THE MINIMUM POWER BROADCAST PROBLEM IN WIRELESS NETWORKS : AN ANT COLONY SYSTEM APPROACH

Short Paper

Arindam K. Das, Robert J. Marks, Mohamed El-Sharkawi, Payman Arabshahi, Andrew Gray

ABSTRACT

Wireless multicast/broadcast sessions, unlike wired networks, inherently reaches several nodes with a single transmission. For omnidirectional wireless broadcast to a node, all nodes closer will also be reached. Heuristics for constructing minimum power trees in wireless networks have been proposed by Wieselthier *et al* [1] and Stojmenovic *et al* [2]. In this paper, we present an Ant Colony System algorithm for solving the minimum power broadcast (MPB) trees in wireless networks. Experiments on randomly generated 10, 25 and 50-node networks indicate that significantly better results can be obtained using the proposed algorithm, and in very little computation time.

I. INTRODUCTION

Broadcasting/multicasting in wireless networks is fundamentally different as compared to wired networks, since multiple nodes can be reached by a single transmission. This, of course, assumes that the nodes are equipped with omnidirectional antennas, so that if a transmission is directed from node i to node j, all nodes which are nearer to i than j will also receive the transmission. This is known as the "wireless multicast advantage" [1] property. For a given node constellation with an identified source node, the minimum power broadcast (MPB) problem in wireless networks is to communicate to all remaining nodes, either directly or hopping, such that the overall transmission power is minimized. We assume that signal reception and processing powers are negligible.

Although previous work in this area focused on a "link-based solution", Wieselthier *et al* [1] note that a "node based" approach is needed for wireless environments. The *broadcast incremental power* (BIP) algorithm suggested in [1] for constructing the MPB tree is a node based minimum-cost tree building algorithm for wireless networks. In this algorithm, new nodes are added to the tree on a minimum incremental cost basis, until all intended destination nodes are included. Other approaches suggested for solving this problem include an internal nodes based broadcasting procedure by Stojmenovic *et al* [2] and an evolutionary approach utilizing the "viability lemma" by Marks *et al* [3]. In this paper, we describe a swarm intelligent approach for solving the MPB problem.

Swarm intelligence appears in biological swarms of certain insect species. It gives rise to intelligent behavior through complex interaction of thousands of autonomous swarm members. Interaction is based on primitive instincts with no supervision. The end result is accomplishment of very complex forms of social behavior and fulfillment of a number of optimization and other tasks [4].

The main principle behind swarm intelligence interactions is *stigmergy*, or communication through the environment. An example is *pheromone* laying on trails followed by ants. Pheromone is a potent form of hormone that can be sensed by ants as they travel along trails. It attracts ants and therefore ants tend to follow trails that have high pheromone concentrations. This causes an autocatalytic reaction, *i.e.*, one that is accelerated by itself. Ants attracted by the pheromone will lay more pheromone on the same trail, causing even more ants to be attracted. In essence, therefore, swarm intelligence paradigms use positive reinforcement as a search strategy.

The Ant Colony System (ACS) algorithm, a swarm based optimization procedure, was first proposed by Dorigo and Gambardella [5] for solving the celebrated traveling salesman problem (TSP). Experimental studies carried out by the authors indicate that the ACS algorithm outperforms other evolutionary techniques like simulated annealing, elastic nets and self-organizing maps on the TSP. Before describing the ACS procedure for solving the MPB problem in wireless networks, we discuss the network model assumed and establish the notation used in this paper.

II. NETWORK MODEL

We assume a fixed N-node network with a specified source node which has to broadcast a message to all other nodes in the network. Any node can be used as a relay node to reach other nodes in the network. All nodes are assumed to have omnidirectional antennas, so that if node i transmits to node j, all nodes closer to i than j will also receive the transmission.The power matrix of the network, $\mathbf{P}$, is an $N \times N$ symmetric matrix whose $(i,j)th$ element represents the power required for node i to transmit to node j and is given by:

$$\mathbf{P}_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2\right]^{\alpha/2} = d_{ij}^{\alpha} \qquad (1)$$

A.K. Das, R.J. Marks and M. El-Sharkawi are with Department of Electrical Engineering, University of Washington, Box 352500, Seattle, WA 98195. e-mails: {arindam, marks, melshark}@ee.washington.edu.

P. Arabshahi and A. Gray are with Jet Propulsion Laboratory, 4800 Oak Grove Drive, MS 238-343, Pasadena, CA 91109. e-mails: {payman, gray}@arcadia.jpl.nasa.gov

where $\{(x_i, y_i) : 1 \leq i \leq N\}$ are the coordinates of the nodes in the network, α ($2 \leq \alpha \leq 4$) is the channel loss exponent and d_{ij} is the Euclidean distance between nodes i and j.

III. NOTATION

The following notation will be used in this paper:

t = time index
t_{MAX} = maximum time index
N_A = number of Type-A ants
N_B = number of Type-B ants
$\tau_{ij}(t)$ = pheromone level on the edge $i \to j$ at time t
η_{ij} = local visibility of node j from node $i \triangleq 1/\mathbf{P}_{ij}$
β_A = tunable parameter to control η_{ij} for Type-A ants, $0 < \beta_A \leq 1$
β_B = tunable parameter to control η_{ij} for Type-B ants, $0 < \beta_B < \beta_A \leq 1$
$T_k(t)$ = tree developed by ant k at time t
$Y_k(t)$ = tree power of $T_k(t)$
ρ = pheromone decay coefficient, $\rho \in (0, 1]$
q = uniformly distributed random variable over the interval [0,1]
q_0 = tunable parameter, $q_0 \in [0, 1]$

Note that the local visibility parameter, η_{ij}, is defined to be the inverse of $\mathbf{P}_{ij}$ and not d_{ij} (see eqn. 1), as was used by Dorigo *et al* [5] for solving the TSP. This definition allows for the effect of the channel loss factor, α, to be incorporated into local visibility.

IV. TREE BUILDING BY AN ANT

We begin by defining the following sets:

V = set of all nodes in the network
s = transmission step number
$\mathbf{NR}^s$ = new nodes reached in transmission step s
$\mathbf{NR}^{0:s}$ = all nodes reached till transmission step s
$\mathbf{NNR}^{0:s}$ = nodes not reached till transmission step s $\triangleq V \setminus \mathbf{NR}^{0:s}$

A node, i, is *newly reached* in step s if $i \in \mathbf{NR}^s$ but $i \notin \mathbf{NR}^{0:s-1}$.

Tree building by an ant is an iterative process which starts with a transmission from the source to a destination node and continues till all the intended destination nodes are reached. For a broadcasting session in an N-node network with an identified source node, the iteration must converge in *at most* $N-1$ iterations (*i.e.*, $s \leq N-1$). It should be noted that, because of the wireless advantage property, whereby multiple nodes are reached by a single transmission, the number of iterations can range from as few as 1 (when the source transmits to the node farthest from it, covering all intermediate nodes in the process) to $N-1$ (this will be the case when exactly one new node is reached during each iteration).

At a given time instant t, the decision rule governing which edge an ant chooses to travel on at step s of the tree building process is $pseudo-random-proportional$, as described in Figure 1. Starting with $s = 0$ and the initialization $\mathbf{NR}^0 = [source]$, this decision rule is executed till all the intended destination nodes are reached, *i.e.*, till $\mathbf{NNR}^{0:s} = \emptyset$.

A couple of points are worth noting in Figure 1. First, when the pheromone distribution on all edges is almost uniform, proper selection of β_A and β_B can effectively alter the way Type-A and Type-B ants choose which edge to travel on, from some node i. For an arbitrary 4-node network, suppose we have one Type-A and one Type-B ant at node 1. Assume that the distances of nodes 2, 3 and 4 from 1 are $d_{12} = 0.5$, $d_{13} = 1.5$, $d_{14} = 2.0$ and $\tau_{12} = \tau_{13} = \tau_{14} = 0.01$. Choosing $\beta_A = 1$ and $\beta_B = 0.1$, the probabilities $\{x_{ij} : i = 1, j = 1, 2, 3\}$ (see eqn. 5) for the two types of ants are as follows:

- Type-A: $x_{12} = 0.63$, $x_{13} = 0.21$, $x_{14} = 0.16$
- Type-B: $x_{12} = 0.36$, $x_{13} = 0.32$, $x_{14} = 0.32$

Clearly, if both the ants are following their exploratory regimen (see Step 4 in Figure 1), while the Type-A ant will choose the nearest node (node 2) 63% of the time, the Type-B ant has almost equal chances of selecting any of the three nodes. Type-B ants, therefore, can select their edges by looking *deeper* into the network, as opposed to Type-A ants which are *mostly greedy* and tend to choose nearby nodes. Because of this reason, we will refer to Type-A ants as $narrow-vision$ ants and Type-B ants as $wide-vision$ ants. It may be noted that the wide-vision ants, because of their ability to make decisions by looking deeper into the network, are better suited for exploiting the wireless advantage property than the narrow-vision ants.

Second, the only condition which needs to be satisfied by a node if it wants to transmit at step s is that it should be reached by step $s-1$ (see eqn. 2). There's no restriction, however, on a node transmitting more than once in the broadcast tree. For example, in an arbitrary 4-node network with node 1 being the source, a broadcast tree generated by an ant could be: $T = \{1 \to 2, 2 \to 3, 1 \to 4\}$. If $d_{14} > d_{12}$, it is obvious that T can be easily improved by replacing the two transmissions from node 1 by the higher-powered transmission $1 \to 4$. An improved tree is therefore: $T_1 = \{1 \to 4, 2 \to 3\}$. In general, multiple transmissions from a node are unnecessary in wireless networks because the highest-powered transmission will also cover the nodes which are reached by lower-powered transmissions from that node. Given a sequence of transmissions constituting a broadcast tree, the primary step in cost reduction is therefore to replace the first transmission from a node by the highest powered transmission from that node. All other transmissions from that node are then deleted from the tree. We will refer to this cost-reducing procedure as *Procedure MTR* (*m*ultiple *t*ransmission *r*emoval).

A second cost-reduction mechanism is *Procedure ET* (*e*dge *t*rimming), whereby redundant transmissions are eliminated from the broadcast tree. The *sth* transmission in a broadcast tree is redundant if no new node is reached by it; *i.e.*, if $\mathbf{NR}^s = \emptyset$. In our above example, if we assume that node 3 is nearer to 1 than 4, the transmission

1. In general, at any step s, an ant can travel $from$ any node which has been reached till step $s-1$, to any node which has not yet been reached till step $s-1$. Let f^s and t^s denote the $from$ and to nodes at step s. We therefore have the conditions:

$$f^s \in \mathbf{NR}^{0:s-1}, \ \ t^s \in \mathbf{NNR}^{0:s-1} \tag{2}$$

2. Let $\mathbf{A}^s(t) = \{a^s_{ij}(t) : i \in \mathbf{NR}^{0:s-1}, \ j \in \mathbf{NNR}^{0:s-1}\}$ be the decision matrix on which the ant bases its decision for selecting an edge at step s, at a given time instant t. The a_{ij}'s are computed as follows:

$$a^s_{ij}(t) = \begin{cases} \frac{[\tau_{ij}(t)][\eta_{ij}]^{\beta_A}}{\sum_{k,l}[\tau_{kl}(t)][\eta_{kl}]^{\beta_A}} : \ k \in \mathbf{NR}^{0:s-1}, \ l \in \mathbf{NNR}^{0:s-1}, & \text{for Type-}A \text{ ants} \\ \frac{[\tau_{ij}(t)][\eta_{ij}]^{\beta_B}}{\sum_{k,l}[\tau_{kl}(t)][\eta_{kl}]^{\beta_B}} : \ k \in \mathbf{NR}^{0:s-1}, \ l \in \mathbf{NNR}^{0:s-1}, & \text{for Type-}B \text{ ants} \end{cases} \tag{3}$$

3. Let q be a random number drawn from a [0,1] uniform distribution.
4. **if**($s == N-1$)
/* Make a deterministic decision by choosing the minimum cost path to the unreached node. */
• Choose $[f^s, t^s]$ such that the cost of including the unreached node in the tree is minimum.
else
if($q < q_0$) /* Choose the strongest trail in $\mathbf{A}^s(t)$ */

$$[f^s, t^s] = \text{argmax}_{i,j}\{a^s_{ij}(t)\} \tag{4}$$

else /* Let the ant explore more. */
• Compute probabilities $\{x^s_{ij}(t)\}$ as:

$$x^s_{ij}(t) = \frac{a^s_{ij}(t)}{\sum_{k,l} a^s_{ij}(t)} : \ k \in \mathbf{NR}^{0:s-1}, l \in \mathbf{NNR}^{0:s-1} \tag{5}$$

• Choose the edge $[f^s, t^s]$ based on these probabilities, using, *e.g.*, a roulette-wheel selection procedure as in Genetic Algorithms.
endif
endif

Fig. 1. The $pseudo-random-proportional$ decision rule governing which edge an ant chooses to travel on at step s of the tree building process.

$2 \rightarrow 3$ is redundant because node 3 will be automatically reached by the transmission $1 \rightarrow 4$. Applying procedure ET to T_1 will therefore yield the further improved tree: $T_2 = \{1 \rightarrow 4\}$.

V. THE ACS ALGORITHM

A high level description of the ACS algorithm for solving the minimum power broadcast problem in wireless networks is provided in Figure 2. We assume that $MTR(T)$ is a function which takes a tree T, applies procedure MTR on it and returns the updated tree. Similarly, $ET(T)$ is a function which takes a tree T, applies procedure ET on it and returns the updated tree. At any time t, the pheromone level $\tau_{ij}(t)$ on the edge $i \rightarrow j$, reflects the cumulative knowledge acquired by the ants till time t on the 'desirability' of moving to node j from node i. A very high pheromone level on any edge, therefore, makes it much more probable for that edge to be included in the final tree. As in [5], we have adopted a two-level pheromone update operation; first after computing and refining each tree $T_k(t)$ (the update step inside the **for** loop in Figure 2) and again after computing all trees $\{T_k(t)\}$ at a given time instant t (the update step after the **for** loop). Note that the latter pheromone update is partly proportional to the quality of the best solution produced till iteration t. Better the best solution, the higher the pheromone amount that is deposited on the set of directed edges in the best broadcast tree. The role of the pheromone decay coefficient, ρ, is to prevent stagnation in the search process, a situation where all or most of the ants end up choosing the same set of edges and hence generating identical trees.

VI. SIMULATION RESULTS

We tested the 1-shrink algorithm on 10, 25 and 50-node networks in a 5 × 5 grid. In each case, 50 networks were randomly generated and the tree powers averaged to obtain the mean tree power. 'α' was chosen to be equal to 2 for all cases. Values of the parameters used in the simulations are given in Table I. A key point to note in Table I is the dynamic nature of the parameters q_0 and β_B with respect to t. Gradually reducing q_0 ensures that the bulk of the exploration work (Step 4 in Figure 1) is carried out during the initial stages of the algorithm, when the pheromone distribution on the edges is not too uneven and "trail conditions" are more suitable for wide-vision ants, as explained in Section IV. Increasing β_B with respect to t has the effect of reducing the local visibility of wide-vision ants so that they start behaving more like their narrow-vision counterparts as iteration progresses. In fact, for $\lfloor 0.75 * t_{MAX} \rfloor + 1 \leq t \leq t_{MAX}$, β_B is equal to β_A, which ensures that all ants concentrate on the best

1. Set $t = 0$.
2. Set $\tau_{ij} = \tau_0 : \forall(i,j), i \neq j, \;\; \tau_0 > 0$.
3. Let T^{best} be the tree grown by the global best ant and Y^{best} its cost.
4. Let $T^{best}(t)$ be the best tree grown by any ant during iteration t and $Y^{best}(t)$ its cost.
5. **while**$(t < t_{MAX})$

  Select β_B and q_0 according to t.

  for$(k = 1 : N_A + N_B)$ /* ant number */

    • Build the tree $T_k(t)$; /* See Section IV */

    • $T_k(t) \leftarrow MTR(T_k(t))$;

    • $T_k(t) \leftarrow ET(T_k(t))$;

    • $Y_k(t) =$ cost of $T_k(t)$;

    • $\tau_{ij}(t) \leftarrow \rho\tau_0 + (1-\rho)\tau_{ij}(t), \;\; \forall(i,j) \in T_k(t)$;

  endfor

  if$(t == 0)$

    $T^{best} \leftarrow T^{best}(t), \;\; Y^{best} \leftarrow Y^{best}(t)$;

  else

    if$(Y^{best}(t) < Y^{best})$

      $T^{best} \leftarrow T^{best}(t), \;\; Y^{best} \leftarrow Y^{best}(t)$;

    endif

  endif

  • $\tau_{ij}(t+1) \leftarrow \rho / Y^{best} + (1-\rho)\tau_{ij}(t), \;\; \forall(i,j) \in T^{best}$;

  • $t \leftarrow t+1$;

endwhile

6. Print T^{best} and Y^{best}.

Fig. 2. High level description of the ACS algorithm for solving the MPB problem in wireless networks.

TABLE I

Parameter values used in the simulations.

Parameter	$N = 10$	$N = 25$	$N = 50$
t_{MAX}	50	50	100
N_A	5	7	13
N_B	5	6	12
ρ	0.2	0.2	0.2
$\tau_{ij}(0)$	0.01	0.005	0.0005
β_A	1	1	1
β_B	$1/\alpha^2$, if $t \leq \lfloor 0.5 * t_{MAX} \rfloor$ $1/\alpha$, if $\lfloor 0.5 * t_{MAX} \rfloor + 1 \leq t \leq \lfloor 0.75 * t_{MAX} \rfloor$ 1, if $\lfloor 0.75 * t_{MAX} \rfloor + 1 \leq t \leq t_{MAX}$		
q_0	0.3, if $t \leq \lfloor 0.5 * t_{MAX} \rfloor$ 0.6, if $\lfloor 0.5 * t_{MAX} \rfloor + 1 \leq t \leq \lfloor 0.75 * t_{MAX} \rfloor$ 0.9, if $\lfloor 0.75 * t_{MAX} \rfloor + 1 \leq t \leq t_{MAX}$		

TABLE II

Mean tree powers for BIP (column 2), BIP followed by sweep as in [1] (column 3) and ACS (column 4). Figures in parentheses in columns 3 and 4 represent the percentage improvement in mean tree power over the BIP solutions. $\alpha = 2$ for all N.

N	BIP	BIP(sweep)	ACS
10	11.57	11.08 (−**4.23**%)	10.06 (−**13.05**%)
25	12.46	12.14 (−**2.57**%)	10.21 (−**18.06**%)
50	11.67	11.45 (−**1.89**%)	10.04 (−**13.93**%)

routes generated and look for better solutions within local neighborhoods during the last stages of the algorithm.

The mean tree powers for the BIP solutions are shown in column (2) in Table II. The mean tree powers for the BIP solutions followed by the sweep algorithm proposed in [1] are shown in column (3). Column (4) lists the mean tree powers obtained by applying the ACS algorithm. The figures in parentheses in columns 3 and 4 represent the percentage improvement in mean tree power over the BIP solutions. Clearly, the ACS algorithm is able to find significantly better broadcast trees than the BIP algorithm.

VII. CONCLUSION

In this paper, we have proposed an Ant Colony System algorithm for solving the minimum power broadcast problem in wireless networks. Although we have not yet tested the algorithm for multicast applications, we conjecture that the same algorithm, with some minor modifications, can be used for such applications also. The algorithm proposed uses a mix of narrow-vision and wide-vision ants. While a narrow-vision ant located at a particular node tends to choose a nearby unreached node to visit next, wide-vision ants are allowed to choose distant nodes to visit next. Simulations indicate that wide-vision ants, with their less greedy approach, are generally better able to exploit the wireless multicast advantage property during the initial exploration phase of the algorithm. Narrow-vision ants, on the other hand, are generally more effective during the latter stages, when they hone in on the best routes generated and look for better solutions within local neighborhoods. Experiments carried out on 10, 25 and 50-node networks confirm that significantly better results can be obtained using the proposed algorithm, and in very little computation time.

REFERENCES

1) J.E.Wieselthier, G.D. Nguyen and A. Ephremides, “On the construction of energy-efficient broadcast and multicast trees in wireless networks”, IEEE INFOCOM 2000, pp. 585-594.
2) Ivan Stojmenovic, Mahtab Seddigh and Jovisa Zunic, “Internal Nodes Based Broadcasting in Wireless Networks”, Proceedings of the 34th Hawaii International Conference on System Sciences, 2001.
3) Robert J. Marks II, Arindam K. Das, Mohamed El-Sharkawi, Payman Arabshahi and Andrew Gray, “Minimum Power Broadcast Trees for Wireless Networks: Optimizing Using the Viability Lemma”, Proceedings of the IEEE International Symposium on Circuits and Systems, Scottsdale, Arizona, 2002.
4) E. Bonabeau, M. Dorigo, and G. Theraulaz, “Swarm intelligence: From natural to artificial systems”, Oxford University Press, 1999.
5) Dorigo M. and L. M. Gambardella , “Ant Colonies for the Traveling Salesman Problem,” BioSystems 43, 1997, pp. 73-81.