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Radar Waveform Optimization to Minimize Spectral Spreading and Achieve Target Detection

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***Abstract*—Spectral spreading in radar systems is a concern that is related to the radar waveform. This paper describes a procedure to optimize a linear frequency-modulated chirp waveform for a desired ambiguity function while meeting spectral requirements. The *minimax* approach used for the optimization is described, and some example results for the optimization are presented. The integration of bench-top spectrum analyzer measurements into the optimization is presented, and the way forward in terms of measurement-based optimization is discussed.**

I. Introduction

Radar systems are presently being required to operate under tighter spectrum allocations. *The National Broadband Plan* of the United States requires that 500 MHz of spectrum be released to wireless broadband applications by 2020. Much of this spectrum is expected to be re-allocated from bands presently assigned to radar. However, the importance of the civilian and military applications of radar, and the related needed precision of these measurements, requires that spectrum engineering must be examined in the context of preserving radar detection capabilities.

In radar and wireless communications, spectral spreading is usually regulated by a governing body. In the United States, government-used spectrum is regulated by the *National Telecommunications and Information Administration* (NTIA), while the Federal Communications Commission regulates commercial spectrum. A spectral mask is used to define the region within must the spectrum of the signal must abide, as shown and described in [1].

Significant attention has been devoted in several cross-disciplinary radar areas to meeting the increasingly stringent spectral requirements. Specifically in the area of waveform optimization, the design of spectrally confined waveforms through variable-modulus techniques [2] and then through constant-modulus techniques such as continuous-phase modulation [3-4] and piecewise linear chirp optimization [5] has been documented. Skolnik discusses the connection of the ambiguity function with the waveform, including properties of frequency-modulated bursts, or *chirps* [6]. Patton demonstrates optimization of the linear frequency-modulated (LFM) chirp design by tuning the nonlinear Fourier Series perturbations to the phase [7]. Holtzman and Thorp use the ambiguity surface as a weighted error criterion for waveform optimization [8]. Use of genetic algorithms to minimize the ambiguity function volume in different regions of the range-Doppler plane is explored by Wong and Chung [9]. Sussman applies least-squares optimization to the radar waveform problem [10]. Blunt *et al.* and Cook demonstrate the use of continuous-phase modulation to minimize the spectral spreading of waveforms [3-4].

The contribution of the work we present is the inclusion of spectral mask considerations into a waveform optimization based on ambiguity function constraints.

II. The Radar Ambiguity Function

The radar ambiguity function is given by the following [6]

$$\chi(\tau', u') = \int_{t=-\infty}^{\infty} x(t') x^*(t'-\tau') e^{-j2\pi u' t'} dt', \qquad (1)$$

where $x(t)$ is the transmitted signal, τ' is the difference in time from the actual time delay associated with the target, and u' is the frequency difference in Hertz from the actual Doppler frequency shift based on the target's true velocity. The ambiguity function represents a measure of the output of a Range-Doppler correlation operation over offsets in time and Doppler frequency from the true time-Doppler state of the target. It measures how well the waveform is able to accurately discern both range and Doppler of a target.

Magnitude plots of some simple ambiguity functions can be used to provide an intuitive feel for this concept. Fig. 1 shows the ambiguity function of a time-domain impulse function. The impulse function is the ideal waveform for range detection of a target, as the ambiguity is only on the u' axis; by the same token, it is a poor choice for a radar where Doppler detection is important due to the large frequency ambiguity. Intuition verifies that the time an impulse occurs is easily discernable with high resolution due to the time narrowness of the signal; however, a frequency shift in a burst of near-zero time length is difficult to discern because the impulse contains all frequencies in equal proportion.

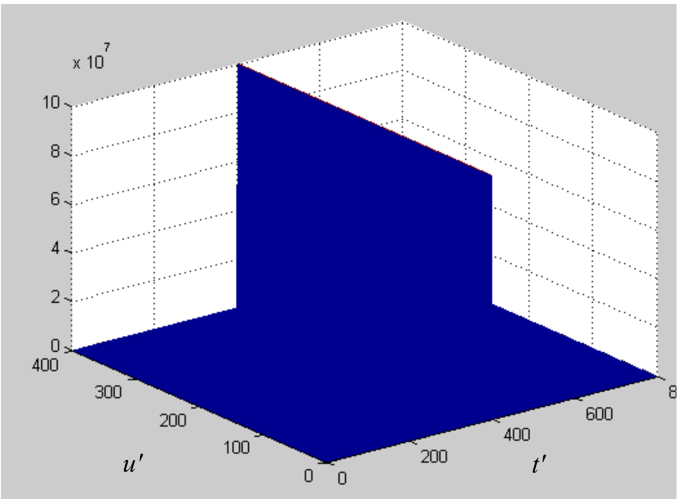


Fig. 1. Ambiguity function magnitude for a time-domain impulse

Fig. 2 displays the ambiguity function of a time-domain sinusoid. The sinusoid used to calculate the ambiguity function shown is time-limited; hence the non-rectangular form of the graph. For the time-domain sinusoid, the ambiguity is aligned along the time-delay axis, indicating the sinusoid provides poor range discernment, but excellent Doppler discernment.

Fig. 3 shows the ambiguity function of a linear frequency-modulated (FM) chirp. For a linear FM chirp, the ambiguity ridge is tilted in the plane, with the slope of the tilt being B/T, where B is the chirp sweep bandwidth in Hz and T is the time width of the chirp in seconds [6].

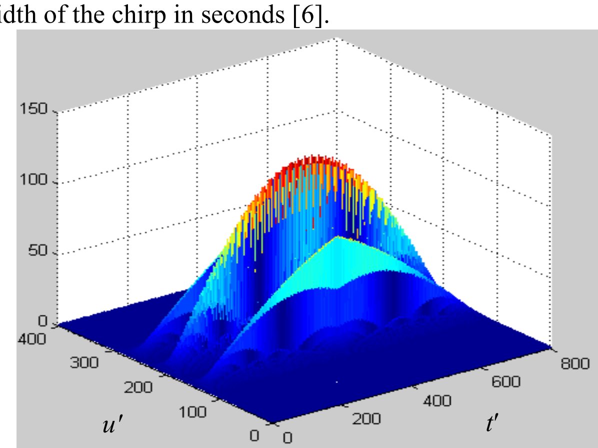


Fig. 2. Ambiguity function magnitude for a time-domain sinusoid

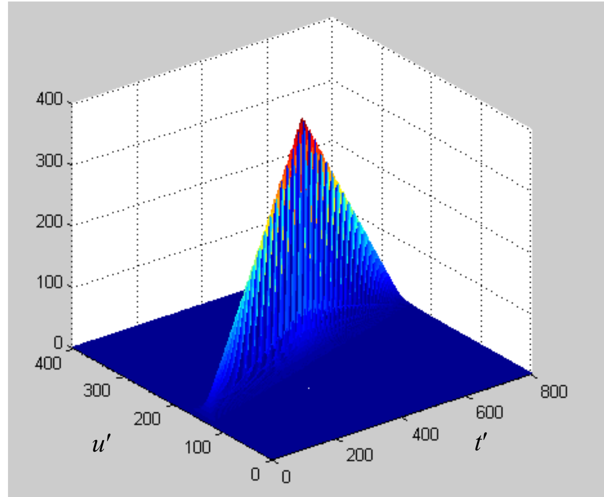


Fig. 3. Ambiguity function magnitude for a linear FM chirp

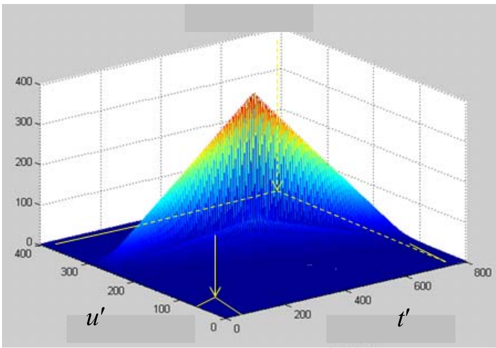
III. Search for the Best Chirp Based on Ambiguity Function and Spectral Compliance

Chirp optimization is performed using linear up- and down-chirps based on (1) minimization of ambiguity at certain range-Doppler combinations and (2) meeting spectral mask criteria. A simulation-based search in MATLAB was performed for the best linear FM chirp taken from a catalogue of candidates. The waveform catalogue for this study consisted of linear FM chirps with bandwidths varying from 0 to 5 MHz in steps of 250 kHz. This means that 21 bandwidth options were available in the search. Furthermore, both up- and down-chirps were allowed, meaning that 42 chirp possibilities were considered in the search. The time width of the chirp was fixed at 10 μs, selected as a reasonable value for the chirp burst width.

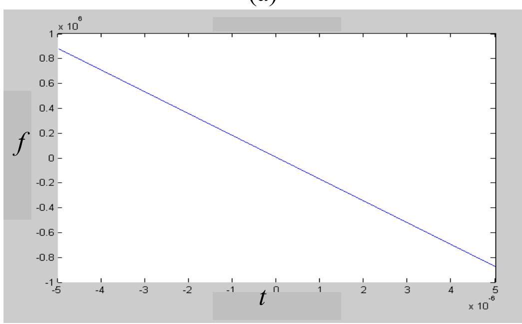
Each chirp under consideration is tested for spectral mask compliance. If the chirp was found to be in violation of the spectral mask criteria, it was no longer considered in the search. Following this criterion, the criterion for optimization is to select the chirp providing the smallest maximum ambiguity function value over specified range-Doppler candidates of interest. The specified range-Doppler candidates of interest in real life would likely come based upon some suspicion of legitimate misinterpretations. For example, a possible interferer or additional target could be located at a point that, if the ambiguity is high, can give false information that it was located at the range/Doppler combination for which information was sought for a primary target.

The optimization approach is *minimax* The minimax approach is chosen because a large ambiguity in one of multiple range/Doppler errors can prove fatal in allowing unwanted or erroneous information to be discerned. Thus it is desired to take the chirp providing the smallest worst-case error over the range/Doppler combinations of interest.

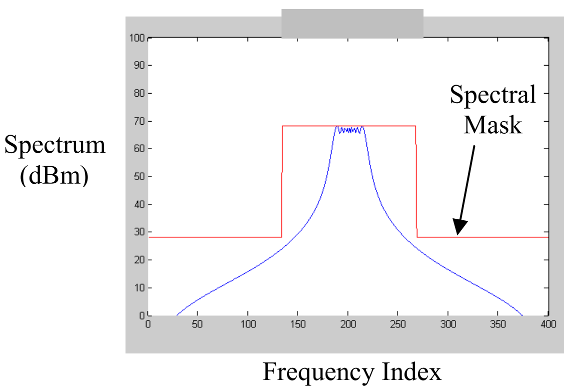
The results from the first example, "Optimization 1," are shown in Fig. 4.



(a)



(b)

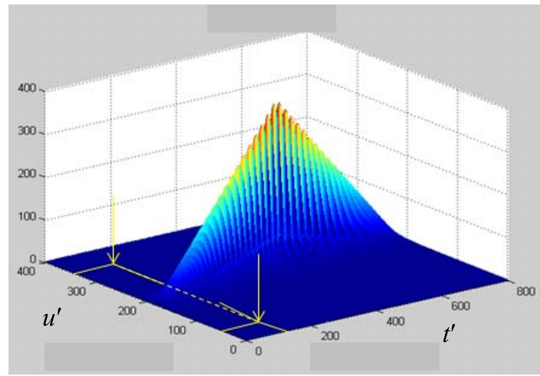


(c)

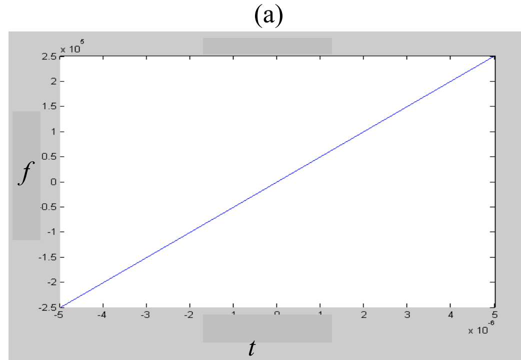
Fig. 4. (a) Ambiguity function magnitude, (b) frequency-versus-time plot, and (c) baseband spectrum and spectral mask for the best chirp in Optimization 1. In (a), arrows denote the range/Doppler combinations for which the ambiguity function value should be minimized.

Fig. 4(a) shows that for Optimization 1, the chirp selected seems to be an intuitively good choice given the minimization points shown. The tilt of the ambiguity ridge in the range-Doppler plane splits the indicated minimization points. Fig. 4(c) shows that the spectrum of the chirp meets spectral mask criteria. From Fig. 4(b), it appears that the chirp has a bandwidth of nearly 2 MHz. Note that both positive and negative frequencies are displayed for this baseband frequency-versus time representation. When upconverted, the frequency value shown in Fig. 4(b) will be added to the assigned frequency of operation.

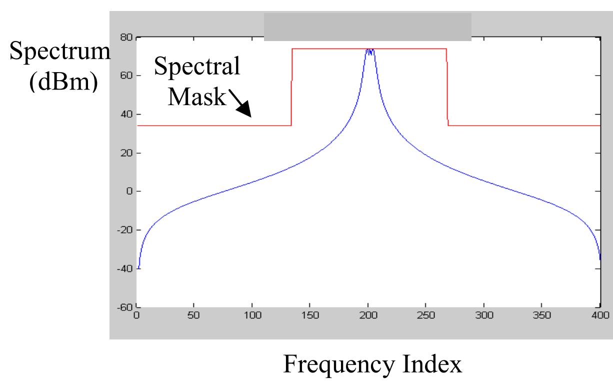
Fig. 5 shows the results for Optimization 2. Once again, the ambiguity ridge for the waveform selected as the optimum seems to "split" between the two designated minimization points. This chirp has an ambiguity ridge that seems to be much more aligned along the time axis than the selected optimum from Optimization 1, and the bandwidth is much smaller (approximately 500 kHz). Fig 5(c) shows that the chirp meets the spectral mask criteria.



(a)



(b)



(c)

Fig. 5. (a) Ambiguity function magnitude, (b) frequency-versus-time plot, and (c) baseband spectrum and spectral mask for the best chirp in Optimization 2. In (a), arrows denote the range/Doppler combinations for which the ambiguity function value should be minimized.

While simulations of spectral mask characteristics are useful, the measured spectral mask compliance is of utmost importance. In Optimization 3, shown in Fig. 6, the best results are ranked according to the simulation results, then are measured using a spectrum analyzer and compared to the spectral mask. The highest ranked signal that passes spectral mask criteria is selected as the optimal waveform.

IV. Conclusions

This paper has provided a waveform optimization approach in which spectral mask considerations are used to select the best waveform from a catalogue of linear FM chirps, along with minimization of the maximum ambiguity function over multiple designated range-Doppler combinations. The results have been demonstrated in simulations using MATLAB, and have been extended to include the bench-top spectrum analyzer measurement of the signal. In future experiments, we expect to include an amplifier in the loop in bench-top spectrum measurements, The measured ambiguity function and spectrum at the output of the amplifier will provide significant information about the waveform actually used in the radar detection. The methodology described in this paper provides the framework within which these measurements will be easily integrated.

Acknowledgments

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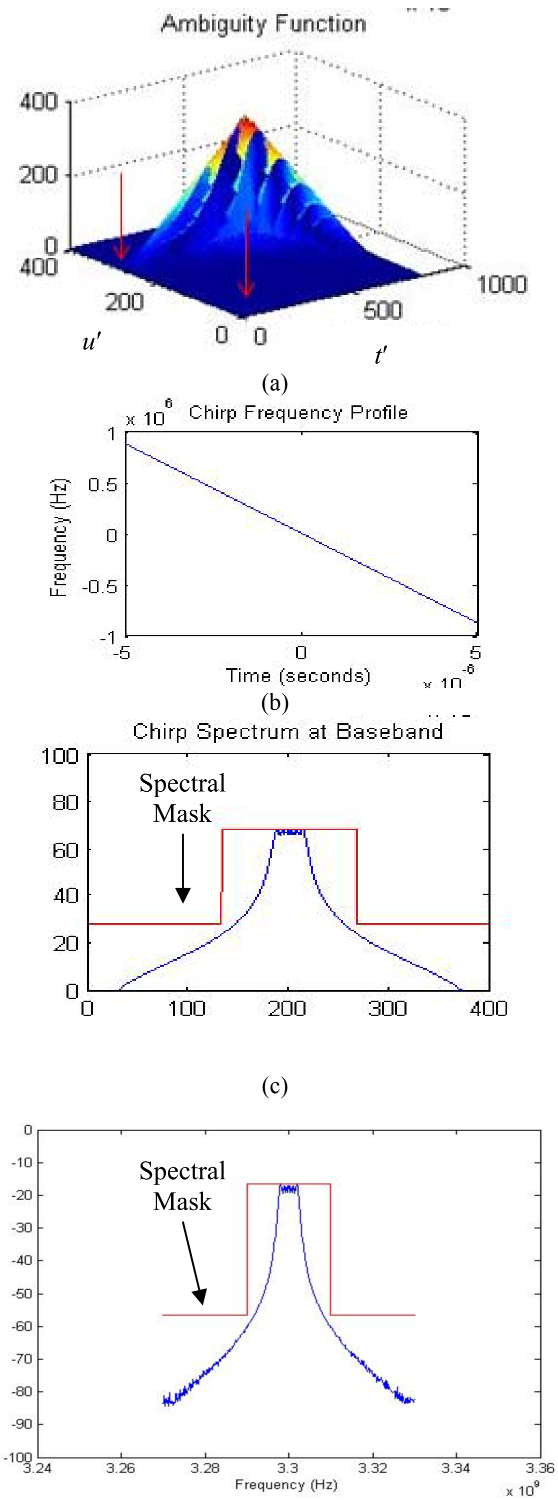

Fig. 6. (a) Simulated ambiguity function magnitude, (b) frequency-versus-time plot, (c) simulated baseband spectrum and spectral mask, and (d) spectrum-analyzer measured spectrum and spectral mask at 3.3 GHz for the best chirp in Optimization 3. In (a), arrows denote the range/Doppler combinations for which the ambiguity function value should be minimized.