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Waveform Synthesis via Alternating Projections with Ambiguity Function, Peak-to-Average Power Ratio, and Spectrum Requirements

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***Abstract* — A method is presented which synthesizes a radar waveform with desirable ambiguity function properties, while simultaneously meeting user-specified requirements on the waveform's peak-to-average power ratio (PAPR) and spectrum. The result is a radar waveform optimization providing desired range-Doppler resolution, spectral compliance, and power efficiency capability. The waveforms are generated via projection and are not constrained to any set of basis functions. A brief overview of the projection process is given, as well as the outline for the methodology to ensuring PAPR and spectral compliance. Simulation and measurement results show successful operation of the approach.**

***Index Terms* — Ambiguity function, radar waveform design, alternating projections.**

I. Introduction

Woodward [1] defines the narrowband ambiguity function (AF) as.

$$\chi(\tau,u)=\int_{-\infty}^{\infty} x(t)x^*(t-\tau)e^{-j2\pi ut}dt. \quad (1)$$

Other literature further describes the AF [2], [3]. The topic of synthesizing a waveform optimized for its AF is a subject which has been extensively studied, notably in early work by Wilcox [4], Sussman [5], and Wolf *et al.* [6]. All of these approaches utilize a set of orthonormal basis functions and attempt to minimize the difference between the synthesized and desired AFs. However, there may be scenarios where the designer cannot easily derive a desired AF and instead only knows a very general desired shape.

An extension valuable for improving the practicality of Wilcox's method is presented by Gladkova and Chebanov [7], who solve the minimization problem for the cross-ambiguity function in specific regions using Hermite waveforms. In practice, the generation of Hermite waveforms might not be the most straightforward process [8], [9].

Noteworthy AF synthesis research considering the waveform's peak-to-average power ratio (PAPR) includes work by Sebt *et al.* [8], a technique based on Sussman's work that optimizes the AF using a set of OFDM basis functions with lowered PAPR, greatly improving the usefulness of OFDM in radar applications.

Examples of research using alternating projections for waveform design include fascinating work by Blunt *et al.* [10] in radar embedded communications, Selesnick and Pillai [11] for notched chirp-like radar waveform, and Kassab *et al.* [12], which focuses on the autocorrelation properties of the waveforms rather than the AF.

This paper is an extension of our earlier work [13], which details the principles for a technique for AF synthesis using alternating projections. Rather than trying to find the best combination of basis functions, an "organic" solution is produced. In the present paper, we describe how PAPR and spectral requirements can be enforced on the synthesized waveform. The end result is a waveform providing desired range-Doppler resolution characteristics while meeting spectral requirements and maintaining PAPR commensurate with desired power efficiency of the system. The results will be useful in real-time radar waveform optimization, allowing radar transmitters to perform their critical functions in crowded spectral environments.

II. Synthesis Technique

Consider a monostatic radar system utilizing a possibly complex baseband waveform $x(t)$ with pulse duration T seconds and bandwidth B Hz.

A. Ambiguity minimization function

We introduce the minimization function, denoted by $M(\tau,u)$, which serves as the goal for AF optimization. A waveform $x(t)$ is desired with AF $\chi_x(\tau,u)$ having normalized magnitude less than $M(\tau,u)$ for all nearby range-Doppler combinations, or

$$\frac{|\chi_x(\tau,u)|}{|\chi(0,0)|}\leq M(\tau,u). \qquad \begin{matrix}-B<u<B\\ -T<\tau<T\end{matrix} \quad (2)$$

Consider a set of two-dimensional minimized functions $\mathbf{R}$ which satisfy (2) for a given $M(\tau,u)$ and a set of ambiguity functions $\mathbf{A}$ satisfy all the properties of AF.

Note that the optimum is reached for AF optimization when $\chi_x \in \mathbf{A} \cap \mathbf{R}$.

B. Review of projections for AF minimization

The projection of $\chi_x(\tau, u)$ onto $\mathbf{R}$ is given by

$$\Phi_x(\tau,u) = P_{\mathbf{R}}(\chi_x) = \begin{cases} \chi_x(\tau,u)\dfrac{M(\tau,u)}{|\chi_x(\tau,u)|}, & (\tau,u) \in \mathbf{B} \\ \chi_x(\tau,u), & (\tau,u) \notin \mathbf{B} \end{cases} \quad (3)$$

where $\mathbf{B}$ is the set of range-Doppler combinations where (2) is not satisfied.

Let the set of temporal waveforms with pulse duration T seconds and bandwidth B Hz be $\mathbf{X}$. The projection from $\mathbf{R}$ onto $\mathbf{X}$ is given by

$$y(t) = P_{\mathbf{X}}^{f}(\Phi_x) = \int_{-\infty}^{\infty} \Phi_x^*(-\tau,u)du \quad (4)$$

$$z(t) = P_{\mathbf{X}}^{t}(\Phi_x) = \mathcal{F}^{-1}\left\{\int_{-\infty}^{\infty} \Phi_x(\tau,u)d\tau\right\}^*. \quad (5)$$

A weighted combination of $y(t)$ and $z(t)$ is then used to form $s(t)$.

C. PAPR and spectral compliance projections

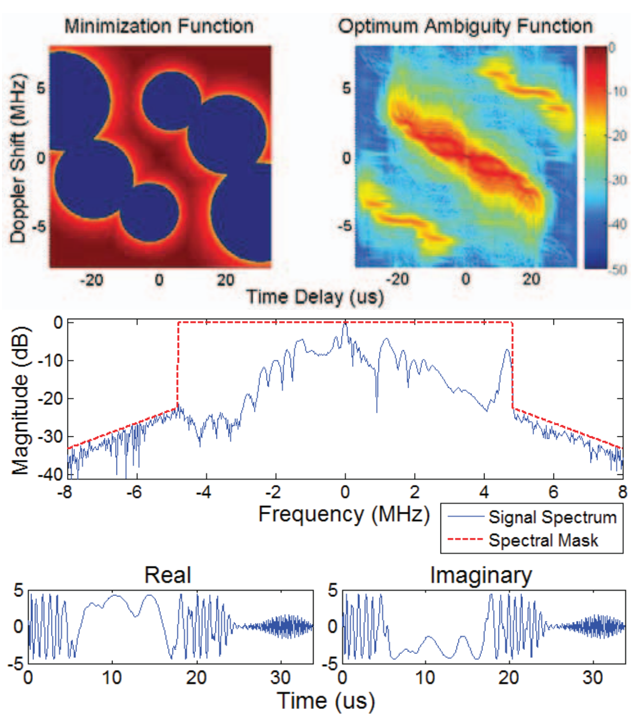

Fig. 1. The minimization function (top-left) and the normalized magnitude of the synthesized AF (top-right) in dB and the spectrum (middle) and time (bottom) domains of the synthesized waveform for trial I.

Once $s(t)$ has been found, the nearest waveform which also meets energy, PAPR, and spectral requirements must be found before projecting back to $\mathbf{A}$. Let $\mathbf{E}$ be the set of waveforms with greater than minimum desired energy E, $\mathbf{P}$ be the set of waveforms with less than maximum desired PAPR, and $\mathbf{S}$ be the set of waveforms whose spectrum fit inside the desired spectral mask. We wish to find $x(t)$ such that

$$x(t) \in \mathbf{X} \cap \mathbf{E} \cap \mathbf{P} \cap \mathbf{S}. \quad (6)$$

Starting with $s(t)$, we project onto $\mathbf{E}$ via

$$s_{\mathbf{E}}(t) = P_{\mathbf{E}}(s) = s(t)\frac{E}{\int_{-\infty}^{\infty}|s(t)|^2 dt}. \quad (7)$$

We then project from $\mathbf{E}$ onto $\mathbf{P}$ with

$$s_{\mathbf{P}}(t) = P_{\mathbf{P}}(s_{\mathbf{E}}) = \begin{cases} s_{\mathbf{E}}(t)\dfrac{|s|_{peak}}{|s_{\mathbf{E}}(t)|}, & |s_{\mathbf{E}}(t)| > |s|_{peak} \\ s_{\mathbf{E}}(t), & |s_{\mathbf{E}}(t)| \le |s|_{peak} \end{cases} \quad (8)$$

where $|s|_{peak} = \sqrt{PAPR/s_{RMS}}$.

$PAPR$ is the linear PAPR desired and s_{rms} is the root mean square of $s_{\mathbf{E}}(t)$. Finally, the projection from $\mathbf{P}$ to $\mathbf{S}$ is done via

$$\mathcal{F}\{s_{\mathbf{S}}(t)\} = P_{\mathbf{S}}(s_{\mathbf{P}}) = \begin{cases} S_{MASK}(f), & |\mathcal{F}\{s_{\mathbf{P}}(t)\}| > S_{MASK}(f) \\ \mathcal{F}\{s_{\mathbf{P}}(t)\}, & |\mathcal{F}\{s_{\mathbf{P}}(t)\}| \le S_{MASK}(f) \end{cases} \quad (9)$$

where $S_{MASK}(f)$ is the user-defined spectral mask. Projections from $\mathbf{E} \to \mathbf{P} \to \mathbf{S}$ are iterated until (6) is satisfied, at which point $s_{\mathbf{E}}(t) = s_{\mathbf{P}}(t) = s_{\mathbf{S}}(t) = x(t)$.

So, to recap, the projections process iterates from $\mathbf{A} \to \mathbf{R} \to \mathbf{X} \cap \mathbf{E} \cap \mathbf{P} \cap \mathbf{S}$ until an optimum is found.

III. Measurement Results

This technique is tested using a Keysight Technologies signal generator and oscilloscope. The quality of χ_x is given by the least-squares and minimax distance functions [13]. Results from two trials are shown in Figs. 1 and 2, with each trial using a different minization function.

The required PAPR and energy for both trials was 2 dB and 6775, respectively. A 1080 sample waveform is generated in a maximum of 100 iterations. Results are summarized in Table I. The baseband signal is created digitally and sent to the signal generator, where it is synthesized and modulated to 3.3 GHz. The resulting physical waveform is then sampled by the oscilloscope and sent back to the PC, where it is demodulated and quantified with the distance functions.

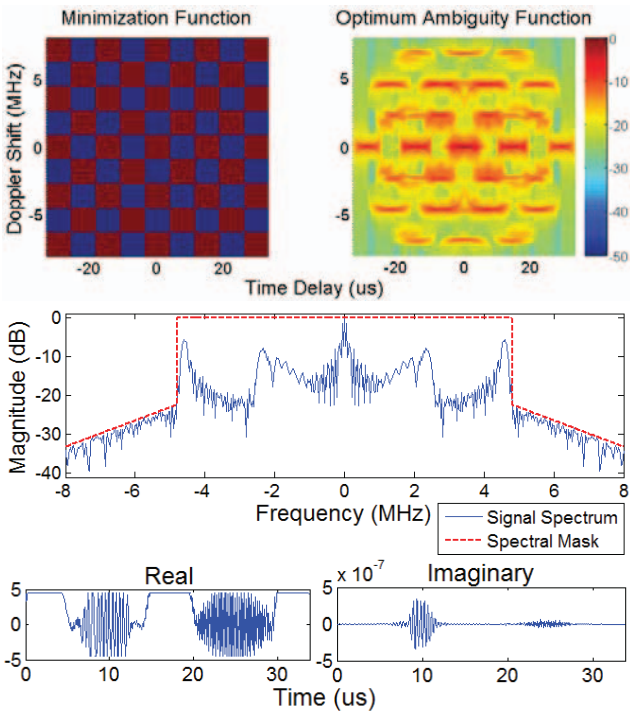

Fig. 2. The minimization function (top-left) and the normalized measured magnitude of the synthesized AF (top-right) in dB and the measured spectrum (middle) and time (bottom) domains of the synthesized waveform for trial II.

TABLE I
NUMERICAL MEASUREMENT RESULTS FROM TRIALS I AND II

	I	II
Optimum Iteration	58	63
PAPR (dB)	1.99953	1.99965
Least-Squares (Simulation)	64.554e-6	119.73e-6
Minimax (Simulation)	0.14794	0.20118
Least-Squares (Measured)	63.253e-6	117.62e-6
Minimax (Measured)	0.14804	0.20027

IV. CONCLUSIONS

A alternating projections method has been introduced which synthesizes a waveform with an optimized AF magnitude, while also meeting PAPR and spectral requirements. Future work is expected to incorporate the design of notched signals and adaptation to nonlinear amplifier distortion, as well as developing a simultaneous joint circuit optimization.

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