Charles Baylis, Joseph Perry, Matthew Moldovan, Robert J. Marks II, and Lawrence Dunleavy,
"Use of a Step-Response Approximation for Thermal Transient Modeling in Power MOSFETs,"
74th ARFTG (Automatic RF Techniques Group) Microwave Measurement Symposium,
December 1st - 4th, Broomfield/Boulder, Colorado, 2009, pp.1-4,

Use of a Step-Response Approximation for Thermal Transient Modeling in Power MOSFETs

Charles Baylis[1], Joseph Perry[1], Matthew Moldovan[1], Robert J. Marks II[1], Lawrence Dunleavy[2,3]

[1]Wireless and Microwave Circuits and Systems Program, Department of Electrical and
Computer Engineering, Baylor University, Waco, Texas 76798, USA

[2]Center for Wireless and Microwave Information Systems,
Department of Electrical Engineering, University of South Florida, Tampa, Florida 33620, USA

[2]Modelithics, Inc., Tampa, Florida 33612, USA

***Abstract* – In a previous ARFTG paper, we presented the measurement of the thermally-induced transient drain voltage response of a power Si MOSFET to a step excitation in gate voltage. The data was fit by assuming the drain voltage takes an exponential shape. This relies on the assumption that the drain current is a step function, which is actually only approximate due to the dependence of the drain current on temperature. In this work, we show that the exponential approximation actually possesses the same asymptotic behavior of a more accurate, model-based solution we have obtained that incorporates thermal feedback. We show that the more theoretically accurate solution and exponential approximation of the solution both provide excellent fits to measured data for the Si MOSFET.**

I. INTRODUCTION

In a paper presented at the Fall 2007 ARFTG conference, we demonstrated a MOSFET transient voltage measurement setup that can be used to estimate the thermal time constant [1]. A very interesting question was raised regarding the validity of our approach to approximate the current excitation as a step function. The approach assumes an electrothermal model for the transistor as shown in Fig. 1 [2]. The analogous "circuit" consists of a parallel RC network describing the time- and frequency-dependent behavior of the channel temperature. A reasonably good fit was obtained for this description of the behavior; following this description, an even more optimal fit was obtained using two parallel RC networks to describe the time dependence, an approach consistent with the multiple RC thermal modeling suggested in [3] and shown in Fig. 2.

II. NATURE OF THE APPROXIMATION

The experimental setup used for the experimentation is shown in Fig. 3 [1]. A step is applied in voltage $v_G(t)$ to "turn on" the transistor from threshold to a significant drain current. As drain current $i_D(t)$ begins to flow, a voltage drop is immediately incurred across the 10.12 Ω resistor. However, as current begins to flow through the transistor, the device begins to heat up more as time increases, causing $i_D(t)$ to decrease and $v_D(t)$ to increase. The rate of this increase in $v_D(t)$ is determined by the electrothermal model parameters R_{th} (thermal resistance) and τ_{th} (thermal time constant). The thermal time constant was estimated by fitting a simple exponential function to the measurement for $v_D(t)$, as reproduced in Fig. 4:

$$v_D(t) = A_2 - A_1 e^{-t/\tau_{th}} \tag{1}$$

where C, D, and τ_{th} were adjusted to fit the measured data. It was then assumed that the resultant value of τ_{th} is can be used to find the C_{th} by equating τ_{th} to $R_{th}C_{th}$. The measured data is rather noisy in this experiment due to the limitations of the oscilloscope when placed in DC coupling mode and measuring voltage waveforms with significant DC components.

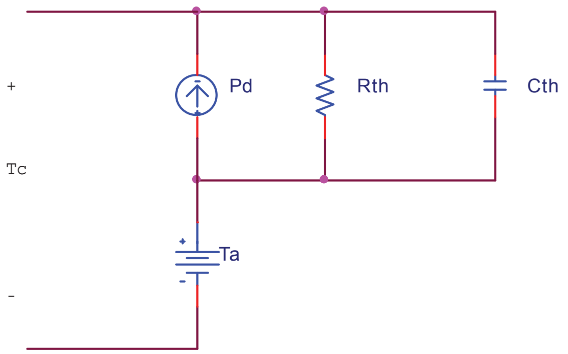

Fig. 1. Thermal Subcircuit Used in Electrothermal Transistor Models [2]

These results are based upon an assumption that the excitation of the thermal equivalent circuit, P_D (and hence the current $i_D(t)$) is represented by a step function. Is this approximation adequate? In actuality, the current $i_D(t)$ is not exactly a step function because it varies as the channel temperature $T_C(t)$ changes. As $i_D(t)$ becomes nonzero, the channel temperature $T_C(t)$ increases, causing a decrease in $i_D(t)$, causing an decrease in $T_C(t)$, causing an increase in

978-1-4244-5714-4/10/$26.00 ©2009 IEEE

$i_D(t)$, and so forth. The assumption that the response of the temperature (and drain voltage) are based on a step in $i_D(t)$ is essential to assume exponential behavior. The fact that $i_D(t)$ (and hence $v_D(t)$) and $T_C(t)$ are interdependent means that this is not exactly correct.

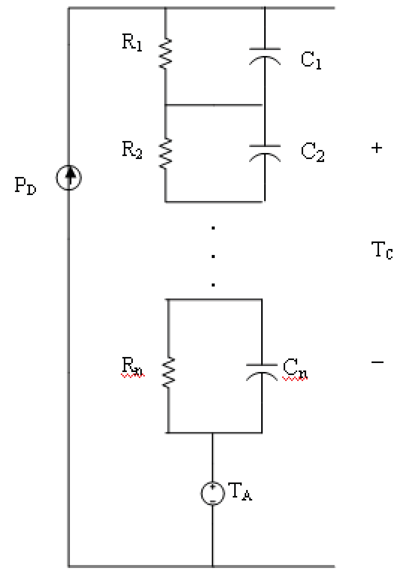

Fig. 2. Thermal Equivalent Circuit with Multiple RC Networks [3]

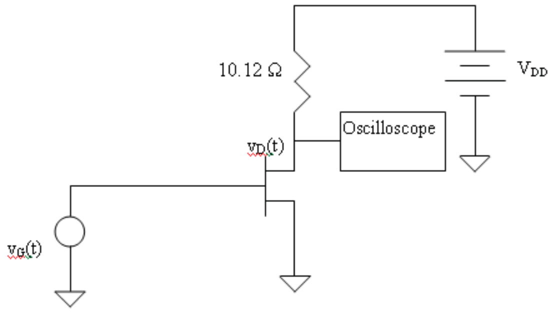

Fig. 3. Experimental Setup for Transient Measurement

We now compare the response of this circuit to behavior of the device based on a more accurate model to examine whether the exponential assumption can be used. This must be derived from the nodal differential equation taken from the single-RC thermal network of Figure 1:

$$\frac{\Delta T(t)}{R_{th}} + C_{th}\frac{d\Delta T(t)}{dt} = V_{DD}\, i_D(t) - R_D\, i_D{}^2(t) \qquad (2)$$

The problem at hand is evident in the nonlinearity of this equation. The form of the time-domain function solving the differential equation for the drain voltage as a function of time is

$$v_D(t) = V_{DD} - \alpha T_A R_D - \beta - \alpha R_D \frac{a(1-e^{ct})}{1-be^{ct}} \; , \qquad (3)$$

where α and β are temperature coefficients of the drain current based on a current-temperature approximation consistent with results given in [4]-[7], T_A is the ambient temperature, R_D is the drain resistance, and a, b, and c are constants; also note that $c < 0$. Applying a simplification suggested by the assumption that the input is similar to a unit step (although for reasons explained earlier, it will not be exactly a unit step in most cases) give the form

$$v_D(t) \approx V_{DD} - \alpha T_A R_D - \beta - \alpha a R_D (1 - e^{ct}) \qquad (4)$$

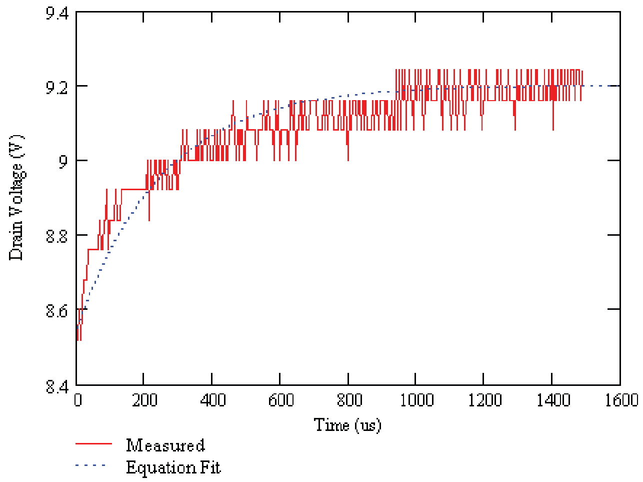

Fig. 4. Si MOSFET Drain Voltage Versus Time for Measured Results (Solid Line) and Exponential Fit (Dotted Line) Using V_{DD} = 16 V and V_{GS} Stepped from 0.3 V to 7.2 V [1]

In comparing the actual solution of equation (3) and the approximation of equation (4), very interesting correspondence can be observed. The limit as *t --> 0* for both the approximation (3) and the actual equation (4) is

$$v_D(0) = V_{DD} - \alpha T_A R_D - \beta \, , \qquad (5)$$

and the limit as t → ∞ for both equations (because $c < 0$) is

$$v_D(t \to \infty) = V_{DD} - \alpha T_A R_D - \beta - \alpha a R_D \, . \qquad (6)$$

This states that, under most conditions, both the approximation and the actual function have the same initial value and asymptotically approach the same value in the steady state. This lends hope for the usefulness of the approximation.

III. MEASUREMENT COMPARISON OF ACTUAL SOLUTION AND APPROXIMATION

A comparison can be performed between using the forms of the approximation and the more detailed differential equation solution for $v_D(t)$ and their usefulness in fitting measured data from the Si power MOSFET. The approximation can be written as

$$v_{Dapprox}(t) = A_2 - A_1 e^{-t/\tau_{th}} \, , \qquad (7)$$

where A_1, A_2, and τ_{th} are adjusted to fit the measurement data. The more detailed differential equation solution can be written as

$$v_{Dexact}(t) = C_4 - \frac{C_1(1-e^{C_2 t})}{1-C_3 e^{C_2 t}} \tag{8}$$

where C_1, C_2, C_3, and C_4 are adjusted to fit the measurement data.

A comparison between the measured data, approximation equation fit, and the fit of the differential equation solution is was performed. A least squares error fit was performed to the measurement data by varying A_1 and A_2 in equation (7) (the single-exponential approximation) and by varying C_1, C_2, C_3, and C_4 in equation (8) (the more detailed model). Fig. 5 shows a comparison between these fits of $v_{Dapprox}$(t) and v_{Dexact}(t) to the measured data. For each fit, the mean-square error (MSE) can be used as a measure of the fit quality. While the more detailed model (MSE = 0.00214) fits the data more exactly than the single-exponential (MSE = 0.00242) derived from the step-response approximation, the step-response approximation MSE is not much larger than the detailed model. Qualitatively it can be noted from Fig. 5 that while the more detailed model shows a better correspondence for low values of time, the step approximation seems to be reasonably accurate in predicting the transient behavior of the device. When the step response approximation is used with an electrothermal circuit containing two RC networks in parallel, the results for the approximation improve even further and more accurately portray the fit of the exact equation, as shown in Fig. 6. For the double-exponential approximation, the MSE (MSE = 0.00200) was even lower than for the more detailed model (MSE = 0.00214); however, this is sensible, as the detailed model is based on an electrothermal model containing a single RC network, while the approximation assumes two RC networks.

IV. CONCLUSIONS

The question of whether it is permissible to assume that the voltage transient measurement of Fig. 3 can be assumed to be a step response (and hence exponential in nature) arising from discussions related to our paper at the Fall 2007 ARFTG Conference has prompted efforts to validate this approximation. The results we have presented in this paper suggest that the approximation of the drain voltage as an exponential step response appears to, with a high level of accuracy, fit both measurement data and an equation derived representing more exact representation of the time dependence. The equation forms of the approximation and more detailed model demonstrate the same initial behavior and asymptotically approach the same final value. Fitting the approximation and more exact model equations to measured data has demonstrated that, while the mean-square error is larger for the single-exponential approximation than for the more exact model, the difference in MSE is reasonably small. It appears that the approximation can be used to represent the actual device behavior in the transient measurement with reasonable success.

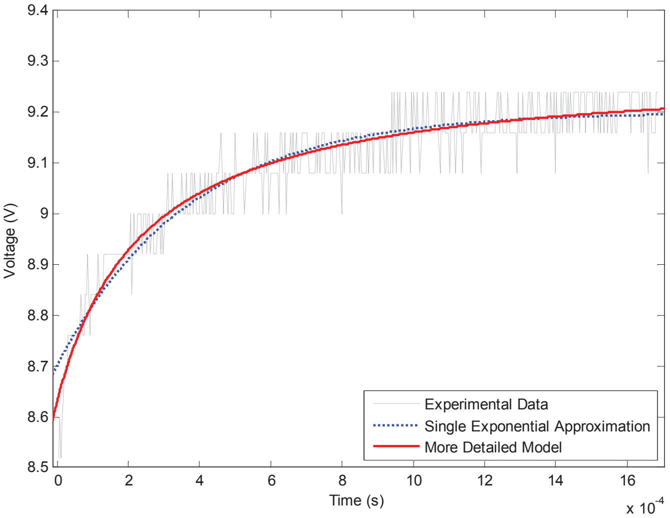

Fig. 5. Comparison of Si MOSFET Measured Data (V_{DD} = 16 V and V_{GS} Stepped from 0.3 V to 7.2 V) with the Step Approximation for a Single-RC Thermal Circuit and Exact Equation Modeling for a Single-RC Circuit of the Thermally-Induced Voltage Transient (Note that the numbers on the horizontal axis are in hundreds of milliseconds.)

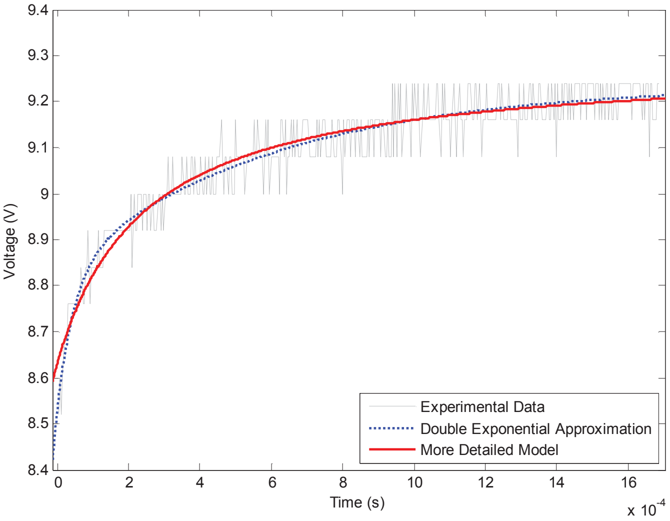

Fig. 6. Comparison of Si MOSFET Measured Data (V_{DD} = 16 V and V_{GS} Stepped from 0.3 V to 7.2 V) with the Step Approximation for a Double-RC Thermal Circuit and Exact Equation Modeling for a Single-RC Circuit of the Thermally-Induced Voltage Transient (Note that the numbers on the horizontal axis are in hundreds of milliseconds.)

ACKNOWLEDGMENTS

The authors would like to thank Dr. Marc Vanden Bossche for stimulating this work. Appreciation is also extended to Dr. Arthur David Snider of the University of South Florida for assistance in developing underlying theoretical justification for the measurements presented in this work.

REFERENCES

[1] C. Baylis and L. Dunleavy, "Voltage Transient Measurement and Extraction of Power RF MOSFET Thermal Time Constants," Automatic RF Techniques Group Conference, Tempe, Arizona, November 2007.

[2] G. Sibinski and W. Sethares, "Thermal Parameter Estimation Using Recursive Identification," *IEEE Transactions on Power Electronics*, Vol. 6, Issue 2, April 1991, pp. 228-239.

[3] Y. Yang, Y. Woo, J. Yi, and B. Kim, "A New Empirical large-Signal Model of Si LDMOSFETs for High-Power Amplifier Design," *IEEE Transactions on Microwave Theory and Techniques,* Vol. 49, Issue 9, September 2001, pp. 1626-1633.

[4] P. Abernathy, Master's Thesis: *A Systematic Nonlinear Device Modeling Procedure for Large-Signal MESFET Applications*, Tampa, Florida: University of South Florida, 1994.

[5] R. Neidert and C. Scott, "Computer Program for Microwave GaAs MESFET Modeling," *NRL Report 8561*, Washington, D.C.: Naval Research Laboratory, 1982.

[6] V. Sunde, Z. Bencic, and Z. Jakopovic, "A Temperature-Dependent Electrothermal MOSFET Model for Calculating Its Current Loadability," *Proceedings of the IEEE International Symposium on Industrial Electronics*, Vol. 2, pp. 12-16, 1999.

[7] B. Streetman and S. Banerjee, *Solid State Electronic Devices*, Fifth Edition, Upper Saddle River, New Jersey: Prentice Hall, 2000.