime.[4]

[4]Node F had no battery power to begin with.

V. Points-to-Points Communication

The wireless broadcasting problem concerns connection of one source node to the remaining $N-1$ nodes. A generalization of this problem

1. increases the number of source nodes to, say, M nodes, and
2. decreases the number of receiver nodes to, say $K < N-1$ nodes.

The generalization to static energy constrained wireless broadcasting (Section III), and dynamic energy constrained wireless broadcasting (Section IV) is straightforward. The only revision is the assessment of cut viability.

VI. Concluding Remarks

We have presented in this paper, to our knowledge, the first models allowing for optimization of both the static and dynamic energy constrained wireless broadcasting. Minimizing the total broadcast power for a viable connection tree has been addressed elsewhere [1, 4]. The static energy constrained problem is a generalization of this approach. The dynamic problem, on the other hand, requires optimization using a more computationally intensive TOCS approach.

VII. Acknowledgement

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VIII. References

1. Robert J. Marks II, Arindam K. Das, Mohamed El-Sharkawi, Payman Arabshahi & Andrew Gray, "Minimum Power Broadcast Trees for Wireless Networks: Optimizing Using the Viability Lemma", Proceedings of the IEEE International Symposium on Circuits and Systems, Scottsdale, AZ (2002).
2. Jae-Byung Jung, Mohamed A. El-Sharkawi, G.M. Anderson, Robert T. Miyamoto, Robert J. Marks II, Warren L. J. Fox & C.J. Eggen, "Team Optimization of Cooperating Systems: Application to Maximal Area Coverage" Proceedings of the International Joint Conference on Neural Networks 2001, Washington D.C. pp. 2212-2217.
3. T.M. Cover, "The best two independent measurements are not the two best", *IEEE Transactions on Systems, Man and Cybernetics*, vol. SMC-4, pp.116-117, January 1974.
4. J.E.Wieselthier, G.D. Nguyen and A. Ephremides, "On the construction of energy-efficient broadcast and multicast trees in wireless networks", *IEEE INFOCOM* 2000, pp. 585- 594.