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Frequency Multiplexing Tickle Tones to Determine Harmonic Coupling Weights in Nonlinear Systems

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***Abstract*— Obtaining the Jacobian parameters for a nonlinear system, such as in X-parameter or S-function measurement, requires excitation of tickle tones either on or slightly off of the main harmonic frequencies. Traditional off-frequency measurements involve simultaneous excitation with tickle tones both slightly above and/or below a single harmonic frequency at a time. This paper describes a multiplexing technique for the excitation of the system with perturbations at all harmonics simultaneously. This technique requires both frequency and cross-frequency noninterference for successful results from the measurements. If these conditions exist for the nonlinear system, the off-frequency multiplexing technique for tickle tones can reduce the measurements necessary to find the Jacobian matrices by a factor equal to the number of harmonics considered in the polyharmonic distortion. These methods provide exact results for memoryless polynomial nonlinearities.**

I. Introduction

In making nonlinear network parameter measurements, such as the X-parameters[1] [1] and the S-functions [2-4], it is necessary to apply a large-signal "operating" condition, followed by small perturbations to this condition known as "tickle" tones. The authors of this paper recently have written a survey paper on nonlinear network parameters that briefly describes measurement options from a practical stance [5]. Two options exist for the tickle-tone measurements: (1) on-frequency tickle tones, where the perturbation is added at the same frequency as the desired stimulus harmonic and (2) off-frequency tickle tones [6], where the perturbation is added at a slightly offset frequency from the appropriate fundamental or harmonic. The excitation approaches have been significantly considered by Betts in recent publication [7]. While a significant percentage of modern nonlinear network analyzers use the on-frequency method, an advantage of the off-frequency method is that the results of the tickle-tone perturbations can be isolated away from large signals overlapping in frequency.

This paper demonstrates that the off-frequency technique can be extended to allow simultaneous application of perturbations at multiple harmonics, a technique herein labeled *frequency multiplexing* of tickle tones. This can significantly reduce the number of measurement iterations necessary to fully determine the matrix of nonlinear network parameters.

We examine the frequency multiplexing approach for periodicity preservation (PP) systems approach in finding the derivatives composing the system Jacobian in the Fourier series coefficient domain. The PP and time-invariant periodicity preservation (TIPP) systems have been introduced in a recently authored paper in process of publication [8], and the limitations of use on some nonlinear systems examined quantitatively in [9]. This theory forms the foundation for experimental nonlinear network analysis.

II. Background

A basic discussion of the system theory describing the context of this measurement is useful. A more complete description of this theory is found in [8]. Let $\mathcal{Z}$ denote a nonlinear operator and

$$v(t) = \mathcal{Z}\{i(t)\}. \tag{1}$$

The signal $i(t)$ is the stimulus and $v(t)$ the response. The affine approximation for a small stimulus perturbation, $\Delta i(t)$, is [8]

$$v_\Delta(t) := \mathcal{Z}\{i(t) + \Delta i(t)\} \approx v(t) + \Delta v(t) \tag{2}$$

where $v(t) + \Delta v(t)$ denotes the affine approximation of the response to the perturbation.

The *periodicity preservation* (PP) class of generally nonlinear systems [5] has the property that periodic stimulation will result in a periodic response with the same period, T and fundamental frequency $f = 1/T$. Temporally, the PP system is represented by the operator $\mathcal{Z}$ with the constraint that the stimulus and response in (1) are zero outside of a period, say the interval $0 \leq t < T$. Each defines a single period of a periodic function. Since $\Delta i(t)$ is nonzero

[1] X-parameters is a registered trademark of Agilent Technologies.

only over $0 \le t < T$, the response perturbation in (2) can be written as

$$\Delta v(t) = \int_0^T \frac{\partial v(t)}{\partial i(\tau)} \Delta i(\tau) d\tau \qquad (3)$$

Let the vector i_m denote the Fourier coefficients of the periodic signal $\hat{\imath}(\tau) = \sum_{p=-\infty}^{\infty} i(\tau - nT)$. That is

$$i_m = \frac{1}{T} \int_0^T i(\tau) e^{-j2\pi m f \tau} d\tau \qquad (4)$$

and

$$i(\tau) = \sum_{m=-\infty}^{\infty} i_m \, e^{j2\pi m f \tau} \, \Pi_T(\tau) \qquad (5)$$

where $\Pi_T(t)$ is one for $0 \le t < T$ and is otherwise zero. The Fourier coefficients are seen to be samples of the Fourier transform of a single period of its replication. That is

$$i_m = f \, I(mf). \qquad (6)$$

where

$$I(\nu) = \int_0^T i(\tau) e^{-j2\pi \nu \tau} d\tau \, ,$$

Likewise, let v_n denote the Fourier coefficients of periodic replication of $v(t)$. Then (3), expressed in terms of Fourier series coefficients, is the harmonic cross coupling expression [8]
`

$$\Delta v_n = \sum_{m=-\infty}^{\infty} \frac{\partial v_n}{\partial i_m} \Delta i_m \qquad (7)$$

where Δv_n and Δi_m are the Fourier series coefficients of $\Delta v(t)$. The *harmonic coupling weights*, $\frac{\partial v_n}{\partial i_m}$, measure the coupling strength between the mth stimulus harmonic and the nth response harmonic. Only a finite number of experiments can be performed and a finite number of harmonic coupling weights experimentally determined. Thus, in lieu of (7), we approximate the perturbation of Fourier series coefficients by

$$\Delta \, v_n \simeq \sum_{m=-M}^{M} \frac{\partial v_n}{\partial i_m} \Delta i_m \, ; \qquad |n| \le N \, . \qquad (8)$$

Thus, with knowledge of the harmonic coupling weights, the Fourier series coefficients of a response perturbation can be estimated for a small albeit arbitrary stimulus perturbation. In the sections to follow, we illustrate a foundational methodology by which the harmonic coupling weights of a model free PP system can be determined experimentally.

III. The Off-Frequency Method

The harmonic coupling coefficients, $\frac{\partial v_n}{\partial i_m}$, which fill the Jacobian matrix, can be estimated by placing small tickle tones a bit off-frequency from each harmonic. The off-frequency method works with locally frequency-invariant PP systems. If a system is *frequency-invariant* [13,14], then for all values of α,

$$\frac{\partial V(u)}{\partial I(\nu)} = \frac{\partial V(u-\alpha)}{\partial I(\nu - \alpha)} \quad . \qquad (9)$$

A system is locally frequency-invariant if (9) is approximately true for small frequency shifts.

This is because, for memoryless nonlinear systems, a scaling of the stimulus, $i(t)$, generates an identical scaling in the response, $v(t)$. Thus, if $v(t) = g\big(i(t)\big)$, then, for any real nonzero scaling factor, $\Gamma \neq 0$,

$$g\left(i\left(\frac{t}{\Gamma}\right)\right) = v\left(\frac{t}{\Gamma}\right)$$

In the frequency domain, the spectra of both the stimulus and response shift together smoothly in regard to small changes in Γ as dictated by the scaling theorem of Fourier transforms [12].

A. Using Real Tickle Tones

When a cosine tickle tone is used, the input has stimulus frequencies at the two harmonic numbers $\pm m$. The off-frequency tickle tone is

$$\Delta_m \, i(\tau) = 2\varepsilon \cos(2\pi(mf + \Delta f)\tau) \qquad (10)$$

$$= \varepsilon \left(e^{j2\pi(mf+\Delta f)\tau} + e^{-j2\pi(mf+\Delta f)\tau} \right)$$

Besides frequency invariance, an additional *harmonic noninterference constraint* is now required. Specifically, consider the response to two perturbations

$$v_{\pm\Delta}(t) = \mathcal{Z}\big(i(\tau) + \varepsilon e^{\pm j2\pi(mf+\Delta f)\tau}\big).$$

The constraint requires

$$V_{+\Delta}(nf - \Delta f) \approx V(nf - \Delta f)$$

and

$$V_{-\Delta}(nf + \Delta f) \approx V(nf + \Delta f).$$

In other words, the input perturbation of $\varepsilon e^{j2\pi(mf+\Delta f)\tau}$ has no effect on the response spectrum at the frequency $u = nf - \Delta f$ and $\varepsilon e^{-j2\pi(mf+\Delta f)\tau}$ has no effect at $u = nf + \Delta f$. Then the following harmonic coupling weights can thus be read directly from the spectrum.

$$\varepsilon \frac{\partial v_n}{\partial i_m} \text{ at frequency } u = nf + \Delta f$$

and

$$\varepsilon \frac{\partial v_{-n}}{\partial i_m} \text{ at frequency } u = nf - \Delta f.$$

The off-frequency technique allows estimation of the harmonic coupling weights $\frac{\partial v_n}{\partial i_m}$ and $\frac{\partial v_{-n}}{\partial i_m}$ with a single experiment for all n. This experiment must be repeated for all values of m. Each experiment finds the harmonic coupling weights for all response harmonics in terms of a single excitation harmonic. Each repetition of the experiment provides these harmonic coupling weights for a different excitation harmonic. This is illustrated in Figure 1. A simulation illustration using a hyperbolic tangent to model an amplifier with saturation is shown in Figure 2. Details are in the captions.

Polynomial nonlinearities, a special case of the memoryless nonlinearity *v(t)* = *g*(*i*(*t*)), have harmonic coupling weights that can be characterized analytically. Define the polynomial

$$g(i) = \sum_{p=0}^{P} \alpha_p i^p \qquad (11)$$

where the α_p's are real coefficients. In Appendix A, we show that

$$\frac{\partial v_n}{\partial i_m} = \sum_{p=0}^{P} \alpha_p p \; i_{n-m}^{(p-1)*} \qquad (12)$$

where $i_n^{(p-1)*}$ is the n-fold autoconvolution of the Fourier series coefficients, i_m. This example will be continued in the next section.

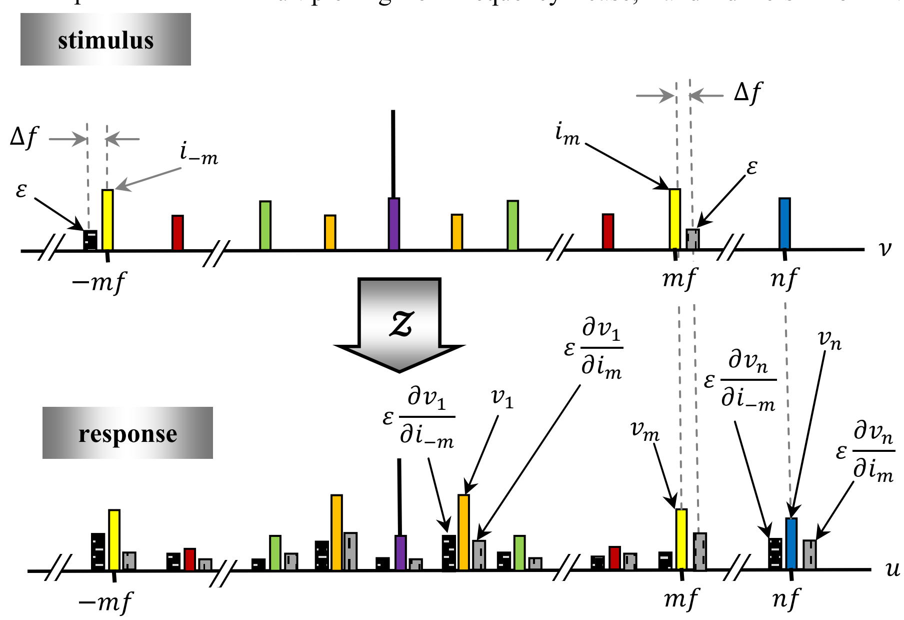

Figure 1: Illustration of the off-frequency method of determining harmonic coupling weights for a PP system. On top is the spectrum of the stimulus, $i(\tau)$, and the spectrum of the response, $v(t)$, is on the bottom. Both signals are periodic with period $T = 1/f$ and therefore have their harmonics at integer values of f. A tickle tone, $\Delta i(\tau) = 2\varepsilon\cos(2\pi(mf + \Delta f)\tau)$, is added and appears in the stimulus spectrum with weight ε at the two frequencies $\pm(mf + \Delta f)$. If the PP system, $\mathcal{Z}$, obeys the harmonic noninterference and frequency invariant constraints, the harmonic coupling weights, $\varepsilon \frac{\partial v_n}{\partial i_m}$ and $\varepsilon \frac{\partial v_n}{\partial i_{-m}}$, can be read at the output at frequencies $nf + \Delta f$ and $nf - \Delta f$ for all values of n.

IV. *MULTIPLEXING OFF-FREQUENCY MEASUREMENTS*

Within the limitations of the model, numerous harmonic coupling weights can be measured simultaneously in a single measurement. In the multiplexing off-frequency measurement, multiple tickle-tones can be applied, with the spacing between the tickle tone and the harmonic frequency different for each harmonic. Specifically, for some range of n *and* m, a single experiment suffices to measure $\frac{\partial v_n}{\partial i_m}$ and $\frac{\partial v_n}{\partial i_{-m}}$.

Let

$$i_\Delta(\tau) = i(\tau) + \Delta i(\tau)$$

where $i(t)$ has a Fourier series in (5). The perturbation, $\Delta i(\tau)$, is assumed to be a trigonometric polynomial with fundamental frequency $f + \Delta f$, *i.e.*

$$\Delta i(\tau) = \sum_{m=-M}^{M} \Delta i_m \, e^{j2\pi m(f+\Delta f)\tau} \qquad (13)$$

where M is the number of harmonic coupling weights to be measured. (13) represents the input polynomial in the multiplexing off-frequency case, and differs from the traditional off-frequency excitation described by (10) in that the m is distributed over the $f + \Delta f$ term. For example, this means that for a situation where the fundamental excitation frequency is 1 GHz and the tickle-tone spacing is 10 kHz, then for the multiplexing case, the first harmonic tickle-tone is placed at 1.000010 GHz, the second harmonic tickle-tone is placed at 2.000020 GHz, and the third harmonic tickle-tone is placed at 3.000030 GHz, and so forth. For the special case where all of the perturbations are the same, *i.e.*

$$\Delta i_m = \varepsilon, \qquad (13)$$

becomes the superposition of tickle tones

$$\Delta i(\tau) = \varepsilon \sum_{m=-M}^{M} e^{j2\pi m(f+\Delta f)\tau}$$

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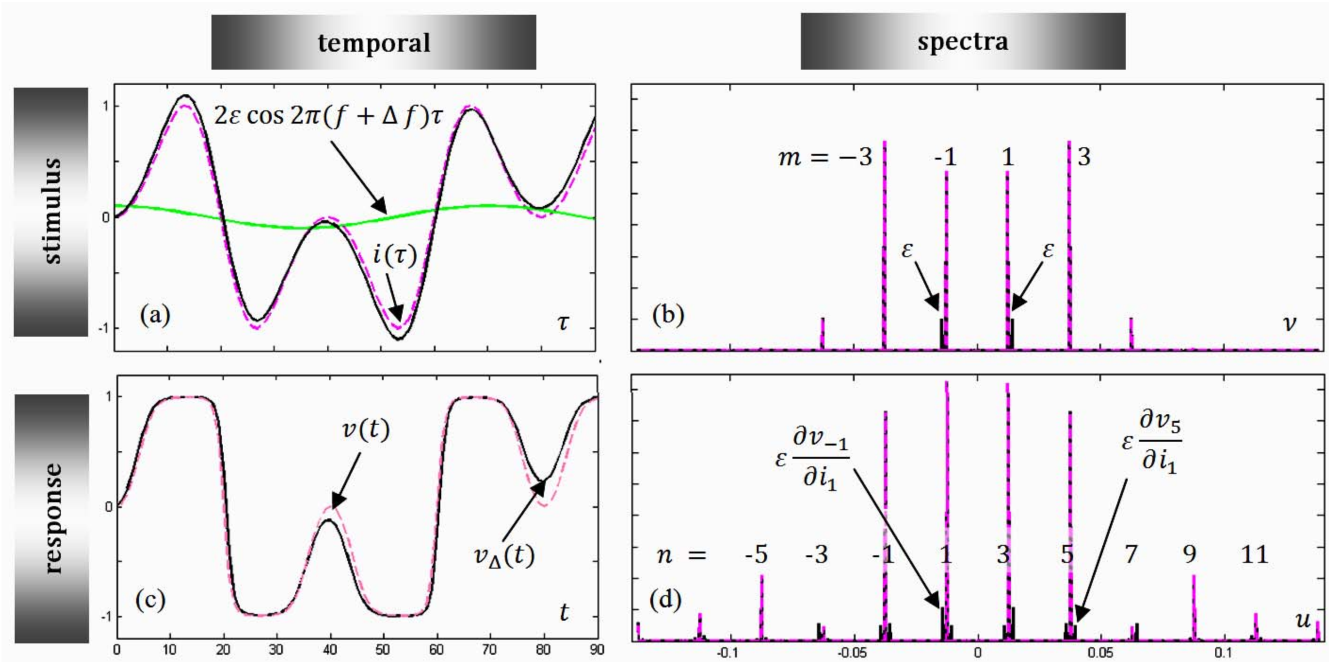

Figure 2: A simulation illustration the off-frequency method of finding harmonic coupling weights. A period of the stimulus, $i(\tau) = \sin(\pi \cos(2\pi f \tau))$ where $f = 50/4000$, is shown in (a) with the tickle tone $2\,\varepsilon \cos(2\pi(f + \Delta f)\tau)$ where $\Delta f = 7/4000$ and $\varepsilon = 0.10$. The sum of the two signals is shown in (a) with the dark bold line. The magnitude of the Fourier transform (spectrum) of this signal, shown in (b), is graphically indistinguishable from the spectrum of $i(t)$ except for the tickle tones with weight appearing at $\pm(f + \Delta f)$. (The signal $i(\tau)$ has only odd harmonics.) Shown in (c) with the light dashed line is the response $v(t) = g(i(t)) = \tanh(3i(t))$. The solid line is the response to the perturbation which is $v_\Delta(t) = \tanh\big(3(i(t) + 2\,\varepsilon \cos(2\pi(f + \Delta f)t))\big)$. The magnitude of the spectra of $v(t)$ and $v_\Delta(t)$, shown in (d), are again graphically indistinguishable except for the small terms at $\pm\Delta f$ from each harmonic. The amplitude of these terms are mined to (for positive frequencies) to obtain the harmonic coupling weights about the operating points of $i(\tau)$ and $v(t)$.

$$= (2M+1)\,\text{array}_{2M+1}\,((f+\Delta f)\tau) \qquad (14)$$

where the array function is defined by [11]

$$\text{array}_K(\tau) := \frac{\sin(\pi K \nu)}{K \sin(\pi \nu)}$$

As $M \to \infty$, $(2M+1)\,\text{array}_{2M+1}(\tau)$ approaches a string of Dirac deltas [12].

Our goal is to use $i_\Delta(t)$ to determine

$$\frac{\partial v_n}{\partial i_m} \text{ and } \frac{\partial v_n}{\partial i_{-m}} \text{ for } 0 \le m \le M.$$

When $\Delta\, i(\tau)$ is real, $\Delta i_{-m} = \Delta i_m^*$. We assume all of the frequency components, as illustrated in Figure 3, obey not only the frequency noninterference constraint, but a cross-frequency noninterference constraint, *e.g.* the tickle tone components at, say, $m = 2$, are assumed to not effect the response perturbation for $m = 1$. This rule requires that there be no substantial intermodulation products sourced from a pair of tickle tones. For the multiplexing to work, the only intermodulation producing substantial products must result from the mixing of a tickle tone with a large-signal harmonic frequency. As an example of unwanted cross-frequency noninterference, the tickle tone input from the third harmonic (in the above example, at 3.000030 GHz), could intermodulate with the second harmonic tickle-tone input (at 2.000020 GHz) to give a response that would interfere with the response from the first-harmonic tickle-tone (at 1.000010 GHz). However, as long as the only substantial intermodulation occurs between a tickle-tone and a harmonic, the multiplexing technique will be successful. If this is the case and the frequency invariant property is also sufficiently present, then the stimulus component at $\nu = m(f + \Delta f)$ with weight Δi_m generates content at the response frequencies $u = nf + m\Delta f$ with amplitude $\Delta i_m \frac{\partial v_n}{\partial i_m}$. Likewise, the $\Delta i_{-m} = \Delta i_m^*$ stimulus terms $\nu = m(f - \Delta f)$ appear in the response with weight $\Delta i_{-m} \frac{\partial v_n}{\partial i_{-m}}$ at frequencies $u = nf - m\Delta f$. Thus, a single experiment suffices to measure numerous harmonic coupling weights.

Figure 4 shows a simulation demonstrating the spectrum of a nonlinear system output for a multiplexed tickle-tone excitation. The plots are described in the caption of the figure. The fundamental frequency of the large-signal excitation is 50 Hz for this demonstration, and the nonlinearity is of a typical saturating device: a *tanh* expression as shown in the caption.

For simplicity, tickle tones are only generated for the first and second harmonics. The center plot shows a slightly more zoomed-in vertical scale, revealing tones resulting from intermodulation between tickle tones and main harmonics. The components resulting from negative-frequency terms are on the lower side of the harmonics, and the components resulting from positive-frequency terms are on the upper side of the harmonics. The bottom plot is still more zoomed-in and reveals all of the terms resulting from the tickle tones. Tickle-outputs resulting from second-harmonic excitation (both positive and negative frequency inputs) are located twice the value of Δf from the harmonic. Thus for two simultaneous tickle-tone excitations, there are 5 terms corresponding to each harmonics (main harmonic, fundamental-frequency related output, and second-harmonic related output) rather than just three as occur in a more traditional tickle-tone excitation (tickle-tone excitation at only one harmonic at a time). In constructing measurement equipment to create this multiplexed tickle-tone excitation, it is anticipated that a multi-tone generator could be used for the excitation.

A. Polynomials

The multiplexing off-frequency approach can be derived analytically for memoryless polynomials. In practice, doing so is unnecessary because, for memoryless nonlinearities, a single set of measurements is needed to specify the defining function.

For the polynomial in (11) we obtain the perturbed response linearity in (16). In Appendix B, we show the weight of the spectrum of $\Delta\, v(t)$ at frequency $nf + m\Delta f$ is

$$\Delta i_m \sum_{p=0}^{P} p\, \alpha_p\, i_{n-m}^{(p-1)*} \Delta i_n = \frac{\partial v_n}{\partial i_m}\, \Delta i_m \qquad (15)$$

Since Δi_m can be measured from the tickle signal, a number of harmonic coupling weights can be generated from a single measurement. For polynomial nonlinearities, the multiplexing off-frequency approach is therefore mathematically exact.

V. Conclusions

A frequency multiplexing technique for tickle-tone excitation has been presented to measure the nonlinear network parameters of periodicity preservation (PP) systems. This approach works for systems where frequency noninterference constraints and cross-frequency noninterference constraints are present, and allows the all of the Jacobian (perturbation) parameters to be found with a

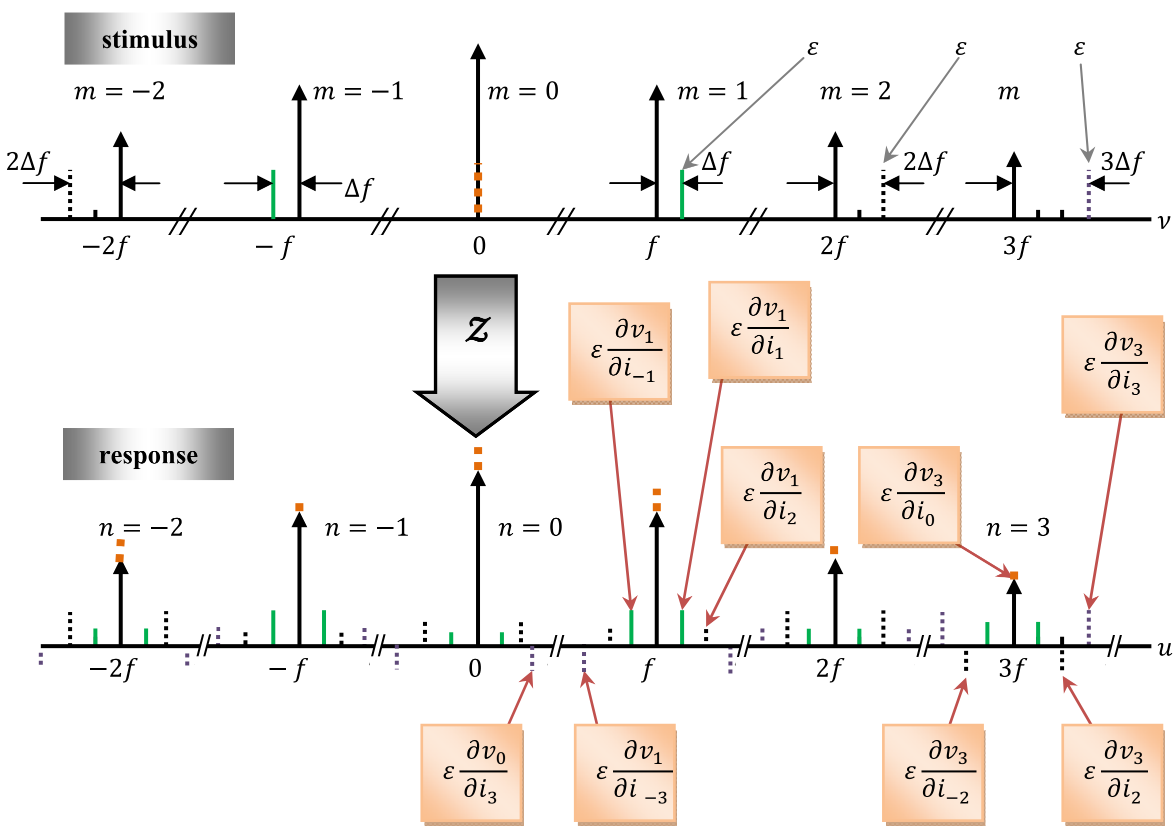

Figure 3. Illustration of simultaneous multiplexed off-frequency measurement of numerous harmonic coupling weights. As is shown in the top figure, the stimulus, $i(t)$, has cosinusoid tickle tones applied at frequencies $m(f + \Delta f)$ there $T = 1/f$ is the period of the periodic signals $i(t)$ and $v(t)$. In this example, all of the amplitudes are ε. If the PP system conforms to frequency invariant and cross interference constraints, then numerous harmonic coupling weights can be measured from the response as shown in the bottom figure. At frequencies $u = nf \pm m\Delta f$ we measure $\varepsilon \frac{\partial v_n}{\partial i_m}$.

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single measurement. This work can significantly eliminate measurement iterations in extractions of nonlinear network parameters such as the X-parameters and the S-functions for devices possessing nonlinearities such that the tickle-tones do not significantly intermodulate with each other, but only intermodulate substantially with harmonics of the large-signal operating condition. The results of this work will decrease the number of measurements required by a factor equal to the number of harmonics being considered in the nonlinear network parameter determination. As a next step, software routines will be constructed to evaluate the nonlinear PP parameters from the spectral output for the multiplexed case. Comparisons will be performed for prediction capability of different nonlinearities from nonlinear PP parameters extracted under (1) single tickle-tone excitation and (2) multiplexed simultaneous tickle-tone excitation. Limitations of this multiplexing approach can be studied in such a scenario.

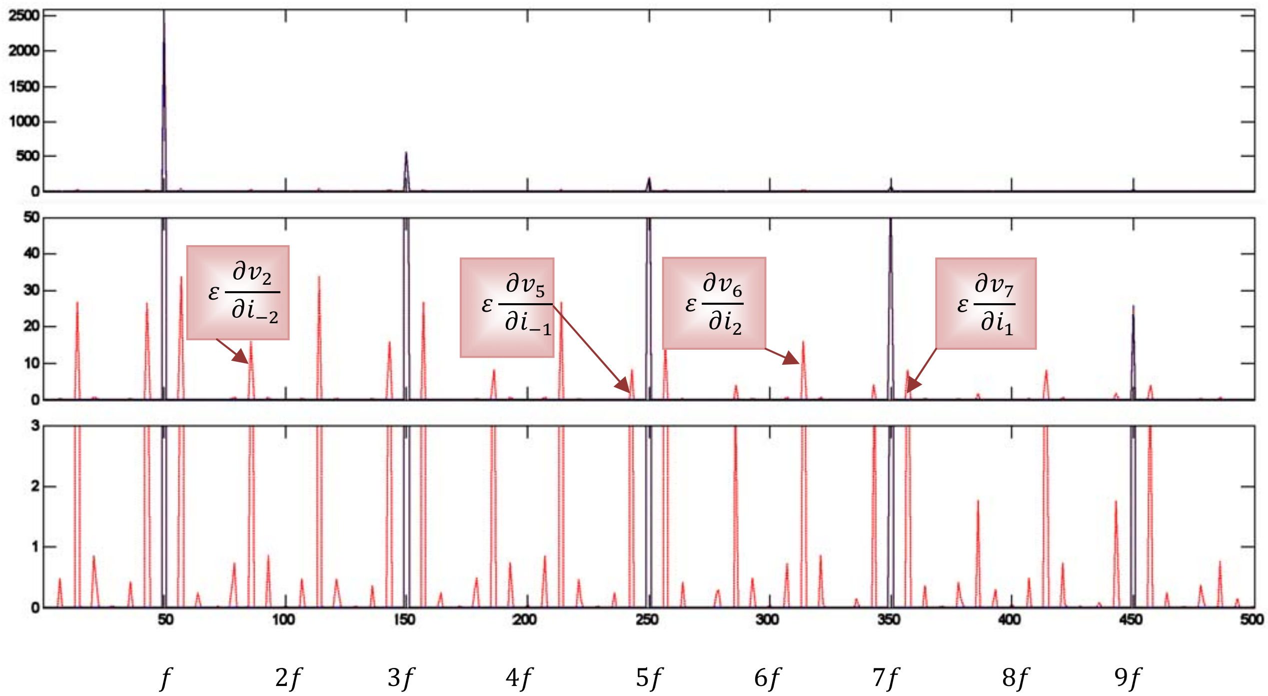

Figure 4. A simulation demonstrating the multiplexing tickle tones illustrated in Figure 3. The input operating point is $i(t) = \cos(2\pi f t)$ where $f = 50$. The input is subjected to a nonlinearity, $g(i) = \tanh(3i)$. Two tickle tones are added to the input equal to $\Delta i(t) = 2\varepsilon\cos(2\pi(f + \Delta f)t) + 2\varepsilon\cos(4\pi(f + \Delta f)t)$ where $\Delta f = 7$ and $2\varepsilon = 0.0250$. The signal $i_\Delta(t) = i(t) + \Delta i(t)$ for $t = 1{:}4000$ is passed through the nonlinearity to give the output, $v_\Delta(t) = g(i(t))$. The magnitude of the Fourier transform (DFT) of the output, $|V_\Delta(u)|$, is shown above. The three plots shown are of the same function, albeit over different ranges. Some harmonic coupling weights are labeled in the middle plot.

VI. APPENDIX

A. Proof of (12)

Since

$$v(t) = g\left(i(t)\right) = \sum_{p=0}^{P} \alpha_p i^p(t)$$

a small input perturbation gives

$$g\left(i(t) + \Delta i(t)\right) \coloneqq \sum_{p=0}^{P} \alpha_p\left(i(t) + \Delta i(t)\right)^p$$

$$\approx \sum_{p=0}^{P} \alpha_p\left(i^p(t) + p i^{p-1}(t)\Delta i(t)\right)$$

$$= v(t) + \Delta v(t) \qquad (16)$$

where the approximation to the output perturbation is

$$\Delta v(t) = \sum_{p=0}^{P} \alpha_p p i^{p-1}(t)\Delta i(t). \qquad (17)$$

In terms of Fourier series coefficients, an equivalent statement is

$$\Delta v_n = \sum_{p=0}^{P} \alpha_p p\, i_n^{(p-1)*} * \Delta i_n$$

where the asterisk denotes discrete convolution and i_m^{p*} denotes the p fold autoconvolution of the Fourier series coefficients of $i(t)$. Writing out the convolution gives

$$\Delta v_n = \sum_{m=-\infty}^{\infty} \left(\sum_{p=0}^{P} \alpha_p p \; i_{n-m}^{(p-1)*} \right) \Delta i_m \qquad (18)$$

so that , from (3), we find the harmonic coupling weights given in (31).

B. Proof of (15)

Substituting the Fourier series in (10) and (45) into (28) gives

$$\Delta v(t) = \sum_{p=0}^{P} p \; \alpha_p \sum_{k=-\infty}^{\infty} i_k^{(p-1)*} \; e^{j2\pi kft} \sum_{m=-M}^{M} \Delta i_m \, e^{j2\pi m(f+\Delta f)t}$$

$$= \sum_{p=0}^{P} p \; \alpha_p \sum_{m=-M}^{M} \Delta i_m \sum_{k=-\infty}^{\infty} i_k^{(p-1)*} \, e^{j2\pi\left((m+k)f+m\Delta f\right)t}$$

Substituting $n = k + m$ in the k sum gives

$$\Delta v(t) = \sum_{p=0}^{P} p \; \alpha_p \sum_{n=-\infty}^{\infty} \left[\sum_{m=-M}^{M} i_{n-m}^{(p-1)*} \Delta i_m \, e^{j2\pi m\Delta ft} \right] \; e^{j2\pi nft}$$

$$= \sum_{m=-M}^{M} \left[\sum_{n=-\infty}^{\infty} \left(\sum_{p=0}^{P} p \; \alpha_p \; i_{n-m}^{(p-1)*} \right) e^{j2\pi(nf+m\Delta f)t} \right] \; \Delta i_m$$

Examining the frequencies of this signal at $u = nf + m\Delta f$ gives (15).

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