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# Chirp Optimization Using Piecewise Linear Approach

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***Abstract*—A chirp optimization for in-band flatness under specific spectral constraints has been simulated in MATLAB using a search through piecewise linear characterizations of the chirp's time-frequency profile. In general, the piecewise linear chirp consists of both up- and down-chirps. An exhaustive search seeks the chirp best satisfying in-band energy and flatness criteria.**

## I. INTRODUCTION

Spectral spreading has become a critical issue in present-day radar systems. Stringent spectral mask requirements determined by the Federal Communications Commission (FCC) and imposed by the National Telecommunications Information Administration (NTIA), due to developing wireless technology, are requiring radar systems to reduce spectral spreading [1]. The National Broadband Plan in the United States will lead to the re-allocation of radar spectrum for wireless broadband applications [2]; as a result, radars will need to operate in narrower spectra. Cognitive radar systems [3,4] and adaptive waveform optimization [5] have been suggested as the radar protocol of the future. Literature shows that chirp optimization has been performed in the area of optics to increase transmission range of quantum-dash based, directly modulated lasers [6]. Radar waveform optimization has also been performed for colored noise mitigation [7]. Other work deals with improving linearity of the transmitter power amplifier as a method to mitigate spectral spreading [8, 9]. This paper discusses a radar chirp optimization to meet spectral constraints while providing optimum in-band energy and flatness.

## II. PIECEWISE LINEAR CHIRP OPTIMIZATION

There are two goals for this chirp optimization: (1) the in-band energy of the chirp signal must be maximized, and (2) the in-band flatness should be as flat as possible while meeting the spectral mask requirements set by the FCC. For the purposes of this research work, the frequency-versus-time characteristic of the chirp is required to be piecewise linear. Lines in the frequency-versus time profile connect frequency candidate points at specific time locations. Figure 1 shows an illustration of a piecewise linear optimization with five specifically set time points and four frequency points. We use an exhaustive search algorithm and examine each possible combination of frequencies at the given time points. Chirps not meeting the spectral mask requirements are not considered. Of those remaining, the chirp maximizing a criterion "cost function" maximizing in-band energy while minimizing in-band flatness is chosen.

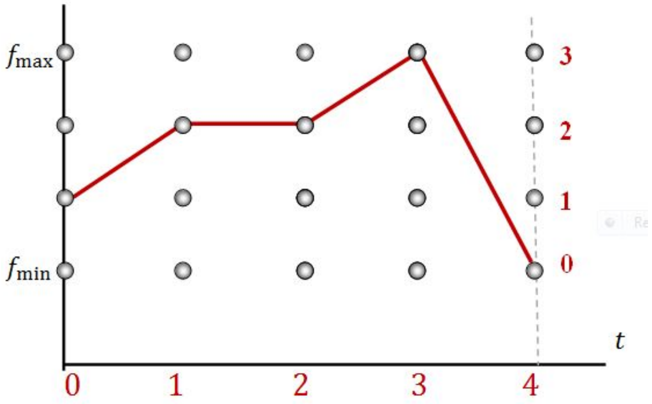

Figure 1. Piecewise Linear Frequency Profile.

## III. FITNESS FUNCTION

A multi-objective optimization for in-band energy and in-band flatness was performed. An initial fitness function for the optimization is defined to maximize a weighted sum of fitnesses of in-band flatness and in-band energy. The fitness function is given by

$$fitness = \alpha E_{IB} + (1-\alpha)(100)e^{-\sigma_{IB}}, \quad (1)$$

where $\alpha$ is the importance factor of the in-band energy and $1-\alpha$ is the importance factor for the in-band flatness. $E_{IB}$ is the energy inside the band and $\sigma_{IN}$ is the standard deviation of the waveform over the desired chirp frequency range. To represent flatness, $100e^{-\sigma_{IB}}$ is used, as a more desirable (smaller) standard deviation $\sigma_{IN}$ will yield a larger value for $e^{-\sigma_{IB}}$. Using MATLAB, a series of simulations was performed, in which different values of $\alpha$ were used while the chirp spectra and frequency-versus-time characteristic were observed. The optimization finds the point on the Pareto frontier that intersects the line defined by (1).

## IV. SIMULATION RESULTS

In the initial MATLAB simulations, a chirp is optimized to try to form a rectangular spectrum between 400 and 600 Hz. For each example, a spectral mask is assigned, and four time points are fixed at 0, 2, 4, and 6 seconds to connect the piecewise lines in the frequency-versus-time characteristic of the spectrum. The total energy of each chirp candidate is set to 1. The spectral masks are pulled out 100 Hz on each side of the chirp spectrum (i.e., to 300 Hz and 700 Hz) in order to provide the chirp with some “wiggle room” and to illustrate how the Pareto trade-off of in-band energy and flatness can be adjusted. In total, there are 14,642 different chirp candidates in the search. Figure 2 shows a plot of the in-band flatness metric $100e^{-\sigma_{IB}}$ versus the in-band (between 400 and 600 Hz) energy for all chirp candidates in the search. The optimum solution should ideally fall in the top right corner of the in-band flatness-versus-in-band energy plot since the main goals are maximum in-band energy and maximum in-band flatness. Final simulations confirm this.

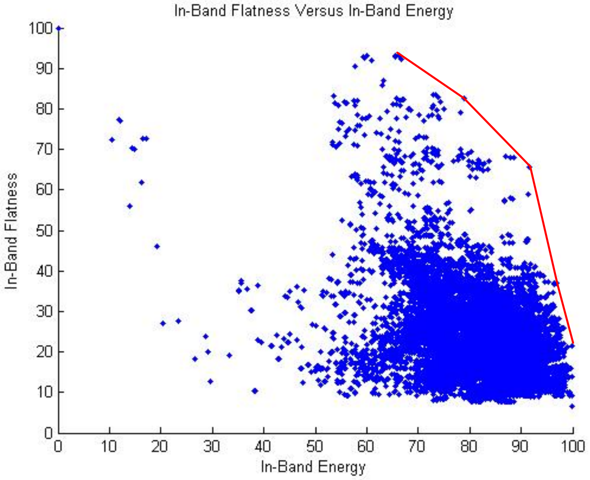

Figure 2. All Solutions in Chirp Optimization.

The chirp candidates that not meeting spectral mask requirements are discarded and each candidate meeting spectral requirements is compared to the previous candidate and saved if its fitness function value is greater than that of the previous chirp candidate. Figure 3 shows all chirp contenders meeting the specific spectral mask requirements that were set in the simulation. In total, there are 2,394 solutions that met spectral mask requirements.

Under the reasonable assumption that the Pareto fronts are concave, the piecewise linear hulls shown in Figs. 2 and 3 provide a Pareto front bound.

The spectral mask requirements can be relaxed by pulling the spectral masks outward and also raising them. Relaxing the spectral mask requirements increases the pool of chirp contenders meeting spectral mask requirements. In actual scenarios, the spectral mask will be determined by government agencies. Once the spectral mask is set, the Pareto optimization can be performed. A series of simulations were performed to illustrate optimum chirp solutions for several different cases of $\alpha$. Four different cases of varying $\alpha$'s are presented here, for which $\alpha$ is 0, 0.5, 0.9 and 1.

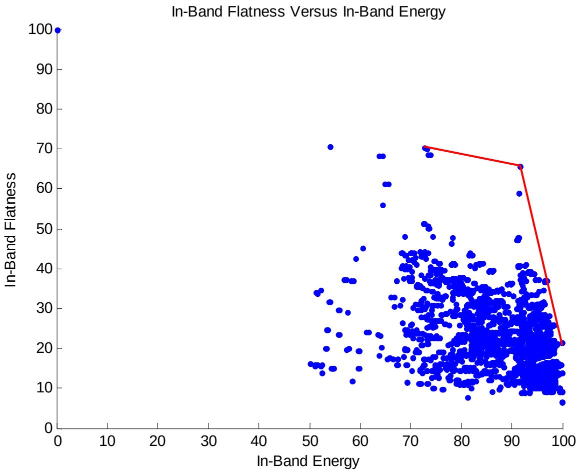

Figure 3. All Solutions That Meet Spectral Mask Requirements.

Case 1: $\alpha = 0$

When $\alpha$ is zero, the Pareto optimization maximizes the flatness inside the band (here between 400 and 600 Hz) and the energy inside the band is not considered. Since this case is rudimentary, the best solution is not a chirp but a tone with zero in-band energy as seen in Figure 4. This impulse function is a tone with a frequency profile in which frequency does not change. The frequency-versus-time characteristic of this chirp is a flat line (constant). The reason this chirp candidate was picked as the optimum solution is because it fell outside the designated optimization bandwidth (i.e., 400 Hz to 600 Hz) but still remained in between the spectral masks. This solution is obviously not desirable and is only possible because of wide spectral masks and that the characteristics outside of the 400 to 600 Hz range are not considered. Enforcing the in-band energy and in-band flatness together, shown in the other examples, is seen to yield much more desirable optimization results.

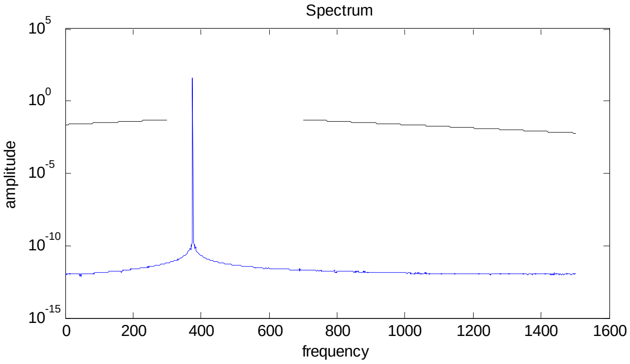

Figure 4. Spectrum of Optimum Chirp and Spectral Mask for Case 1.

Notice the point in Figure 3 located in the top left corner of the plot. The point is the optimum solution of the Pareto optimization when $\alpha$ is zero, which yields a zero in-band energy and a 100 % in-band flatness. However, this flatness is the flatness of the noise floor; while flat, it does not produce the desired in-band energy. It seems this case does not present practically useful results.

### Case 2: α = 0.5

In this next optimization, the in-band energy and the in-band flatness share the same importance, which yields a winning chirp candidate in the top right quadrant of the chirp candidate pool seen in Figure 3. This spectral spreading of this chirp, as seen in Figure 5, is significantly close to the spectral masks and it possesses a significant amount of energy outside the 400 to 600 Hz chirp range. Figure 6 shows the frequency-versus-time characteristic of the chirp. The in-band energy value for the optimization is 91% and the in-band flatness metric is 67.

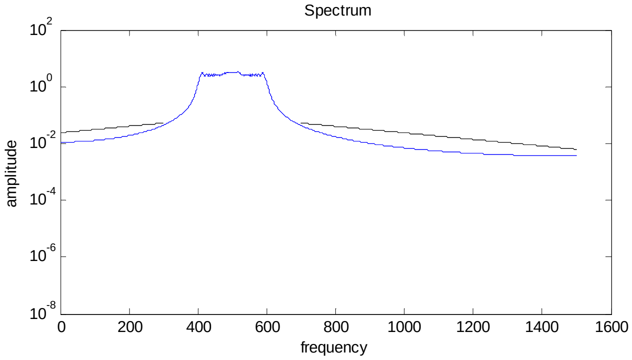

Figure 5. Spectrum of Optimum Chirp and Spectral Mask for Case 2.

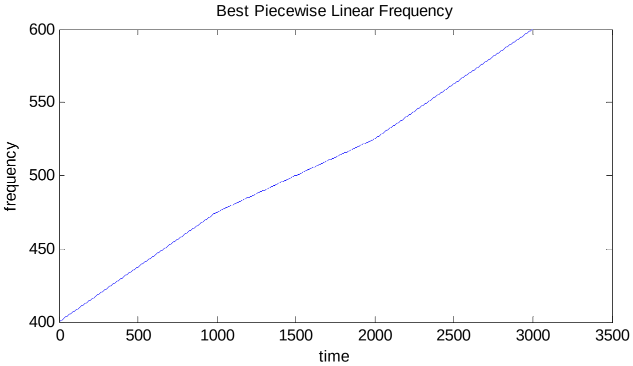

Figure 6. Optimum Chirp Frequency Profile for Case 2.

### Case 3: α = 0.9

Case 3 explores a Pareto optimization in which the in-band energy gets 90% of the importance and the in-band flatness 10 %. The winning chirp candidate seen in Figure 7 falls between 400 Hz and 600 Hz, illustrating that most of the energy is packed inside the band. Figure 8 shows the frequency-versus-time characteristic of the waveform. This chirp clearly contains a significantly larger percentage of its energy inside the band than the chirp of Figure 5. This trend goes along with the fact that higher values of $\alpha$ place more value on in-band energy and less importance on in-band flatness. Simulation results show that the optimum in-band energy is approximately 99.7976% and the in-band flatness metric value is approximately 21.3471.

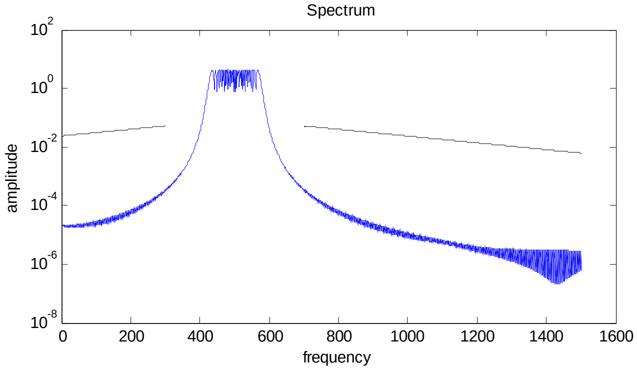

Figure 7. Spectrum of Optimum Chirp and Spectral Mask for Case 3.

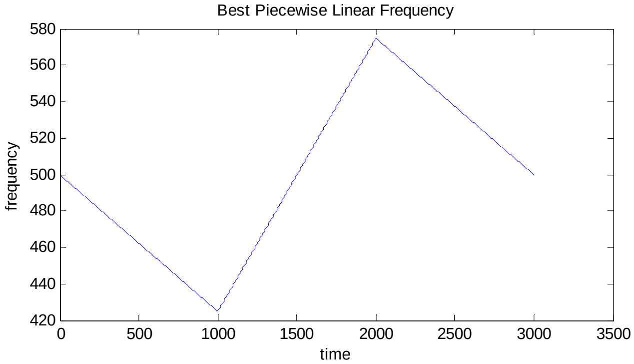

Figure 8. Optimum Chirp Frequency Profile for Case 3.

Case 4: $\alpha = 1$

This case explores the multi-objective optimization in which the in-band energy gets 100% importance and the in-band flatness is completely disregarded. The in-band energy should theoretically be 100% while the in-band flatness is zero. Figure 9 illustrates that the optimum waveform in this case is an impulse function located inside the 400 to 600 Hz band. The frequency-versus-time characteristic of this tone is a constant. Notice that the total energy of the tone is located inside the band, which confirms the accuracy of the optimization's choice. The in-band energy and in-band flatness are 100% and 6.6, respectively. The value of $\alpha$ can be decreased in order to achieve a chirp signal instead of an impulse function.

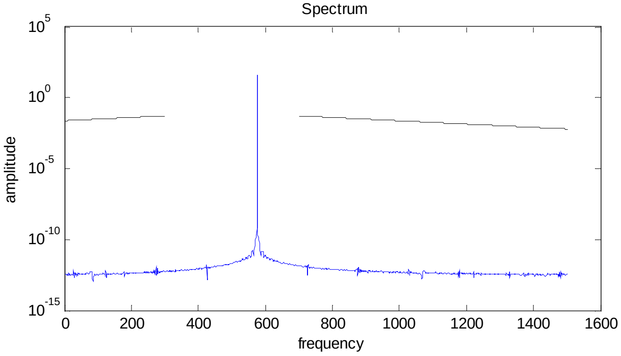

Figure 9. Spectrum of Optimum Chirp and Spectral Mask for Case 4.

## V. CONCLUSIONS

A piecewise linear multi-objective optimization has been performed to achieve optimum in-band flatness while meeting spectral requirements. An exhaustive search was performed in which approximately 15,000 piecewise linear chirp candidates were tested. The optimal solution was required to fall under the spectral mask. In future work, the optimization will be scaled up to higher frequencies that are compatible with radar systems. Also, the resolution of the optimization will be significantly increased. In addition, a computationally intelligent search will be implemented in order to minimize the run-time of the simulations.

## VI. ACKNOWLEDGMENTS

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## REFERENCES

[1] J. de Graaf, H. Faust, J. Alatishe, and S. Talapatra, "Generation of Spectrally Confined Transmitted Radar Waveforms," Proceedings of the IEEE Conference on Radar, 2006, pp. 76-83.

[2] "Broadband Plan": http://www.broadband.gov.

[3] S. Haykin, "Cognitive Radar: A Way of the Future," *IEEE Signal Processing Magazine*, January 2006, pp. 30-40.

[4] J. Guerci, *Cognitive Radar: The Knowledge-Aided Fully Adaptive Approach*, Artech House, 2010.

[5] S. Haykin, "Optimal Waveform Design for Cognitive Radar, 42nd Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, California, October 2008.

[6] Pollakowski, M.; Ermert, H.; , "Chirp signal matching and signal power optimization in pulse-echo mode ultrasonic nondestructive testing," *Ultrasonics, Ferroelectrics and Frequency Control, IEEE Transactions on* , vol.41, no.5, pp.655-659, Sep 1994

[7] Bergin, J.S.; Techau, P.M.; Don Carlos, J.E.; Guerci, J.R.; , "Radar waveform optimization for colored noise mitigation," *Radar Conference, 2005 IEEE International* , vol., no., pp. 149- 154, 9-12 May 2005

[8] C. Baylis, L. Wang, M. Moldovan, J. Martin, H. Miller, L. Cohen, and J.de Graaf, "Designing for Spectral Conformity: Issues in Power Amplifier Design," IEEE Waveform Diversity Conference, Niagara Falls, Ontario, Canada, August 2010.

[9] C. Baylis, L. Wang, M. Moldovan, J. Martin, H. Miller, L. Cohen, and J. de Graaf, "Designing Transmitters for Spectral Conformity: Power Amplifier Design Issues and Strategies," *IET Radar, Sonar, and Navigation*, June 2011.