

# Common Control Channel Assignment in Cognitive Radio Networks Using Potential Game Theory

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**Abstract**—In a cognitive radio network, it is indispensable to assign common control channels for group operations of the secondary users of the spectrum. The assignment requires that multiple secondary users establish the least amount of frequency channels among them while each chooses a channel that has minimum interference to its nearby primary users. We model this problem as a strategic game and design its utility function such that the game is a potential game. A set of pure Nash equilibria are found by locating the local optima of the potential function. We develop sequential and asynchronous updates of game players' strategies using the best response dynamic. In order for the search to escape the local optimum and reach the global optimum of the potential function, we adopt simulated annealing in the sequential and asynchronous updates of the strategies. The optimal assignment of the common control channel is obtained accordingly and the convergence property is analyzed for these updating schemes.

## I. INTRODUCTION

In a cognitive radio network, multiple secondary users share the spectrum with the primary users for efficient use of the radio spectrum. The incumbent primary users have licensed access to the spectrum. The secondary users actively monitor the radio environment and restrict their transmissions according to the interference-power constraint set by each primary user [1]. For a wireless network that consists of multiple secondary users, a common control channel is indispensable for various group operations such as channel access negotiation, spectrum management, network cooperation, and adjustment upon changes in primary user activity or network topology. For example, previous researches on dynamic spectrum access of the cognitive radio network rely on the existence of a common control channel among the secondary users [2]–[4]. Other researches on distributed spectrum allocation [5] and distributed spectrum sharing [6] have assumption that the secondary users can exchange information by some means.

To establish the common control channel, the secondary users continuously sense the spectrum for “high quality” frequency channels that have no or minimum primary user activity. A common control channel may have a limited coverage area due to spectrum heterogeneity caused by the distribution of diverse primary users. A group of closely located secondary users may agree on one frequency to be used as a control channel while other secondary users may choose another frequency. It is usually desirable to increase the coverage of the common control channels, because that reduces the number of control channels and incurs less control signaling overhead. In addition, it is important that the common control

channel can be quickly reassigned to adapt to changes of the primary user activity. Doerr *et al.* proposed a method for control channel assignment that simulates the behavior of a school of fish [7]. Their method is characterized by cohesion and obstacle alignment. Chen *et al.* proposed an algorithm for control channel assignment that is based on ant colony optimization in swarm intelligence [8]. Their algorithm uses the Hello message sent by each secondary radio as the pheromone.

In this paper, we propose a scheme to assign common control channels for multiple secondary users from a game-theoretic perspective. The task is to assign as few as possible frequency channels as common control channels in the secondary user network. Each secondary user prefers the frequency channels with no or minimum primary user activity perceived by itself. The problem of common control channel assignment is modeled as a potential game. It has a global potential function onto which the incentive of all game players, i.e. the secondary users, can be mapped. The secondary users update their game strategies by choosing available frequency channels as control channels. A set of pure Nash equilibria can be found by locating the local optima of the potential function. In order for the search to escape the local optimum and reach the global optimum of the potential function, we adopt a probabilistic method called simulated annealing in the process of updating each secondary user's strategy. Using the best response dynamic with simulated annealing, the updating process has a better chance to achieve the optimal assignment of the common control channel that is Pareto efficient. The convergence property is analyzed for these updating schemes.

## II. SYSTEM MODEL

In a cognitive radio network, the secondary users (SUs) sense the spectrum and exploit the frequency channels that are not used by the primary users (PUs). Every SU senses the spectrum holes and makes its choice of possible control channels. It also periodically sends its choice to other SUs through the permissible channels. We assume that the SUs can establish some primitive pair-wise connections through frequency hopping and rendezvous [9]. The overhead of these primitive pair-wise communications can be high. Each SU has its choice of preferred frequency channels and receives the preferences of other SUs. Based on such information, the SUs need to agree on as few as possible frequency channels as their common control channels. Each SU tends to agree

on the frequency channel commonly available to the largest number of its neighbors. Due to spectrum heterogeneity caused by the distribution of diverse PUs, geographic clusters of SUs may form. The SUs within a cluster use one frequency channel as the common control channel. The communication between the SUs of two different clusters that use two different common control channels can be achieved through a gateway SU. However, more gateways generate more communication overhead within the network.

We consider  $N$  SUs sharing the spectrum with the incumbent PUs in a cognitive radio network. Suppose that the shared spectrum can be divided into  $L$  frequency channels. Let  $\mathcal{L}$  denotes the set of all the channels,  $\mathcal{L} = \{1, 2, \dots, L\}$ . Each SU can monitor the PU activity and sense the channel usage in its vicinity. The  $i$ th SU assigns a value of channel quality to each of the  $L$  frequency channels, that is  $q_i(l)$  with  $i \in \mathcal{N} = \{1, 2, \dots, N\}$  and  $l \in \mathcal{L}$ . The channel quality is a metric that indicates the interference-power constraint imposed by the PUs. With a higher channel quality, the interference constraint is less tight such that the channel is more available for secondary user transmission. Each SU prefers the frequency channels with better quality as the control channel. In this paper, we assume that the distributions of the channel qualities are similar in geographic proximity, but the distributions differ more with farther distance between two locations. In our future work, we will take into account the communication channel quality of two SUs as whether they are in close proximity or far apart. It should be noted that, when two distant SUs sense a same frequency channel with good quality, they may not be able to use that channel as the common control channel because a direct link between them is difficult to establish.

The problem of common control channel assignment can be modeled as a strategic game

$$\mathcal{G} : \langle \mathcal{N}, \{S_i\}, \{u_i\} \rangle \quad (1)$$

- $\mathcal{N}$  is the finite set of  $N$  SUs as the  $N$  game players. Player  $i$  is the  $i$ th SU,  $i \in \mathcal{N}$ .
- $S_i$  ( $i \in \mathcal{N}$ ) is the set of strategy available to player  $i$ . It is the available frequency channels that the  $i$ th SU can choose as the control channel. The overall strategy set of all the players, i.e. the strategy space of the game, is  $\mathcal{S} = \times_{i \in \mathcal{N}} S_i$ .
- $u_i : \mathcal{S} \rightarrow \mathbb{R}$  ( $i \in \mathcal{N}$ ) is the utility function of player  $i$ . For every strategy combination  $\{s_1, \dots, s_N\} \in \mathcal{S}$ , with  $s_i \in S_i$  ( $i \in \mathcal{N}$ ),  $u_i(s_1, \dots, s_N) \in \mathbb{R}$  is the utility of the  $i$ th SU.

Given a strategy combination  $S = \{s_1, s_2, \dots, s_N\} \in \mathcal{S}$ ,  $s_i$  is the strategy chosen by player  $i$ ,  $i \in \mathcal{N}$ . Let  $s_{-i}$  denote the combination of strategies of the other  $(N - 1)$  players. The strategy of player  $i$ , i.e. the  $i$ th SU, is to choose a frequency channel for use as the common control channel with other SUs. That is,  $s_i := l, l \in \mathcal{L}$ . Let  $s_i^*$  denote the best strategy player  $i$  can respond to the strategy combination  $s_{-i}$  of other players, such that the utility  $u_i(s_i^*, s_{-i})$  is maximized for a given  $s_{-i}$ . A pure Nash equilibrium is reached at the strategy combination

$S^* = \{s_1^*, s_2^*, \dots, s_N^*\}$  such that no player can have any gain in its utility by changing its strategy unilaterally [10].

### III. COMMON CONTROL CHANNEL ASSIGNMENT USING POTENTIAL GAME THEORY

#### A. Utility Function Design

A player's utility is determined by its own strategy as well as other players' strategies. Suppose that, before the common control channel is established, the  $i$ th SU can be informed of other SUs' strategies through pair-wise communications using frequency hopping and rendezvous. The utility function of the  $i$ th SU is given by

$$u_i(s_i, s_{-i}) = \lambda \sum_{j=1, j \neq i}^N \mathbb{I}(s_i = s_j) + \mu(1 - \lambda)q_i(s_i) \quad (2)$$

where  $\mathbb{I}\{\cdot\}$  is the indicator function that equals to 1 when  $(s_i = s_j)$  is true and 0 otherwise. The utility function contains two criteria. The first criterion reflects the common interest of multiple players. That is, each SU intends to use a channel that is most commonly available to other SUs as the control channel. The second criterion reflects the selfish concern of each player. That is, each SU wants to use a channel that has the best quality (perceived by this particular SU) as the common control channel. The trade-off parameter  $\lambda \in [0, 1]$  determines the relative weights of the two criteria in the utility function. The normalization factor  $\mu$  is to ensure that the two utility criteria are approximately equally weighted when  $\lambda = 0.5$ . We have

$$\mu = \frac{\mathbb{E} \left[ \sum_{j=1, j \neq i}^N \mathbb{I}(s_i = s_j) \right]}{\mathbb{E}[q_i(s_i)]}. \quad (3)$$

#### B. Potential Game

According to the definition of the utility of each player in (2), the incentive of player  $i$  of changing its strategy from  $s_i = k$  to  $s_i = l$  ( $k, l \in \mathcal{L}$ ) is given by

$$u_i(l, s_{-i}) - u_i(k, s_{-i}) = \lambda \sum_{j=1, j \neq i}^N (\mathbb{I}(s_j = l) - \mathbb{I}(s_j = k)) + \mu(1 - \lambda)(q_i(l) - q_i(k)). \quad (4)$$

We define a global utility function called the potential function of Game  $\mathcal{G}$  as

$$\Phi(S) = \lambda \sum_{i \in \mathcal{N}} \sum_{j=1}^{i-1} \mathbb{I}(s_i = s_j) + \mu(1 - \lambda) \sum_{i \in \mathcal{N}} q_i(s_i) \quad (5)$$

where  $S$  is the strategy combination,  $S = \{s_1, s_2, \dots, s_N\}$ . The potential function  $\Phi(S)$  contains two criteria. The first criterion reflects the collective benefit of the SU network. The second criterion reflects the sum of individual SU benefits. The value of the indicator that relates the strategies of any two players is counted only once in the sum in the first criterion of the potential function.

*Proposition 1:* Game  $\mathcal{G}$  is an exact potential game with the utility functions  $\{u_i\}_{i \in \mathcal{N}}$  given in (2) and the potential function  $\Phi(S)$  given in (5).

*Proof:* Define a matrix  $\mathbf{A} \in \mathbb{R}^{N \times N}$  as the indicator matrix. If the indicator  $I(s_i = s_j) = 1$  for  $i, j \in \mathcal{N}$  and  $j < i$ , the element  $A_{ij} = 1$ . Otherwise the element of  $\mathbf{A}$  is zero. Matrix  $\mathbf{A}$  is a lower triangle matrix. Overall, the potential function of Game  $\mathcal{D}$  can be written as

$$\Phi(S) = \lambda \mathbf{1}^T \mathbf{A} \mathbf{1} + \mu(1 - \lambda) \sum_{i \in \mathcal{N}} q_i(s_i) \quad (6)$$

where  $\mathbf{1}$  is a length- $N$  vector with all ones. When the strategy of player  $i$  is changed from  $s_i = k$  to  $s_i = l$  ( $k, l \in \mathcal{L}$ ), the  $i$ th row and the  $i$ th column of  $\mathbf{A}$  change. The accumulated change in the  $i$ th row is  $\sum_{j=1}^{i-1} (I(l = s_j) - I(k = s_j))$ , and the accumulated change in the  $i$ th column is  $\sum_{m=i+1}^N (I(s_m = l) - I(s_m = k))$ . Therefore, the change in the potential function is given by

$$\begin{aligned} & \Phi(l, s_{-i}) - \Phi(k, s_{-i}) \\ &= \lambda \left( \sum_{j=1}^{i-1} (I(l = s_j) - I(k = s_j)) + \sum_{m=i+1}^N (I(s_m = l) - I(s_m = k)) \right) + \mu(1 - \lambda)(q_i(l) - q_i(k)) \\ &= \lambda \sum_{j=1, j \neq i}^N (I(s_j = l) - I(s_j = k)) + \mu(1 - \lambda)(q_i(l) - q_i(k)). \end{aligned} \quad (7)$$

We have

$$\Phi(l, s_{-i}) - \Phi(k, s_{-i}) = u_i(l, s_{-i}) - u_i(k, s_{-i}). \quad (8)$$

Therefore, Game  $\mathcal{D}$  is an exact potential game.  $\blacksquare$

Since the incentives of all the players are mapped onto the change of the potential function, each player individually adjusting its strategy will cause a change in its utility and in the global potential function with the same amount. Each player sequentially updates its strategy to maximize its utility, i.e. given  $s_{-i}$  the  $i$ th SU chooses a  $s_i$  that maximizes  $u_i(s_i, s_{-i})$ . The potential function will eventually reach a local maximum. At this moment, the potential game stops at a pure Nash equilibrium. In fact, every finite ordinal potential game possesses a pure-strategy equilibrium [11].

#### IV. SIMULATED ANNEALING AND APPROXIMATION TO THE GLOBAL OPTIMUM

There may be multiple Nash equilibria for potential Game  $\mathcal{D}$ . When each SU sequentially updates its strategy  $s_i$  given  $s_{-i}$ ,  $i \in \mathcal{N}$ , such that the utility function  $u_i(s_i, s_{-i})$  is maximized, these  $N$  SUs may reach at a stable state (a Nash equilibrium) and the potential function  $\Phi(S)$  is at a local optimum but not the global optimum. Since the search space is discrete, that is, the strategy sets  $\{S_i\}_{i \in \mathcal{N}}$  are discrete, in order to locate a good approximation to the global optimum of the potential function  $\Phi(S)$ , simulated annealing (SA) can be used as an efficient scheme for game updating [12], [13].

The SA algorithm evolves a discrete-time inhomogenous Markov chain  $x(n)$ . The state  $x(n) = \{s_1, \dots, s_N\}$  is the strategy combination of the  $N$  SUs at discrete-time  $n$ . For the

$i$ th SU, the strategy  $s_i$  can be staying at the current frequency channel or changing to one of the other  $(L - 1)$  frequency channels. In order to simulate the heat (randomness), we assume that the  $i$ th SU can randomly change its current strategy to using one of the other  $(L - 1)$  frequency channels with equal probability  $q_{s_i, s'_i} = 1/(L - 1)$ , where  $s'_i = l$ ,  $l \in \mathcal{L} - \{s_i\}$ . Each of the  $N$  SUs updates its strategy sequentially according to the following rules.

- If  $u_i(s'_i, s_{-i}) \geq u_i(s_i, s_{-i})$ , then  $x(n + 1) = \{s'_i, s_{-i}\}$ .
- If  $u_i(s'_i, s_{-i}) < u_i(s_i, s_{-i})$ , then  $x(n + 1) = \{s'_i, s_{-i}\}$  with probability

$$\begin{aligned} \rho &= \exp \left\{ \frac{u_i(s'_i, s_{-i}) - u_i(s_i, s_{-i})}{T(n)} \right\} \\ &= \exp \left\{ \frac{\Phi(s'_i, s_{-i}) - \Phi(s_i, s_{-i})}{T(n)} \right\} \end{aligned}$$

$x(n + 1) = x(n) = \{s_i, s_{-i}\}$  otherwise.

Therefore, the transition probability is given by

$$\begin{aligned} P[x(n + 1) = \{s'_i, s_{-i}\} | x(n) = \{s_i, s_{-i}\}] \\ = \frac{1}{L - 1} \exp \left\{ \frac{\min(0, \Phi(s'_i, s_{-i}) - \Phi(s_i, s_{-i}))}{T(n)} \right\} \end{aligned} \quad (9)$$

where  $T(n)$  is called the temperature at time  $n$ . The temperature  $T(n) > 0$  and it gradually decreases during the updating process. The time-series  $\{T(n)\}$  is called a cooling schedule. It is almost random to choose the next strategy when  $T$  is large, whereas a better choice with a larger  $\Phi$  is more likely to be made as  $T$  goes to zero.

The allowance of moves that generate a smaller  $\Phi$  leads to a decrease in the potential function. Nevertheless, such ‘‘irregular’’ moves can potentially help the updating process of the SU strategies escape from the local maxima.

#### V. ASYNCHRONOUS UPDATE AND ITS CONVERGENCE

As the  $i$ th SU receives the strategies of the other SUs,  $s_{-i}$ , through high-overhead pair-wise communications, it revises its strategy that maximizes its own utility. This is called the best response dynamic of the strategy updates. The SUs can update their strategies in a sequential round robin fashion or at random. Without a centralized synchronization mechanism or a round robin chain, it is likely that the SUs update their strategies asynchronously. Suppose that the SUs are myopic players such that each one only cares about its current utility. An improvement path of player  $i$  is a path of strategies taken by myopic player  $i$ , such that at every time instant  $k$  of the update process,  $u_i(s_i^{(k)}, s_{-i}^{(k-1)}) > u_i(s_i^{(k-1)}, s_{-i}^{(k-1)})$ . A finite potential game has improvement paths of finite length [11].

For potential Game  $\mathcal{D}$ , the best response takes an improved path toward a Nash equilibrium. In fact, the best response dynamic is equivalent to a local search on the potential function of a potential game. In any finite potential game, sequential updates with best response dynamic always converge to a Nash equilibrium [14]. For asynchronous updates, the following proposition can be proved along the line with Monderer and Shapley’s work [11].

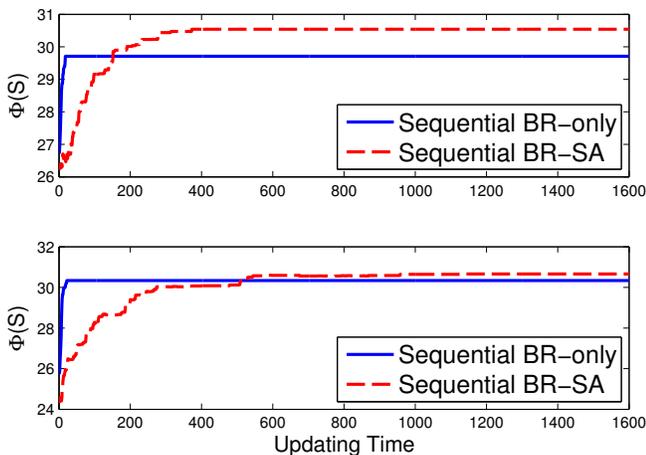


Fig. 1. Game playing with strategy updates using sequential best response and using sequential best response with simulated annealing.  $N = 8$  SUs and  $L = 8$  available frequency channels.

*Proposition 2:* For finite potential Game  $\mathcal{D}$ , the asynchronous updates using the best response dynamic converge to a Nash equilibrium almost surely<sup>1</sup> in finite time and every maximal improvement path terminates on an equilibrium point.

In terms of convergence time, when the best response of each player can be found in polynomial time, Fabrikant *et al.* showed that finding a pure Nash equilibrium in a potential game is Polynomial Local Search (PLS)-complete [15]. In our game of common control channel assignment, each updating SU has only  $L$  choices of strategies that correspond to the  $L$  frequency channels that can be used as the control channel. Each SU picks up one channel out of these  $L$  channels that maximizes its current utility given by (2). Therefore, the convergence to a Nash equilibrium is fairly quick.

Using the scheme of simulated annealing, we add randomness to the update process of the SU strategies. The process is “heated” up before “cooling” down, such that there is a better chance for the potential function to escape a local optimum and converge to the global optimum. The cooling schedule with respect to the heat that is applied should be regulated such that the process will eventually “freeze”, i.e. converge. The convergence of the update process with best response dynamic and simulated annealing, if it indeed converges, is destined to be slower than the update process with just best response dynamic.

For our potential Game  $\mathcal{D}$  of common control channel assignment, let  $S^* \in \mathcal{S}$  denote the optimal strategy combination that maximizes the potential function  $\Phi(S^*)$ . Since each SU can remain at its current strategy or change its strategy by changing its current control channel to any of the other  $(L - 1)$  frequency channels, for any two arbitrary strategy combinations there always exists a path starting with one

<sup>1</sup>If only one player can revise its strategy at a time and the same player is chosen to revise its strategy again and again, the group will never converge to a Nash equilibrium.

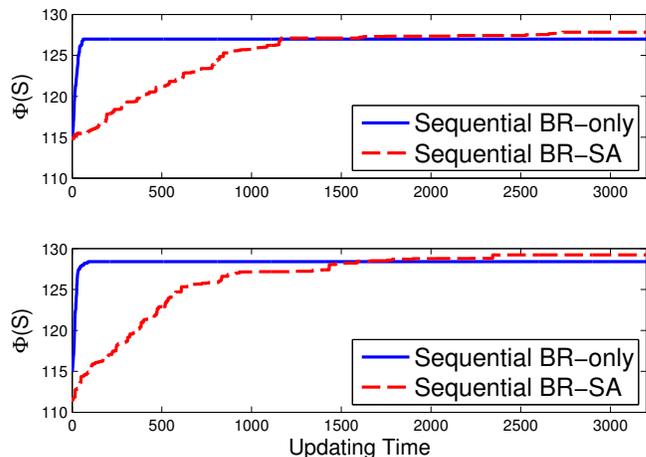


Fig. 2. Game playing with strategy updates using sequential best response and using sequential best response with simulated annealing.  $N = 16$  SUs and  $L = 16$  available frequency channels.

and ending at the other one, regardless of using sequential updates or asynchronous updates. The following proposition on the convergence of Game  $\mathcal{D}$  with simulated annealing can be derived from the convergence theory proved by Hajek [16].

*Proposition 3:* If a strategy combination  $S' \in \mathcal{S}$  has a path to the optimal strategy combination  $S^* \in \mathcal{S}$ ,  $d$  is defined as the depth such that the smallest value of the potential  $\Phi$  along the path is  $\Phi(S') - d$ . For every strategy combination  $S \in \mathcal{S}$  that has a path to  $S^*$ ,  $d^*$  denotes the maximum depth. The updating process of Game  $\mathcal{D}$  with simulated annealing converges if and only if

$$\lim_{n \rightarrow \infty} T(n) = 0 \quad \text{and} \quad \sum_{n=0}^{\infty} e^{-d^*/T(n)} = \infty. \quad (10)$$

The convergence is in probability such that

$$\lim_{n \rightarrow \infty} \Pr[x(n) = S^*] = 1 \quad (11)$$

where the state  $x(n)$  is the strategy combination of the  $N$  SUs at discrete-time  $n$ . The depth  $d$  can be regarded as a measure of “heat disturbance” that can cause  $x(n)$  to escape from a local maximum of the potential function and eventually become the optimal state  $S^*$ .

Given Proposition 3, a cooling schedule can be designed as

$$T(n) = \frac{\beta}{\log(n)} \quad (12)$$

where  $\beta$  is a positive constant and the convergence is guaranteed if and only if  $\beta \geq d^*$ .

## VI. NUMERICAL RESULTS

We present simulation results of the proposed game-theoretic approach to common control channel assignment in a cognitive radio network. The channel quality perceived by the SU is assumed to be a random variable uniformly distributed in  $[0, 1]$ , i.e.  $q_i(l) \sim \mathcal{U}[0, 1]$ . The utility function of each SU is formulated according to (2), and the normalization factor  $\mu$

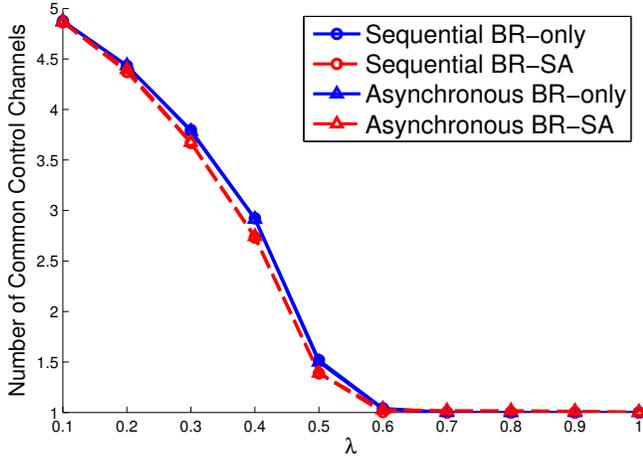


Fig. 3. Number of common control channels at convergence with the first cooling schedule.

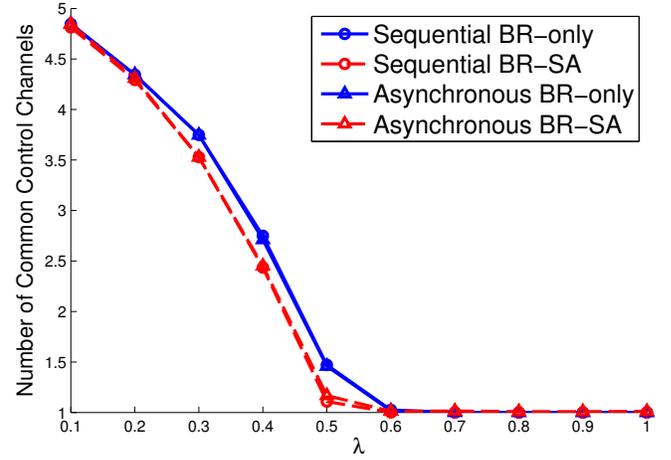


Fig. 4. Number of common control channels at convergence with the second cooling schedule.

is selected according to (3) such that the two utility criteria have similar effects on the utility when  $\lambda = 0.5$ .

For potential Game  $\mathcal{D}$ , each SU is a myopic player that updates its strategy only to maximize its current utility. When the sequential updating process is adopted, each SU updates its strategy in one time unit in a round robin fashion. The update schemes are based on the best response dynamic or the best response dynamic with simulated annealing. With the scheme of simulated annealing, the strategies of the  $N$  SUs define a state of strategy combination that can be transferred to another state with a transition probability given by (9). Two cooling schedules are used for the updating process. For the first cooling schedule, the temperature is given by

$$T(n) = \max(\beta - \rho n, \beta_0), \quad n = 1, 2, \dots \quad (13)$$

where  $\beta = 0.5$ ,  $\beta_0 = 0.00001$ , and the temperature decreases linearly with time at a slope  $\rho = 0.006$ . As time goes by, the temperature approaches 0 and flats out at approximately 0. For the second cooling schedule, the temperature is given by

$$T(n) = \frac{\beta}{\log(n+1)}, \quad n = 1, 2, \dots \quad (14)$$

where  $\beta = 0.5$ . The temperature decreases following the inverse of logarithm of time as in (12).

Fig. 1 and Fig. 2 show the evolution of the potential function  $\Phi(S)$  as the SUs update their strategies sequentially. Fig. 1 shows the game of common control channel assignment with  $N = 8$  SUs in the network and  $L = 8$  available frequency channels. Fig. 2 shows the game with  $N = 16$  SUs and  $L = 16$  channels. The data are of individual simulation runs. In the top sub-figures, the updating processes with simulated annealing (SA) use the first cooling schedule. In the bottom sub-figures, the updating processes with SA use the second cooling schedule. It is revealed that the best response (BR) dynamic with SA has a longer convergence time than the BR-only dynamic. Nevertheless, when it converges, the BR-SA scheme has a better chance to reach the global maximum of

the potential function whereas the BR-only scheme can be stuck at an inferior local maximum. For the SA schemes, the temperature  $T(n)$  starts with a large value which makes the strategy update almost random. With the decrease of  $T(n)$ , the transition becomes concentrated onto the ‘‘correct direction’’.

Fig. 3 and Fig. 4 show, at convergence of the updating process, the number of common control channels used in the SU network (when  $N = 8$ ) versus the trade-off parameter  $\lambda$  in the utility function. When  $\lambda$  is small, the incentive to use a common channel is weak. Therefore, Game  $\mathcal{D}$  reaches a Nash equilibrium where almost every SU chooses a different frequency channel as the control channel. The chosen channel has the best quality perceived by each individual SU. When  $\lambda$  is large, the incentive to use a common channel is strong such that Game  $\mathcal{D}$  reaches a Nash equilibrium where almost all the SU choose the same channel as their common control channel. Fig. 5 and Fig. 6 show the potential function  $\Phi(S)$  at convergence versus the parameter  $\lambda$ . The figures reveal that, when  $\lambda \approx 0.5$  the two criterion of the utility are equally important, the scheme using SA arrives at a better solution with fewer common control channels and a larger potential function. In the simulation, both sequential and asynchronous updates are used. When the asynchronous updates are used, the updating events occur according to a Poisson distribution with an average rate of 8 updating time units ( $\sim \text{Pois}(8)$ ). The updating process of one SU is independent to the updating process of any other SU. The asynchronous updates are shown to have the same strategy results as the sequential updates.

Fig. 7 illustrates the relationship between the common benefit (the first part) and the individual benefit (the second part) in the potential function as in (5). The solid-line represents the case where there are  $N = 8$  SUs and  $L = 8$  available frequency channels. The dashed-line represents the case where there are  $N = 16$  SUs and  $L = 16$  available frequency channels. The sequential BR-only dynamic is used as the updating scheme. With a varying parameter  $\lambda$ , the trade-off between these two benefits is clearly shown.

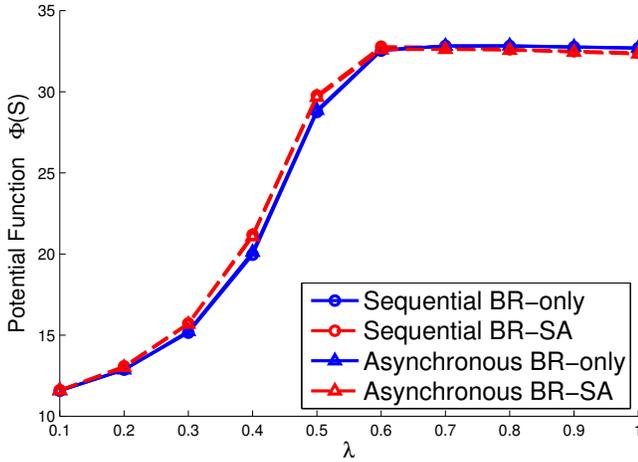


Fig. 5. Potential function at convergence with the first cooling schedule.

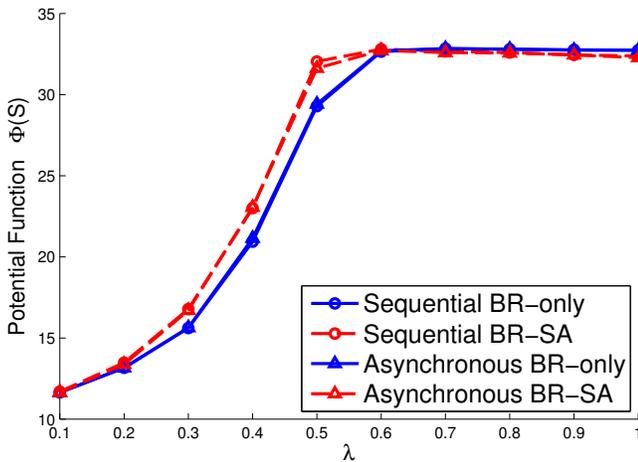


Fig. 6. Potential function at convergence with the second cooling schedule.

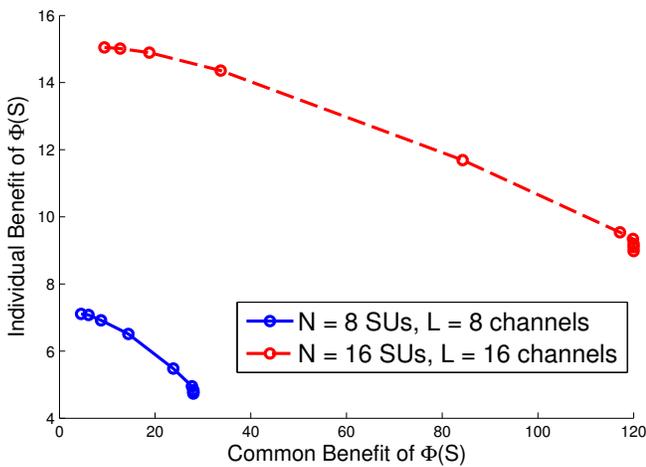


Fig. 7. Trade-off between the common benefit and the individual benefit in the potential function.

## VII. CONCLUSION

In a cognitive radio network, the common control channel of multiple secondary users is essential for effective network operations. In this paper, the problem of assigning common control channels is modeled as a potential game. Each secondary user is a game player and its game strategy is to choose an available frequency channel as the control channel. The utility of each secondary user reflects its common interest and selfish benefit. A potential function is designed onto which the incentives of all the secondary users can be mapped. According to the characteristics of potential games, sequential and asynchronous strategy updates are developed that use the best response dynamic to reach a Nash equilibrium. Moreover, simulated annealing is adopted in the updating process in order to escape a Nash equilibrium which is not Pareto efficient. The proposed scheme has a better chance to obtain the optimal assignment of the common control channel within finite time.

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